Decision aid methodologies in transportation

Lecture 6: Miscellaneous Topics

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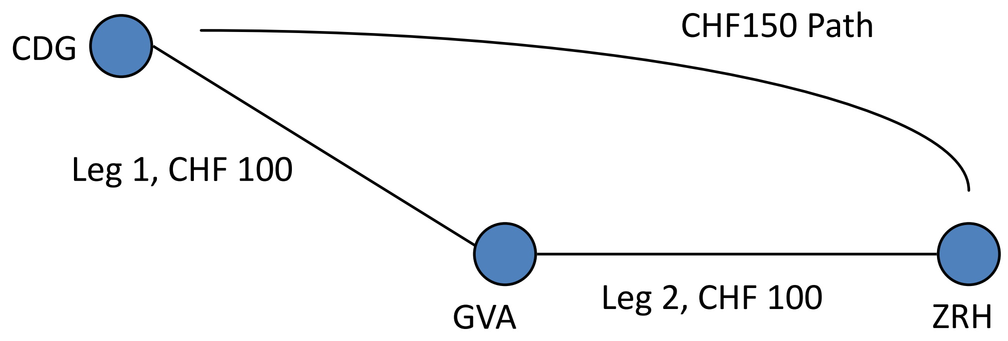
Summary

- We learnt about the different scheduling models
- We also learnt about demand-supply interactions in the form of revenue management concepts
- Today, we will see further application of revenue management to airline industry
- Some more examples of integer programming formulations
- Lastly, some new applications

Revenue Management: H&S Airline



- Given
  - A passenger intends to book a seat on CDG-GVA
- Question
  - Should you sell it or should you wait to sell the ticket for a passenger intending to book CDG-ZRH for a higher revenue?
- Complexity
  - Millions of itinerary

Airline Revenue Management

- Leg Optimization - Set explicit allocation levels for accepting bookings on each flight leg
- Network Optimization - Determine the optimal mix of path-class demand on the airline network

Airline RM: Network Optimization Model

- LP model to maximize revenue subject to capacity and demand constraints
- Network consists of all legs departing on a given departure date (a few thousands) and any path-class with a constituent leg departing on this date (up to a million)
- Model considers the following to determine demand:
  - cancellation forecast
  - no show forecast
  - upgrade potential
- The displacement cost of a leg/cabin is the “shadow price” of the corresponding capacity constraint of the LP

Airline RM: Network Optimization Formulation

- n Path-Classes: $f_1, f_2, \ldots, f_n$ fares

  $d_1, d_2, \ldots, d_n$ demand

  $x_1, x_2, \ldots, x_n$ decision variables

- m Legs: $c_1, c_2, \ldots, c_m$ capacities

- Incidence Matrix $A=[a_{ji}]_{mxn}$

  $a_{ji} = 1$ if leg j belongs to path i, 0 otherwise

- LP Model:

  Maximize $\Sigma f_i x_i$

  Subject To $\Sigma a_{ji} x_i \leq c_j$ $j = 1,2, \ldots, m$ capacity constraints

  $0 \leq x_i \leq d_i$ $i = 1,2, \ldots, n$ demand constraints

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Integer Programming: More Formulations

- Consider the following mathematical formulation:

$$\boxed{\begin{array}{l} \min c^T x \\ Ax \le b \\ x \ge 0 \end{array}}$$

- View this formulation as the one where x indicate different options and c^T the corresponding costs. However if an option is selected, a fixed cost is incurred by default
- PROBLEM: $x = 0$ or $x \ge k$
- How to formulate this?

Integer Programming: More Formulations

- Use a binary auxiliary variable y = $\begin{cases} 0\text{, for } x = 0 \\ 1\text{, for } x \geq k \end{cases}$

- Add the following constraints:

$$\boxed{\begin{array}{l} x \leq M \cdot y \text{ (M is an upperbound on x)} \\ \mathrm{x} \geq \mathrm{k} \cdot \mathrm{y} \\ \mathrm{y} \in \{0,1\} \end{array}}$$

Integer Programming: More Formulations

- This can be applied even when x is not necessarily an integer

$$
\begin{aligned}
&\text{minimize } C(x)\\
&Ax = b\\
&x \geq 0
\end{aligned}
\qquad
\text{where :}\quad C(x_i) = \begin{cases} 0 & \text{for } x_i = 0, \\ k_i + c_i x_i & \text{for } x_i > 0. \end{cases}
$$

- Use auxiliary variable $y_i = \begin{cases} 0, \text{ for } x_i = 0 \\ 1, \text{ for } x_i > 0 \end{cases}$

- Add these constraints

$$
\begin{aligned}
&x_i \leq M \cdot y_i\\
&C(x_i) = k_i \cdot y_i + c_i \cdot x_i\\
&y_i \in \{0,1\}
\end{aligned}
$$

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Integer Programming: More Formulations

