# Decision aid methodologies in transportation

Lecture 5: Revenue Management

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* Presentation materials in this course uses some slides of Dr Nilotpal Chakravarti and Prof Diptesh Ghosh

# Summary

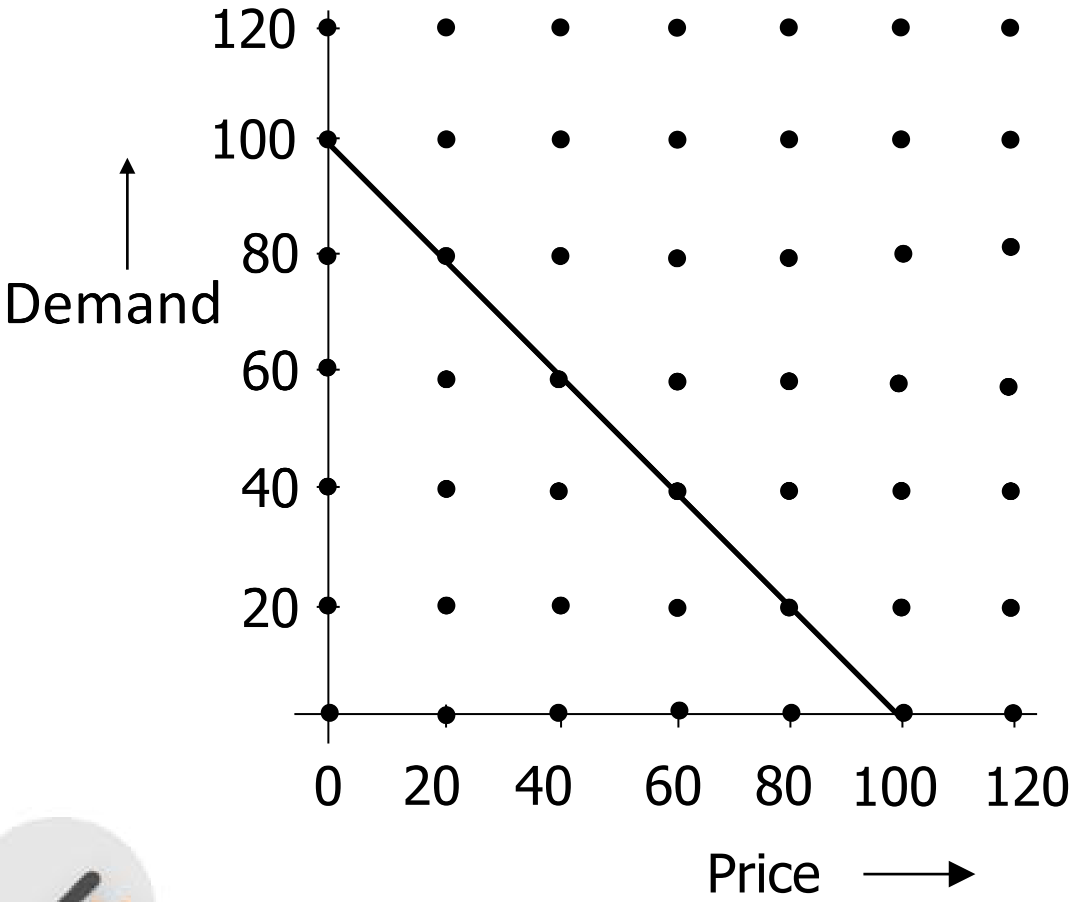
- We learnt about the different scheduling models
- We learnt to formulate these sub-problems into mathematical models
- We learnt to solve problems with different techniques such as heuristics, branch and bound, tree search and column generation
- The models that we learnt so far assumed a fixed system capacity and a known demand pattern
- Eventually capacity is assigned to the demand in such a way that the revenue (or profits) are optimized
- So the moral of the story so far – demand is a "holy cow" while it is only the supply that can be "flogged around"!

