Decision aid methodologies in transportation

Lecture 6: Revenue Management and Integer Program: Tips and Tricks

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* Presentation materials in this course uses some slides of Dr Nilotpal Chakravarti and Prof Diptesh Ghosh

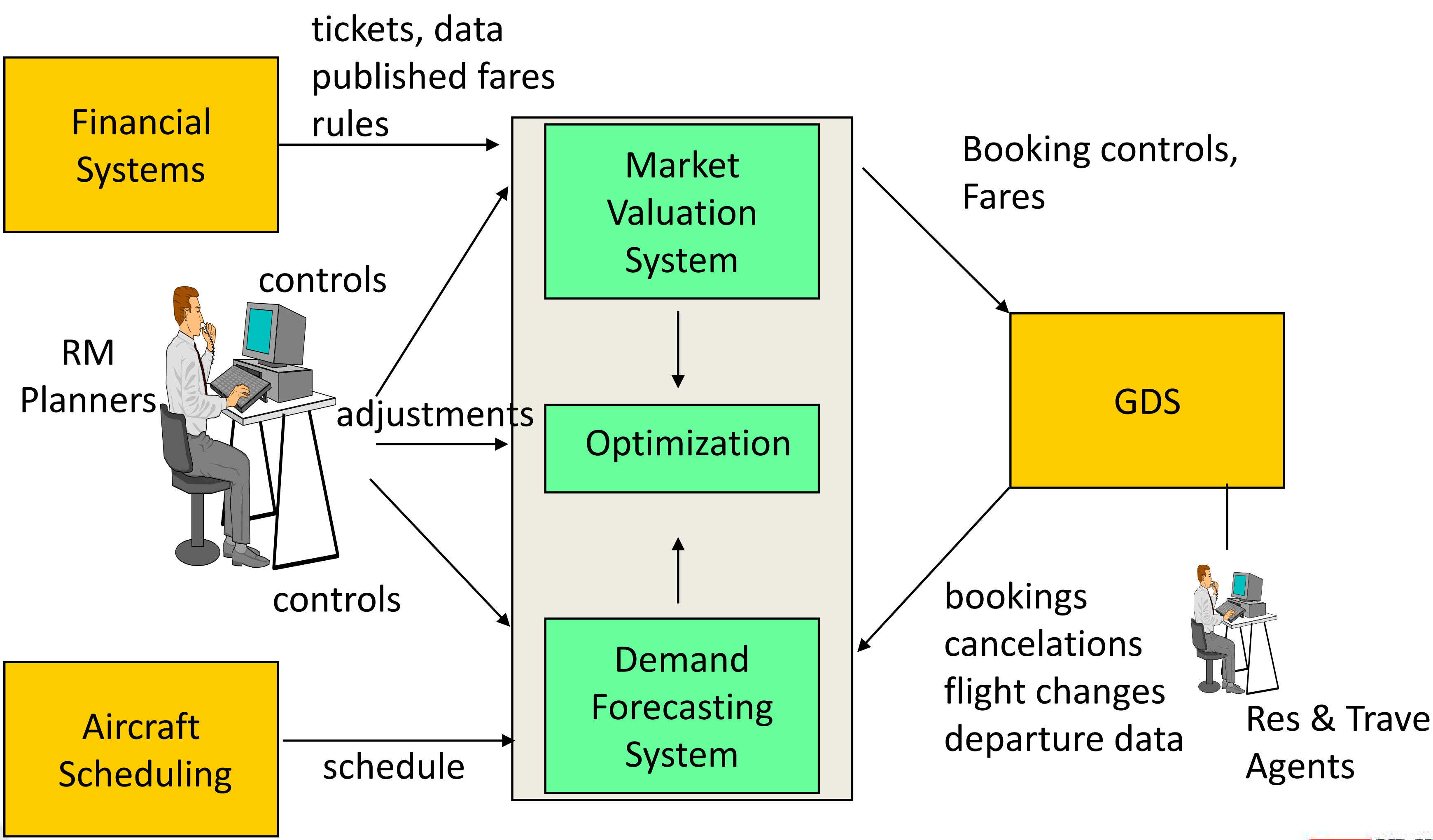
Summary

- We learnt about the different scheduling models
- We also learnt about demand-supply interactions in the form of revenue management concepts
- We learnt to mimic expectations and use solver with Spreadsheets
- Today, we will see further application of revenue management to airline industry
- We will see how to bring the concepts to practice?
- Lastly, we will see some more examples of integer programming formulations
- In the lab today, we will learn ways to implement models using MATHPROG