- Consider the following constraint: $\boxed{x_1 + x_2 \leq 5}$
- If the constraint has to be absolutely satisfied, it is called a **hard** constraint
- However in some situations, you may be able to violate a constraint by incurring a penalty
- Such constraints are called **soft** constraints and they can be modeled as:

$$\boxed{\begin{array}{l} \text{minimize } c^T x + Y \cdot 100 \\ x_1 + x_2 \leq 5 + Y \\ \ldots \\ x \geq 0, Y \geq 0 \end{array}}$$

Integer Programming: More Formulations

- How to consider variables with absolute values? Consider this:

$$
\begin{aligned}
&\min \sum_j |y_t| \\
&\sum_j a_j x_{j,t} = b_t + y_t \\
&x_{j,t} \geq 0, y_t \text{ free}
\end{aligned}
$$

- How to solve this type of formulation?

$$
\begin{aligned}
&y_t = y_t^+ - y_t^- \\
&\Rightarrow |y_t| = y_t^+ + y_t^-
\end{aligned}
$$

$$
\begin{aligned}
&\min \sum_t (y_t^+ + y_t^-) \\
&\sum_j a_j x_{j,t} = b_t + y_t^+ - y_t^- \\
&x_{j,t} \geq 0, y_t^+ \geq 0, y_t^- \geq 0
\end{aligned}
$$

Integer Programming: More Formulations

- How to treat disjunctive programming?
- A mathematical formulation where we satisfy only one (or few) of two (or more) constraints

$$
\begin{aligned}
&\min \sum_j w_j x_j \\
&x_k - x_j \geq p_k \\
&\text{or} \\
&x_j - x_k \geq p_j \\
&\ldots
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
&\min \sum_j w_j x_j \\
&x_k - x_j \geq p_k - M_1 \cdot y \\
&x_j - x_k \geq p_j - M_2 \cdot (1 - y) \\
&\ldots \\
&y \in \{0,1\}
\end{aligned}
$$

Integer Programming: More Formulations

- We started describing MIP with Transportation Problem
- But the problem can be solved with SIMPLEX method. Yes!
- Consider a mathematical formulation $\boxed{\begin{array}{l}\min c^T x \\ Ax \leq b \\ x \geq 0\end{array}}$
- Suppose all coefficients are integers and constraint matrix A has the property of TUM (Total UniModularity)
- TUM implies that every square sub-matrix has determinant value as 0, -1 or 1
- There exists an optimal integer solution x* which can be found using the simplex method

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Optimization at Airports

Airport Gate Assignment: Objectives

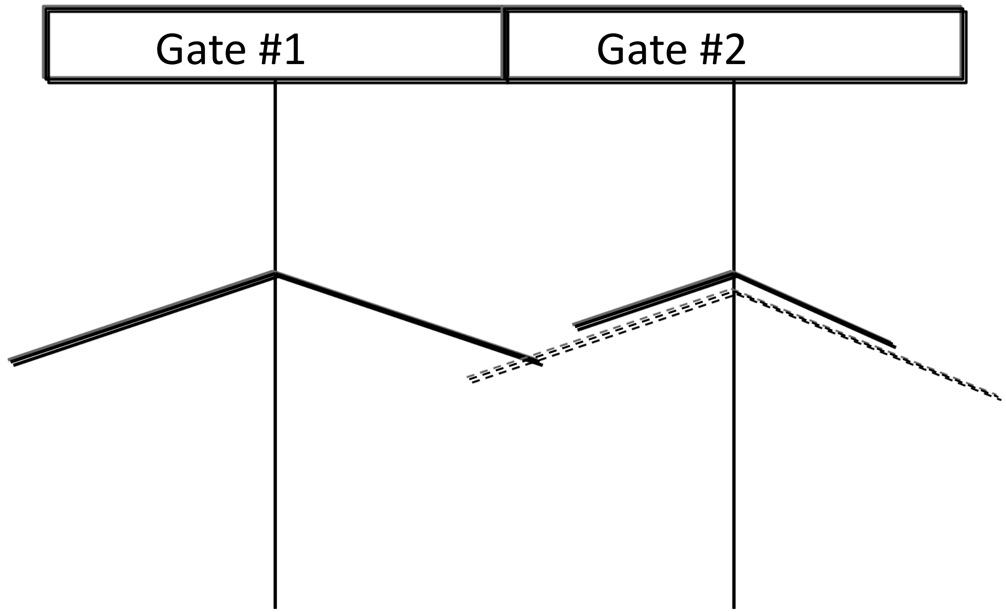
- Given a set of flight arrivals and departures at a major hub airport, what is the **best** assignment of these incoming flights to airport gates so that all flights are gated?
- Gating constraints such as adjacent gate, LIFO gates, gate rest time, towing, push back time and PS gates are applicable

Airport Gate Assignment: Problem Instance

- One of the largest in the world
- Over 1200 flights daily
- Over 25 different fleet types handled
- 60 gates and several landing bays
- Around 50,000 connecting passengers