# What is Revenue Management?

- Let us dissect our “holy cow” with a new dimension
- Revenue Management in most literature is defined as the art or science of selling the **right** supply (seats, tickets, etc.) to the **right** demand (customers) **at the right time**
- So far, we only talked about supply assignment to demand, but now what is this “right” qualifier?
- What is the right timing?

# Revenue Management: Example

- Consider the following simple example:

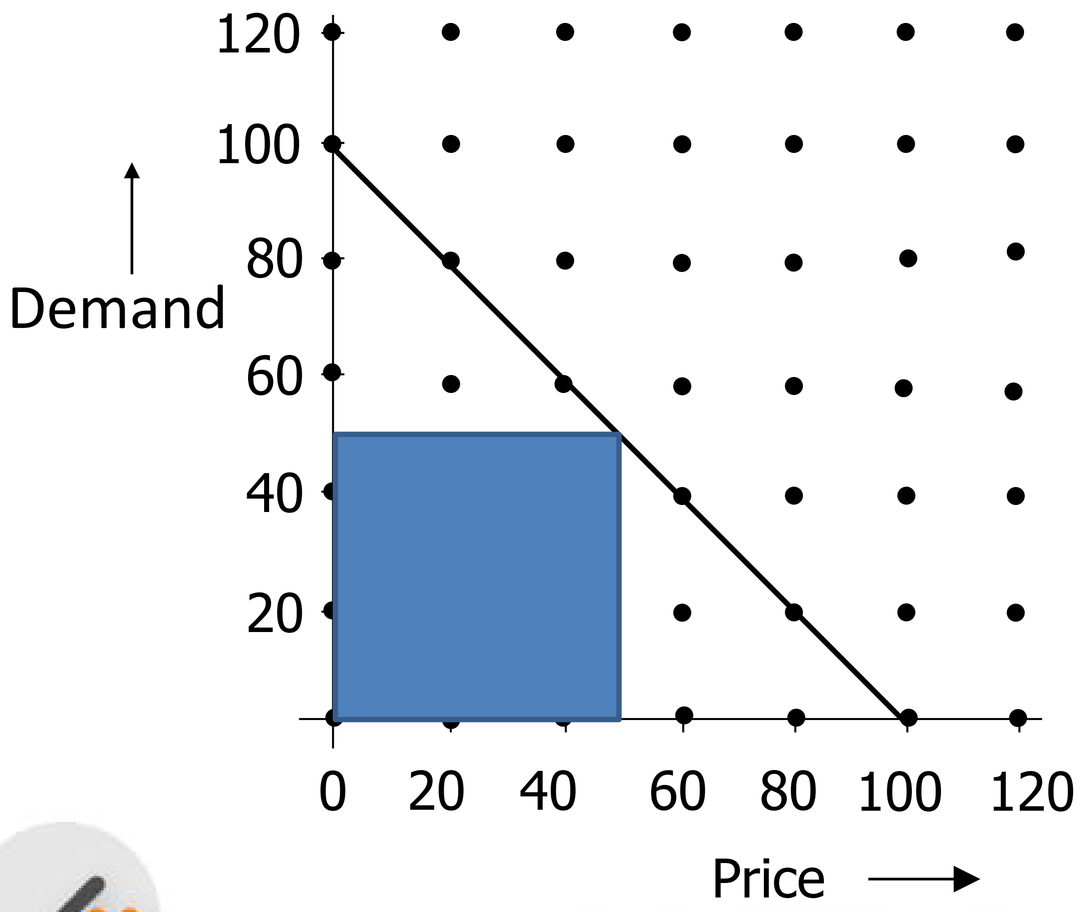



Downward sloping demand curve
D = 100 - P

What price will maximize revenue ?