Revenue Management: H&S Airline

- An airline flies a stopover (through) flight from CDG to ZRH via GVA
- Thus a passenger can book on three potential markets: CDG-GVA, GVA-ZRH or CDG-ZRH
- Let us say the average fare for CDG-GVA is CHF 100, GVA-ZRH is CHF 100 and CDG-ZRH is CHF 150 per seat
- Let us say a passenger comes to you to book a seat on CDG-GVA. Should you sell it or should you wait to sell the ticket for a passenger intending to book CDG-ZRH for a higher revenue?
- Imagine the decision making process for an airline that flies a few thousand flights and builds close to a million itinerary

A Real Revenue Management System



Airline Revenue Management

- Leg Optimization - Set explicit allocation levels for accepting bookings on each flight leg
- Network Optimization - Determine the optimal mix of path-class demand on the airline network

Airline RM: Network Optimization Model

- LP model to maximize revenue subject to capacity and demand constraints
- Network consists of all legs departing on a given departure date (a few thousands) and any path-class with a constituent leg departing on this date (up to a million)
- Model considers:
  - cancellation forecast
  - no show forecast
  - upgrade potential
- The displacement cost of a leg/cabin is the “shadow price” of the corresponding capacity constraint of the LP

Airline RM: Network Optimization Formulation

- n Path-Classes: $f_1, f_2, \ldots, f_n$ fares

  $d_1, d_2, \ldots, d_n$ demand

  $x_1, x_2, \ldots, x_n$ decision variables
- m Legs: $c_1, c_2, \ldots, c_m$ capacities
- Incidence Matrix $A=[a_{ji}]_{mxn}$

  $a_{ji} = 1$ if leg j belongs to path i, 0 otherwise
- LP Model:

  Maximize $\Sigma f_i x_i$

  Subject To $\Sigma a_{ji} x_i \leq c_j$ $j = 1,2, \ldots, m$ capacity constraints

  $0 \leq x_i \leq d_i$ $i = 1,2, \ldots, n$ demand constraints

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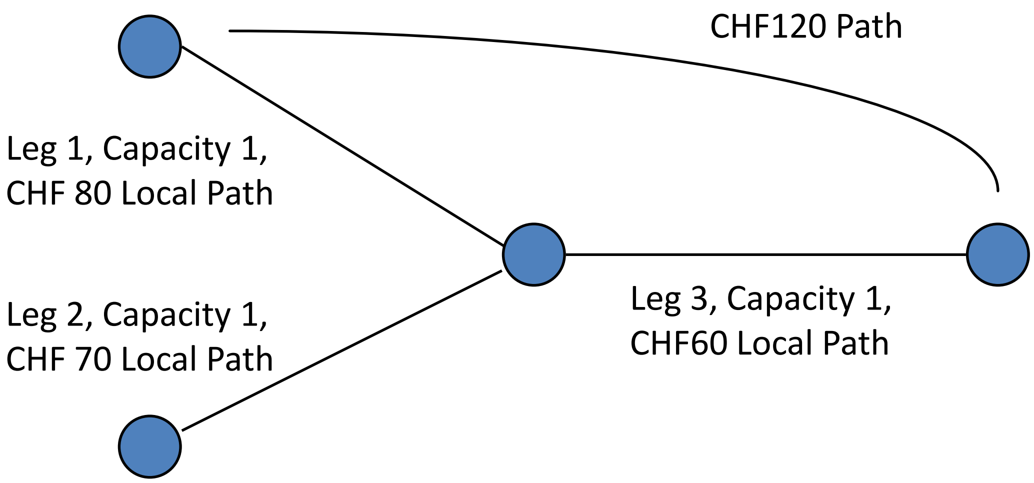
Airline RM: Leg Optimization

- Note that the solution to LP for path level protection levels would also give the shadow prices for legs (from the dual). These shadow prices are referred to as "displacement costs"



Leg 1 Displacement Cost = CHF80, Leg 2 Displacement Cost = CHF70, Leg 3 Displacement Cost = CHF60

Airline RM: Why Segmentation Helps?