Terminology

- Adjacent Gates: Two physically adjacent gates such that when one gate has a wide bodied aircraft parked on it, the other gate cannot accommodate another wide body



Terminology

- Market: An origin-destination pair
- Turns: A pair of incoming and outgoing flights with the same aircraft or equipment
- Gate Rest: Idle time between a flight departure and next flight arrival to the gate. Longer gate rest helps pad any minor schedule delays, though at the cost of schedule feasibility
- PS Gates: Premium Service gates are a set of gates that get assigned to premium markets – typically where VIPs travel

Mathematical Model

- Parameters
  - a_i: scheduled arrival time of turn
  - b_i: scheduled departure time of turn
  - (*k,l*): two gates restricted in the adjacent pair
  - E_k^1, E_l^1 : sets of equipment types such that when an aircraft of a type in E_k^1 is occupying *k*, no aircraft of any type in E_l^1 may use *l*; and vice versa.
- Decision variables
  - $x_{ik} \in \{0,1\}$: 1 if turn i is assigned to gate k; 0 otherwise
  - $y_i \in \{0,1\}$: 1 if turn i is not assigned to any gate; 0 otherwise

Mathematical Model

Maximize $$\sum_{i\in T}\sum_{k\in K} C_{ik}x_{ik} - C\sum_{i\in T} y_i$$

subject to:

$$\sum_{k\in K} x_{ik} + y_i = 1 \qquad i \in T$$

$$x_{ik} + x_{jk} \le 1 \qquad i, j \in T; k \in K : a_i < b_j + \alpha, a_j < b_i + \alpha, i \neq j$$

$$x_{ik} + x_{jl} \le 1 \qquad i, j \in TURNS; k, l \in GATES; (k,l) \in ADJACENT : a_i < a_j \wedge a_i < b_j \wedge a_j < b_i, i \neq j; e_i \in E_k^1; e_j \in E_l^1$$