# Revenue Management: Example

- Consider the following simple example:




Downward sloping demand curve
D = 100 - P

Revenue is maximized when price = 500

Demand = 500

Revenue = 50 x 50 = 2,500

# Revenue Management: Example

| PRICE | DEMAND |
|---|---|
| 1 | 99 |
| 2 | 98 |
| … | … |
| 98 | 2 |
| 99 | 1 |

# Revenue Management: Example

- Suppose we could sell the product to each customer at the price he is “willing” to pay!
- Then total revenue would be 99 + 98 + … + 1

  = 4,950

# Revenue Management: Example

- Even partial segmentation helps:

| PRICE | DEMAND |
|---|---|
| 80 | 20 |
| 60 | 20 |
| 40 | 20 |
| 20 | 20 |
| TOTAL REVENUE | 4000 |

# Revenue Management: Success Stories

- National Car Rental reported annual incremental revenue of $ 56 million on a base of $ 750 million – a revenue gain of over 7%
- RM allowed National Car Rental to avoid liquidation and return to profitability in less than one year

# Revenue Management: Success Stories

- Delta Airline reported annual incremental revenue of $ 300 million from an investment of $ 2 million – a ROI of 150%
- American Airlines reported revenue gain of $ 1.4 billion over a 3 year period.
- Austrian Airlines reported revenue gains of 150 million Austrian Schillings in 1991-92, in spite of a decrease in Load Factor
- People's Express did not use RM – and ceased to exist

# Revenue Management: Success Stories

- National Broadcasting Corporation implemented a RM system for about $ 1 mio.
- It generated incremental revenue of $ 200 mio on a base of $ 9 bio in 4 years. This is a revenue gain of over 2% and ROI of 200%

# Hotels, Cruise, Casinos, Cargo, Railways…

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# Revenue Management: When it works

- Perishable product or service
- Fixed capacity
- Low marginal cost
- Demand fluctuations
- Advanced sales
- Market Segmentation

# Revenue Management: Exercise

| | Fare | Allocation |
|---|---|---|
| Y | 300 | ? |
| B | 120 | ? |
| | | 140 |

- Your first chance for hands on RM!
- How many seats should be allocated to Y and B fare classes respectively? You decide!

# Revenue Management: Demand Forecasting

- Before you can determine the allocations to buckets you need to forecast the demand for each
- Do we need to forecast the demand for both Y and B classes?
- If Y demand came first RM would be unnecessary
  - Just sell seats on a First Come First Served basis!
- Since B demand comes first we need to forecast Y demand and allocate inventory accordingly
- Forecasts should be accurate
  - High forecasts ⟶ spoilage
  - Low forecasts ⟶ spillage

# Revenue Management: Demand Forecasting

- Objective: Obtain quick and robust forecasts.
- Number of forecasts:  Typically around
  - 10,000 fare class demand forecasts, or
  - 2,000,000 OD demand forecasts
  - every night for medium-sized airlines