- Segmentation helps because:
  - Discounted pricing stimulates demand and expands the market
  - People who are willing to pay more are given the product at the right price
  - Extraction of consumer surplus

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Airline RM: Differential Pricing

- A key component of Revenue Management
  - Customer Segmentation and Differential pricing
- How can we justify charging different prices to customers for the same product?
- Differentiate your product and offer the “right product to the right customer”
- How can we build “fences” to segment customers?

Airline RM: Physicial Fences

- Hotels charge according to room types (e.g., ocean view, pool view etc.)
- Airlines charge according flight characteristics (e.g., direct flights vs. stopovers)
- Differential pricing by time-of-day or day-of-week is practiced in many industries (airlines, hotels, cinemas, theme parks, resorts)
- Broadcasters charge more for advertising slots during popular shows
- Price low demand flights low to stimulate demand
- Price peak demand flights high to improve revenue

Airline RM: Logical Fences

- Length of stay (hotels, airlines)
- Flexibility (e.g., discounted pax may not be allowed cancellations or changes or need to pay a high charge)
- Conditions (e.g., discounted pax are often not allowed frequent flyer privileges)
- Time of purchase (advance purchase, promotions)
- Bulk contracts (broadcasting, cargo), Group bookings (airlines, hotels)
- Point of sale

Airline RM: A Real Airline Fare-Sheet

Roundtrip Fare ($)	Class	Advance Purchase	Minimum Stay	Change Fee?	Restrictions
289	W	14 days	Sat. Night	Yes	Non-stops only
294/354	V	14 Days	Sat. Night	Yes	Sales ends 12/04
592/646	H	21 Days	Sat. Night	Yes	Mon-Thurs / Fri-Sun
790/862	M	14 Days	Sat. Night	Yes	Mon-Thurs / Fri-Sun
1265	N	7 Days	Sat. Night	Yes	None
1998	B	None	None	No	2 X OW Fare
2058	Y	None	None	No	2 X OW Fare
3026	C	None	None	No	None
3472	F	None	None	No	None

Kennedy Center of Performing Arts

RM Implementation: Washington Opera

- Washington Opera – a top US professional opera company – was loosing money in 1993-94
- There were 3 pricing levels, \$47, \$63, \$85
- Prices were based on seat location
- They thought of raising prices across the board by 5% but feared that this would lead to sharp slump in sales
- Instead they decided to introduce several levels of prices between \$29 and \$150
- Prices were based on seat location and time
- Result – revenue increase of over 9%
- Washington Opera returned to profitability

Disney World!!!

RM Implementation: Disney World

- Disney had a problem with its hotel occupancy
  - Very high occupancy rates during Christmas week, (all rooms usually filled by early September)
  - Very low occupancy rates during the first week of January
- Disney began launching special events (e.g., Disney marathon) in the first week of January
- That helped a bit but not enough
- Disney at that point of time did not have a RM system

RM Implementation: Data Analysis Helps

- Disney analyzed its data
  - realized that there was considerable unmet demand for the last week of December
- Tried to channel some of the demand to the first week of January using length-of-stay controls
  - They denied reservations to those staying < 3 nights
- Rooms didn’t fill up in September, not even in October, early November …
- However ultimately room occupancy in the first week of Jan increased 10% and room revenue by $ 1.5 million

Integer Programming: More Formulations

- Consider the following Integer Program:

$$\boxed{\begin{array}{l} \min c^T x \\ Ax \leq b \\ x \geq 0 \end{array}}$$

- View this formulation as the one where x indicate different options and c^T the corresponding costs. However if an option is selected, a fixed cost is incurred by default
- PROBLEM: $x = 0$ or $x \geq k$
- How to formulate this?