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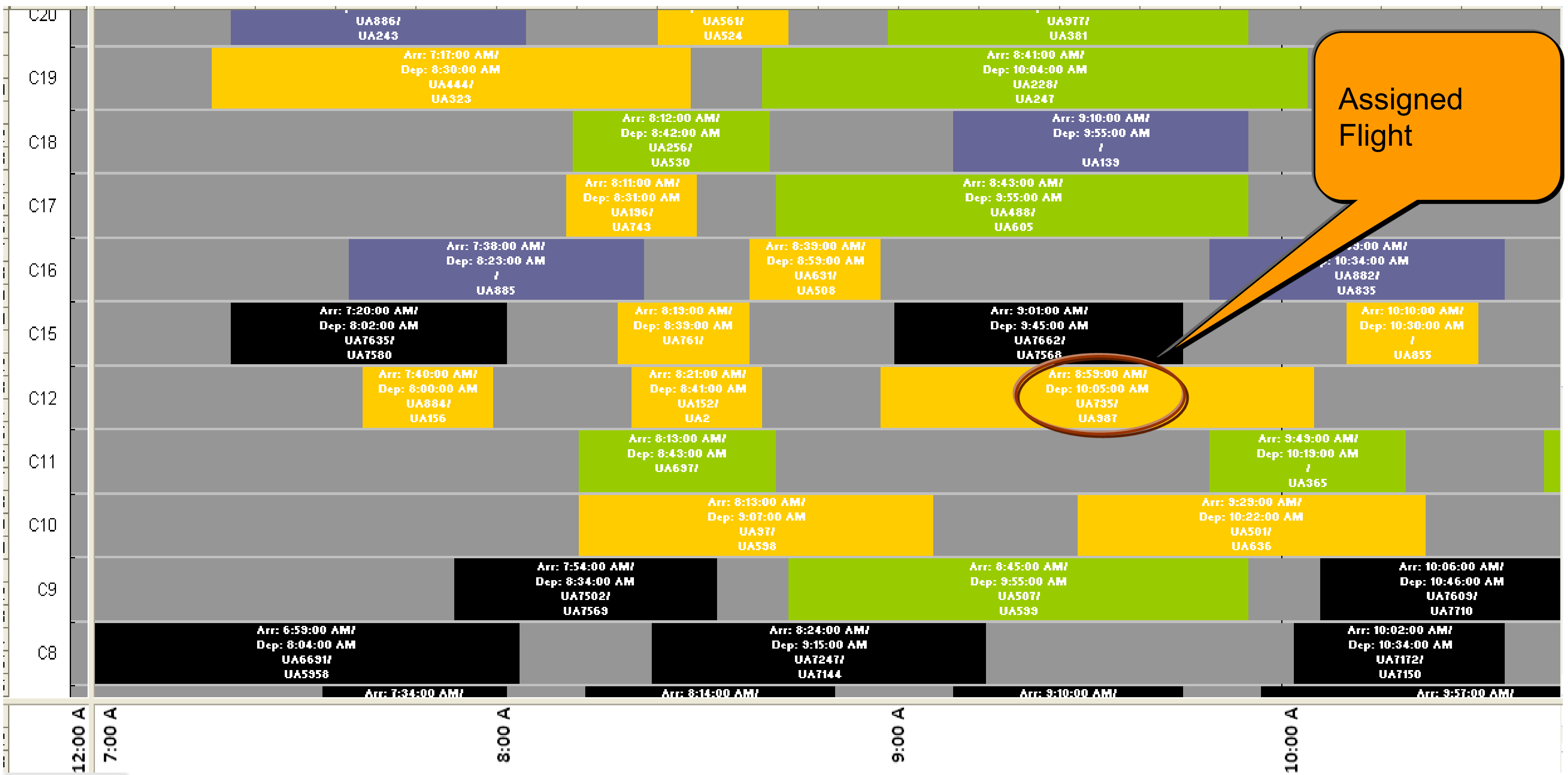
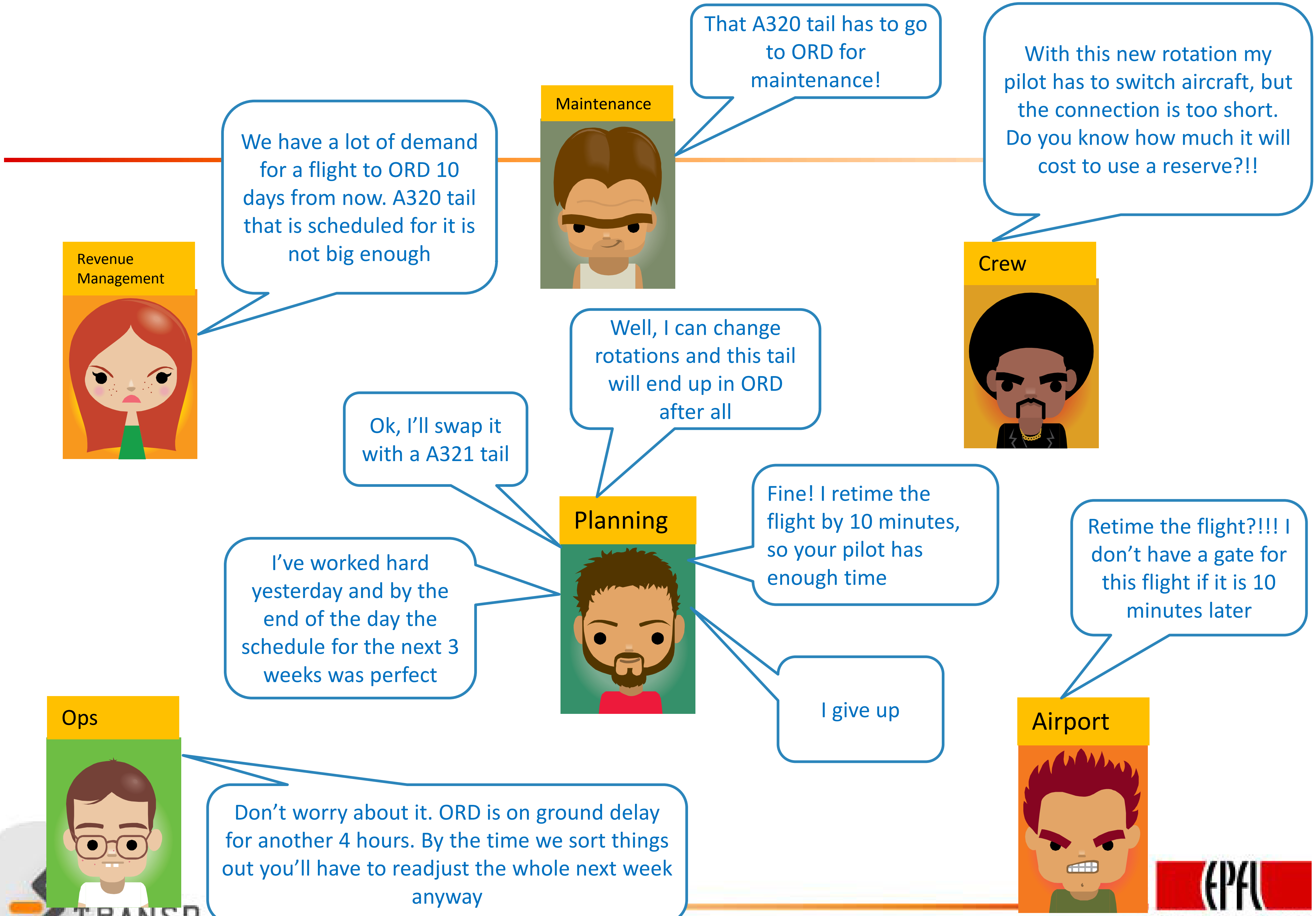
Output: Gantt Charts