# Revenue Management: Demand Forecasting

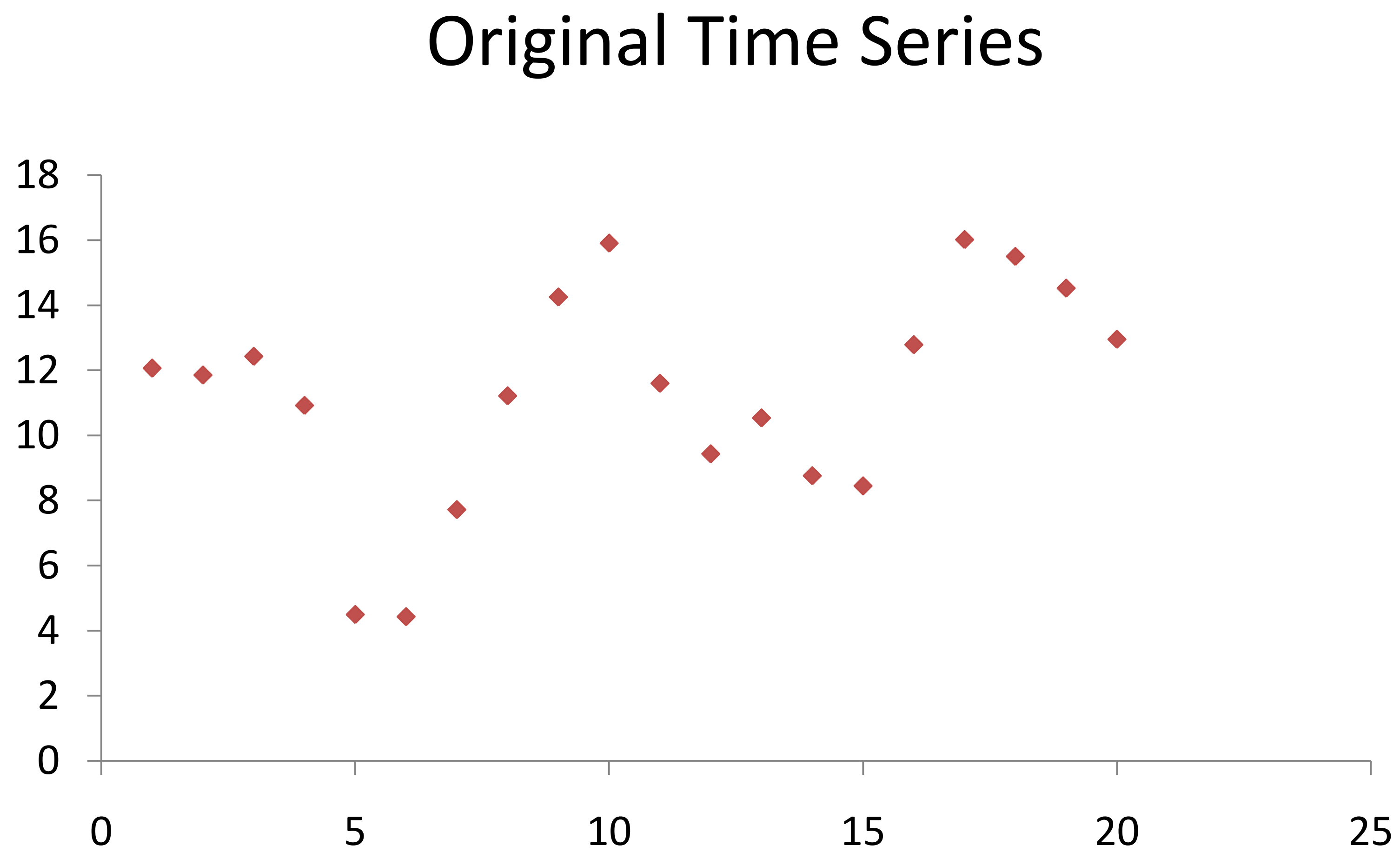
What do we forecast?

- Booking curve, Cancellation curve
- No-shows, Spill, and Recapture
- Revenue values of volatile products
- Up-selling and cross-selling probabilities
- Parameters in the demand function
- Price elasticity of demand

# Revenue Management: Demand Forecasting

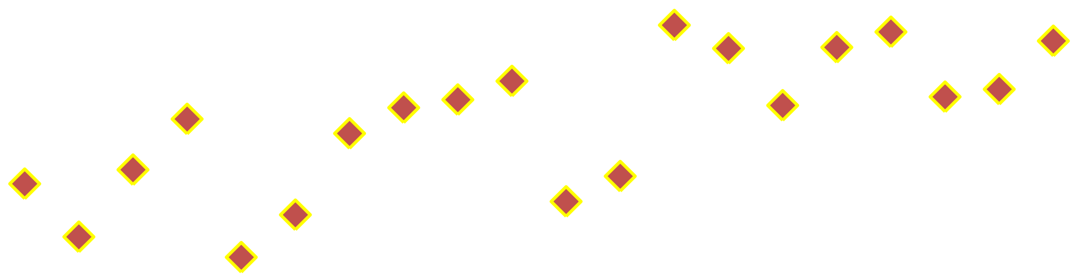
- Time Series Methods
  - Moving Averages
  - Exponential Smoothing
- Regression
- Pick-Up Forecasting
- Neural Networks
- Bayesian Update Methods

# Forecasting Methods



# Forecasting Methods

Time Series (Seasonality Removed)