Integer Programming: More Formulations

- Use a binary auxiliary variable y = $\begin{cases} 0\,, \text{for } x = 0 \\ 1\,, \text{for } x \geq k \end{cases}$

- Add the following constraints:

$$\boxed{\begin{array}{l} x \leq M \cdot y \text{ (M is an upperbound on x)} \\ x \geq k \cdot y \\ y \in \{0,1\} \end{array}}$$

Integer Programming: More Formulations

- This can be applied even when x is not necessarily an integer

$$\begin{aligned} &\text{minimize } C(x) \\ &Ax = b \\ &x \geq 0 \end{aligned}$$

$$\text{where:} \quad C(x_i) = \begin{cases} 0 & \text{for } x_i = 0, \\ k_i + c_i x_i & \text{for } x_i > 0. \end{cases}$$

- Use auxiliary variable $y_i = \begin{cases} 0, \text{ for } x_i = 0 \\ 1, \text{ for } x_i > 0 \end{cases}$

- Add these constraints

$$\begin{aligned} &x_i \leq M \cdot y_i \\ &C(x_i) = k_i \cdot y_i + c_i \cdot x_i \\ &y_i \in \{0,1\} \end{aligned}$$

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Integer Programming: More Formulations

- Consider the following constraint: $\boxed{x_1 + x_2 \leq 5}$
- If the constraint has to be absolutely satisfied, it is called a **hard** constraint
- However in some situations, you may be able to violate a constraint by incurring a penalty
- Such constraints are called **soft** constraints and they can be modeled as:

$$\boxed{\begin{array}{l} \text{minimize } c^T x + Y \cdot 100 \\ x_1 + x_2 \leq 5 + Y \\ \ldots \\ x \geq 0, Y \geq 0 \end{array}}$$

Integer Programming: More Formulations

- How to consider variables with absolute values? Consider this:

$$\boxed{\begin{aligned}&\min \sum_j |y_t| \\ &\sum_j a_j x_{j,t} = b_t + y_t \\ &x_{j,t} \geq 0, y_t \text{ free}\end{aligned}}$$

- How to solve this type of formulation?

$$\boxed{\begin{aligned}&y_t = y_t^+ - y_t^- \\ &\Rightarrow |y_t| = y_t^+ + y_t^-\end{aligned}}$$

$$\boxed{\begin{aligned}&\min \sum_t (y_t^+ + y_t^-) \\ &\sum_j a_j x_{j,t} = b_t + y_t^+ - y_t^- \\ &x_{j,t} \geq 0, y_t^+ \geq 0, y_t^- \geq 0\end{aligned}}$$

Integer Programming: More Formulations

- How to treat disjunctive programming?
- A mathematical formulation where we satisfy only one (or few) of two (or more) constraints

$$
\begin{aligned}
&\min \sum_j w_j x_j \\
&x_k - x_j \geq p_k \\
&\text{or} \\
&x_j - x_k \geq p_j \\
&\ldots
\end{aligned}
\quad \Longrightarrow \quad
\begin{aligned}
&\min \sum_j w_j x_j \\
&x_k - x_j \geq p_k - M_1 \cdot y \\
&x_j - x_k \geq p_j - M_2 \cdot (1 - y) \\
&\ldots \\
&y \in \{0,1\}
\end{aligned}
$$

Integer Programming: More Formulations

- We started describing MIP with Transportation Problem
- But the problem can be solved with SIMPLEX method. Yes!
- Consider a mathematical formulation $\boxed{\begin{array}{l} \min c^T x \\ Ax \leq b \\ x \geq 0 \end{array}}$
- Suppose all coefficients are integers and constraint matrix A has the property of TUM (Total UniModularity)
- TUM implies that every square sub-matrix has determinant value as 0, -1 or 1
- There exists an optimal integer solution x* which can be found using the simplex method