Additional Objectives

- Maximize Connection Revenues
  - This gating objective identifies connections at risk for a hub station and gates the turns involved such that connection revenue is maximized
- Maximize Schedule Robustness
  - Flights must be gated based on the past pattern of flight delays to provide adequate gate rest between a departing flight and the next arriving flight
- Maximize Manpower Productivity
  - While gating the flights, employees could potentially waste a lot of time travelling between gates



Courtesy: Sergey Shebalov, Sabre Technologies

Optimization in Railways

Applications in Railways

- Locomotive Assignment
- Locomotive Refueling
- Revenue Management
- Locomotive Maintenance
- Platform Assignment
- Train-design
- Block-to-Train Assignment

Locomotive Assignment

- Basic Inputs
  - Train Schedule over a period of planning horizon
  - A set of locomotives, their current locations and properties
- Output
  - Assignment schedule of locomotives to trains
- Constraints
  - Locomotive maintenance
  - Tonnage and HP requirement of train
  - Several other constraints
- Objective
  - Cost minimization

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Locomotive Assignment: Some Features

- A train is typically assigned a group of multiple locomotives called a **consist** that usually travels together
- Each train has a different **HP** and **Tonnage requirement** that depends on the number of cars attached
- Locomotives can either pull trains actively or **deadhead** on them.
- Locomotives can also **light travel**.
- Trains need not have the same daily schedule.

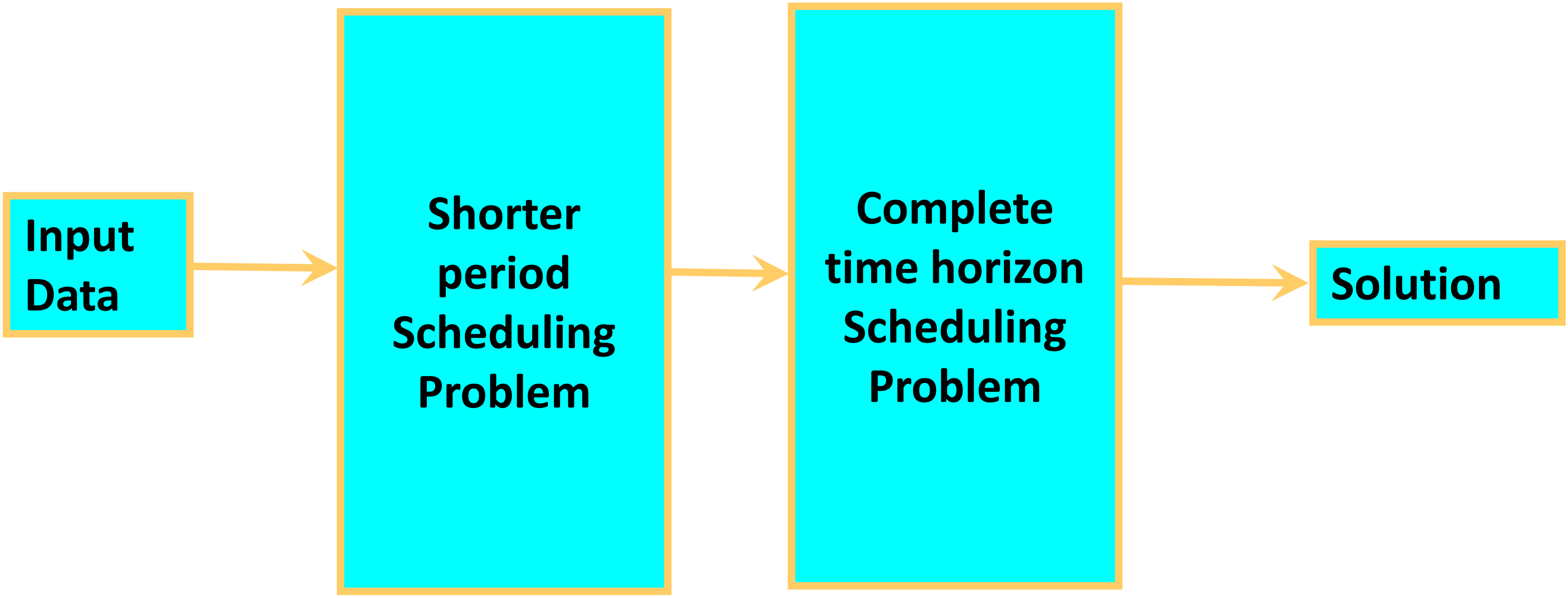
Locomotive Assignment: Mathematical Model

- Decision Variables
  - Locomotive-Train assignment schedule
  - Active locomotives
  - Deadhead locomotives
  - Light travel locomotives
- Parameters
  - Locomotive availability, maintenance schedule and features
  - Train schedule / time-table and train features
  - Infrastructure features for sections and yards