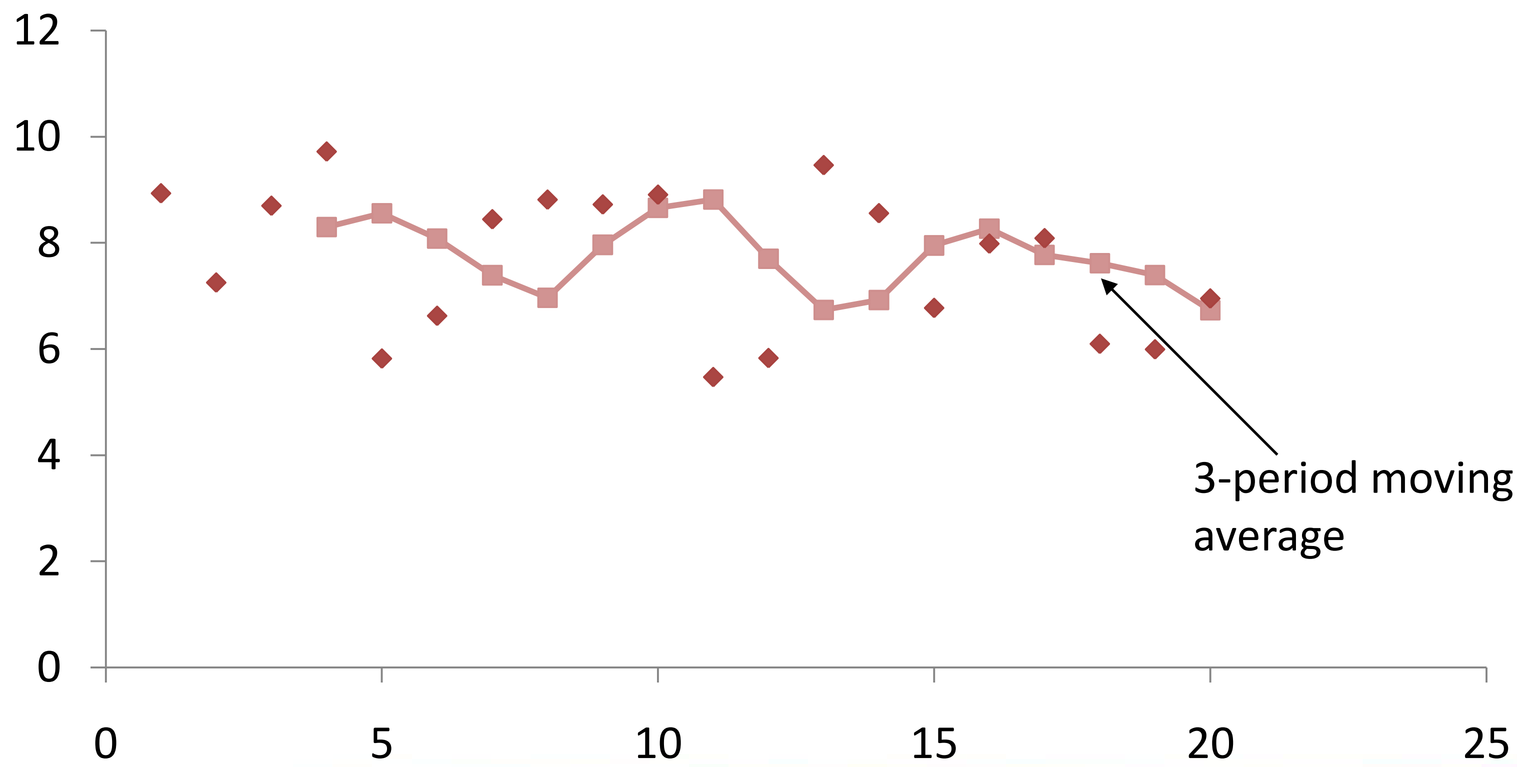
# Forecasting Methods

## Time Series (Trend Removed)

# Forecasting Methods

## Moving Average

*k* period moving average: Take the average of the last *k* observations to predict the next observation