Locomotive Assignment: Hard Constraints

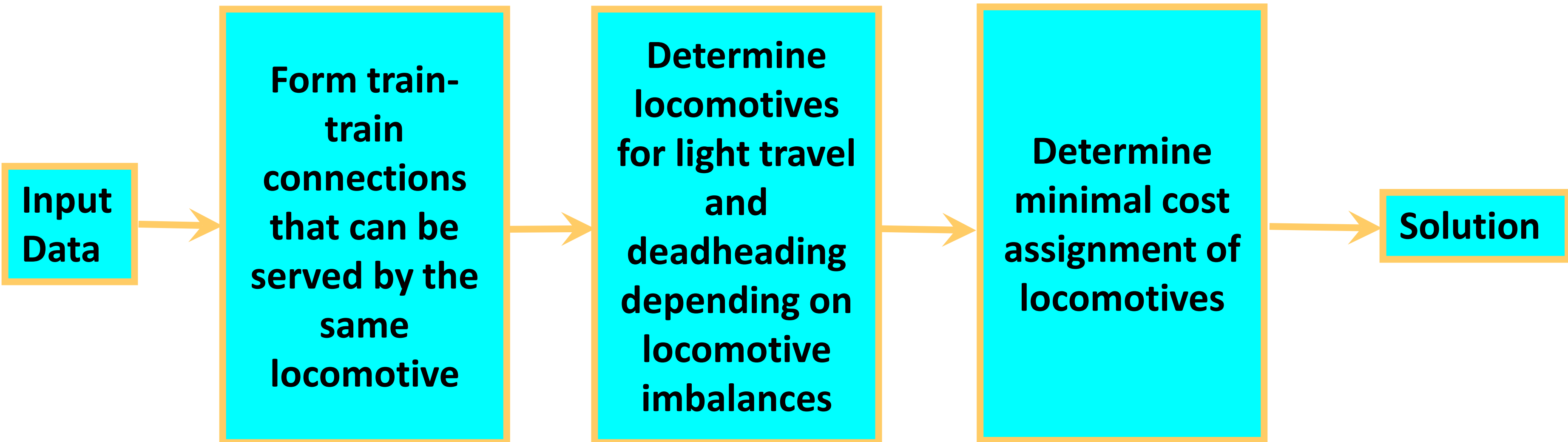
- Horsepower requirements
- Tonnage requirements
- Fleet size limitations
- Consistency of the assignments
- Locomotive availability at yards and sections
- Repeatability of the solution
- Solution robustness and recoverability

Locomotive Assignment: Solution Methodology



Two-stage optimization allows us to reduce the problem size substantially while giving an opportunity to maintain consistency