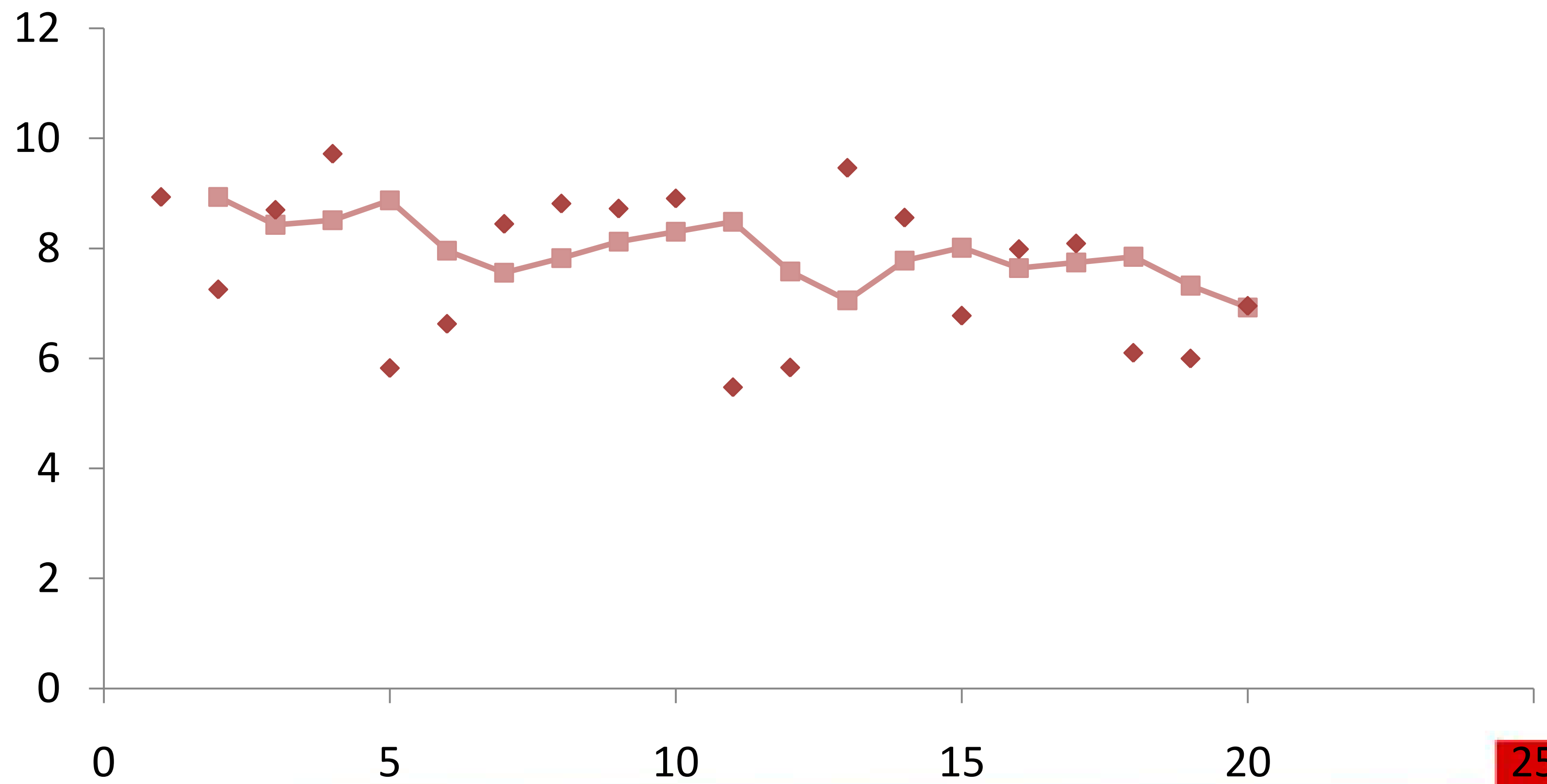
# Forecasting Methods

Exponential Smoothing

Tomorrow's forecast =
Today's forecast +
$\alpha \times$ Error in today's forecast.