Locomotive Assignment: Solution Methodology

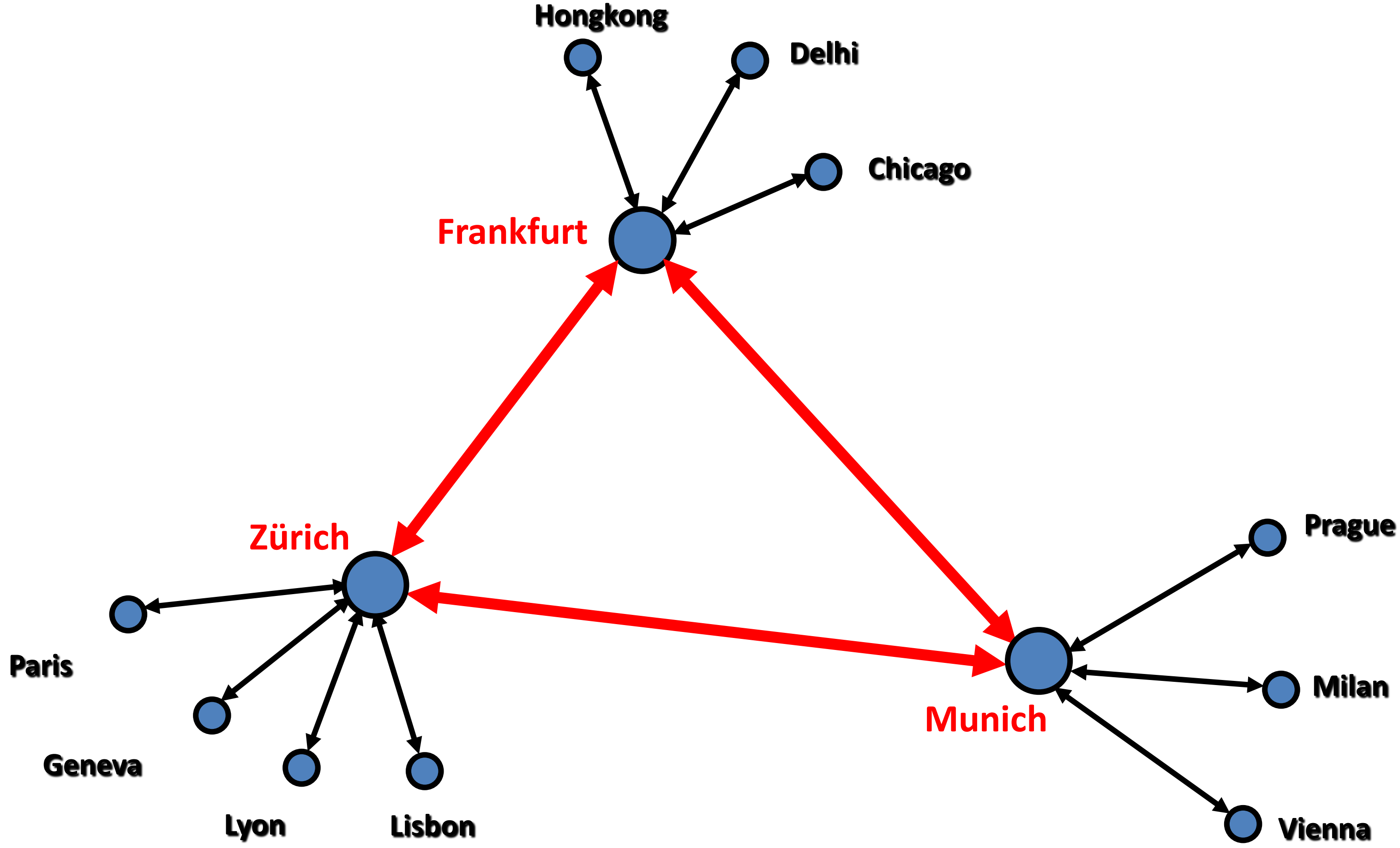


- **Determine the three sets of decision variables using a sequential process.**

Railroad Blocking Problem

- Problem:
  - Origin-Destination of shipments given
  - Each shipment contains different number of cars
  - Train routes and time table known
  - Capacity of the network and trains known
- Magnitude:
  - Thousands of trains per month
  - 50,000 – 100,000 shipments with an average of 10 cars (Ahuja et al)
- Design the network on which commodities flow

Comparison with Airline Schedule Design



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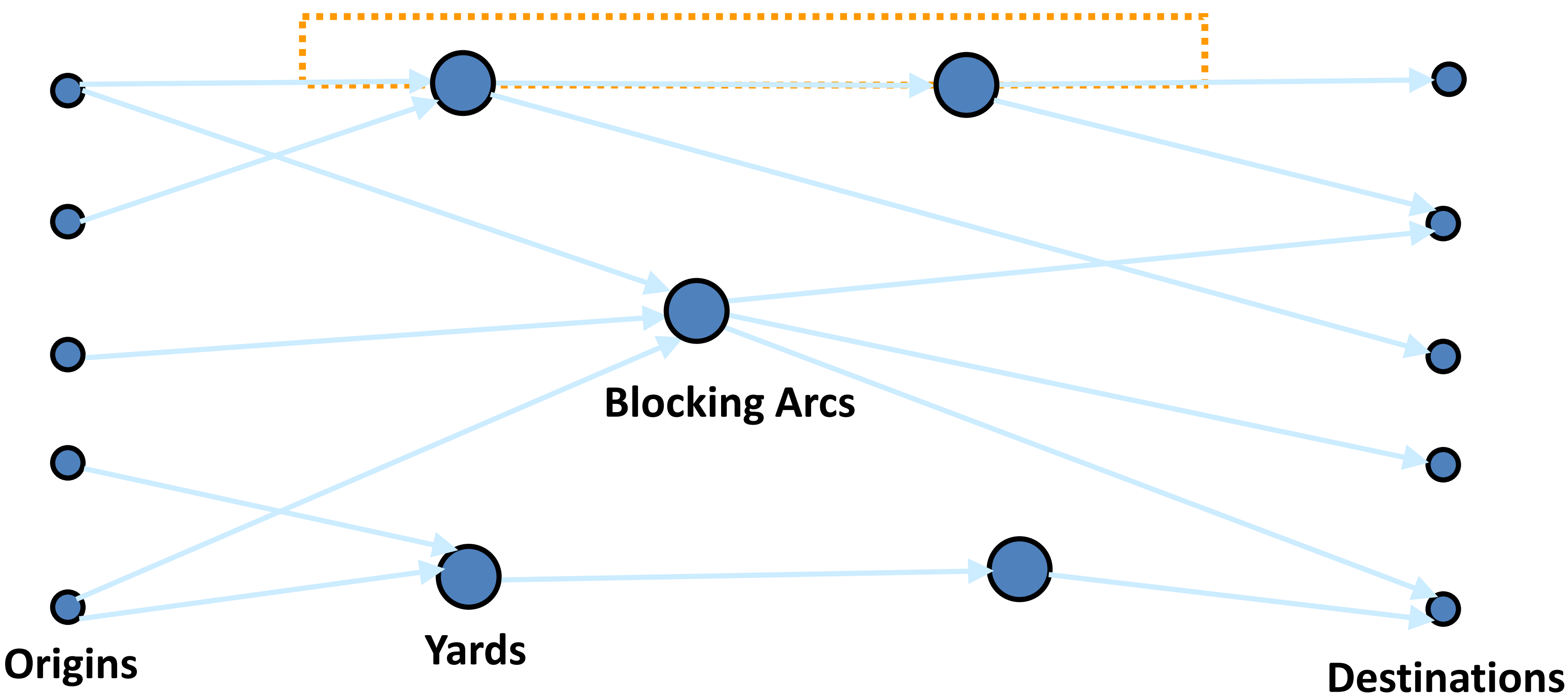
Railroad Blocking Problem



Railroad Blocking Problem: Model

- Decision Variables:
  - Blocking arcs to a yard with origin (or destination) selected, or not
  - Route followed by the shipments along the blocking arcs
- Constraints:
  - Number of blocking arcs at each node
  - Volume of cars passing through each node
  - Capacity of the network and train schedule
- Objective Function:
  - Minimize the number of intermediate handling and the sum of distance travelled (different objectives can be weighted)

Railroad Blocking Problem: Problem Scale

- Network size:
  - 1,000 origins
  - 2,000 destinations
  - 300 yards
- Number of network design variables:
  - 1,000x300 + 300x300 + 300x2,000 ≈ 1 million
- Number of flow variables:
  - 50,000 commodities flowing over 1 million potential arcs

Railroad Blocking Problem: Complexity

- Network design problems are complex for many reasons. Apart from the large number of variables, there can be several competing solutions with the same value of the objective function
- Problems with only a few hundred network design variables can be solved to optimality
- Railroads want a near-optimal and implementable solution within a few hours of computational time.

Reference: Ahuja et al: Railroad Blocking Problems

Railroad Blocking Problem: Solution Approach

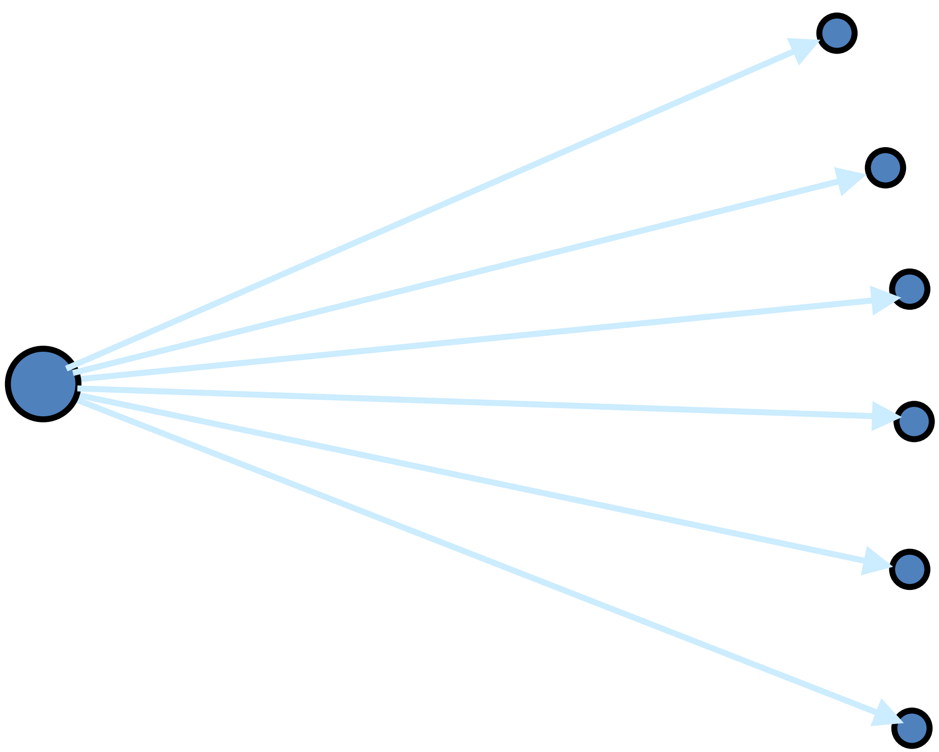
- Integer Programming Based Methods
  - Slow and impractical for large scale instances

- Network Optimization Methods
  - Start with a feasible solutions
  - Gradually improve the solution – one node at a time

Railroad Blocking Problem: Solution Approach

- Start with a feasible solution of the blocking problem
- Optimize the blocking solution at only one node (leaving the solution at other nodes unchanged) and reroute shipments
- Repeat as long as there are improvements.

Railroad Blocking Problem: Solution Approach



Out of about 3,000 arcs emanating from a node, select 50 arcs and redirect up to 50,000 shipments to minimize the cost of flow.

Problem instance could be solved for one node using CPLEX in one hour.

Reference: Ahuja et al: Railroad Blocking Problems

Railroad Blocking Problem: Future

- This is one of the ongoing research open problems that is currently being tackled by the railroad industry
- Of course there are many such interesting problems in railways and we could give example of only two in this lecture