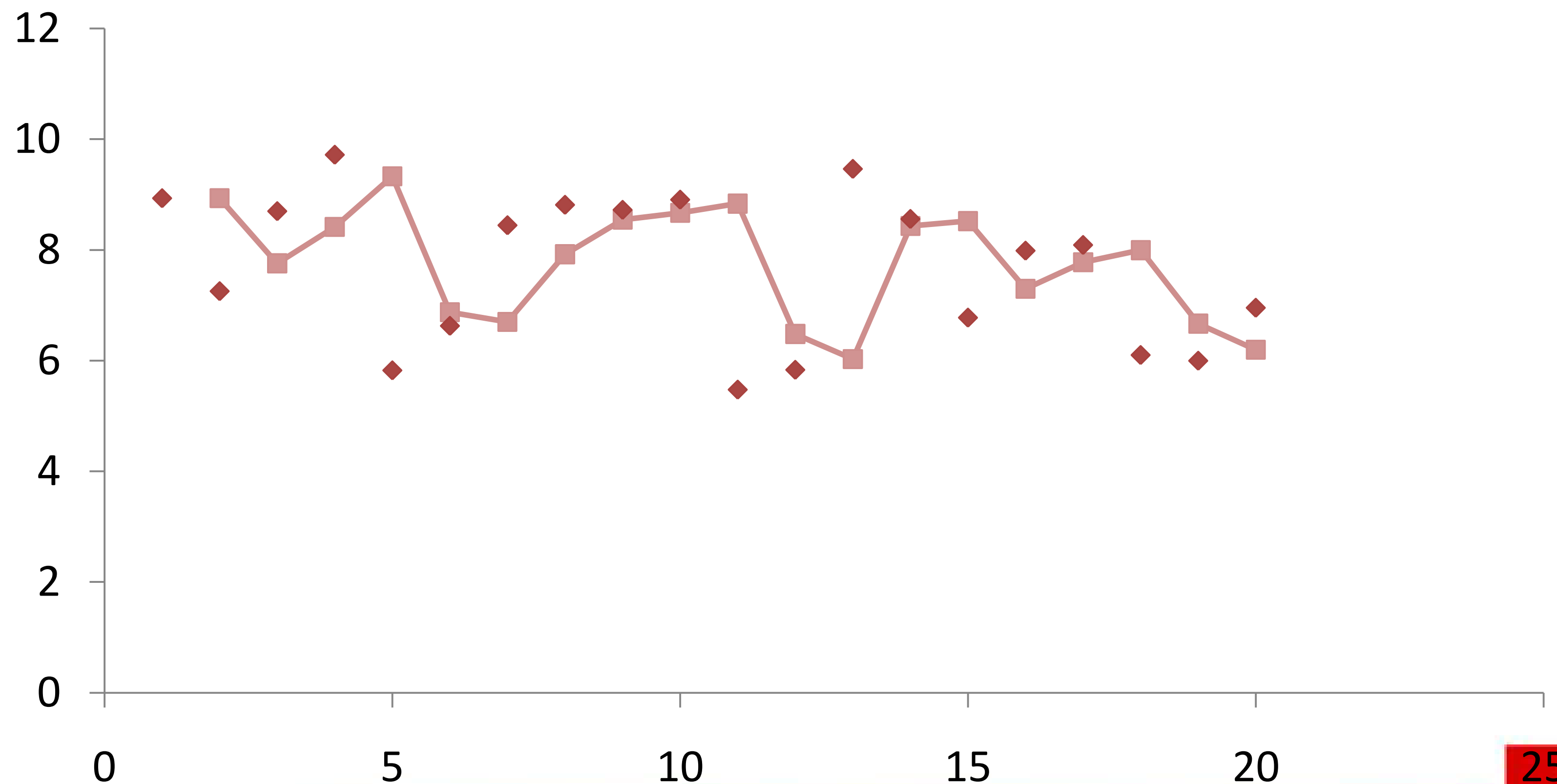
# Forecasting Methods

Exponential Smoothing ($\alpha$=0.3)



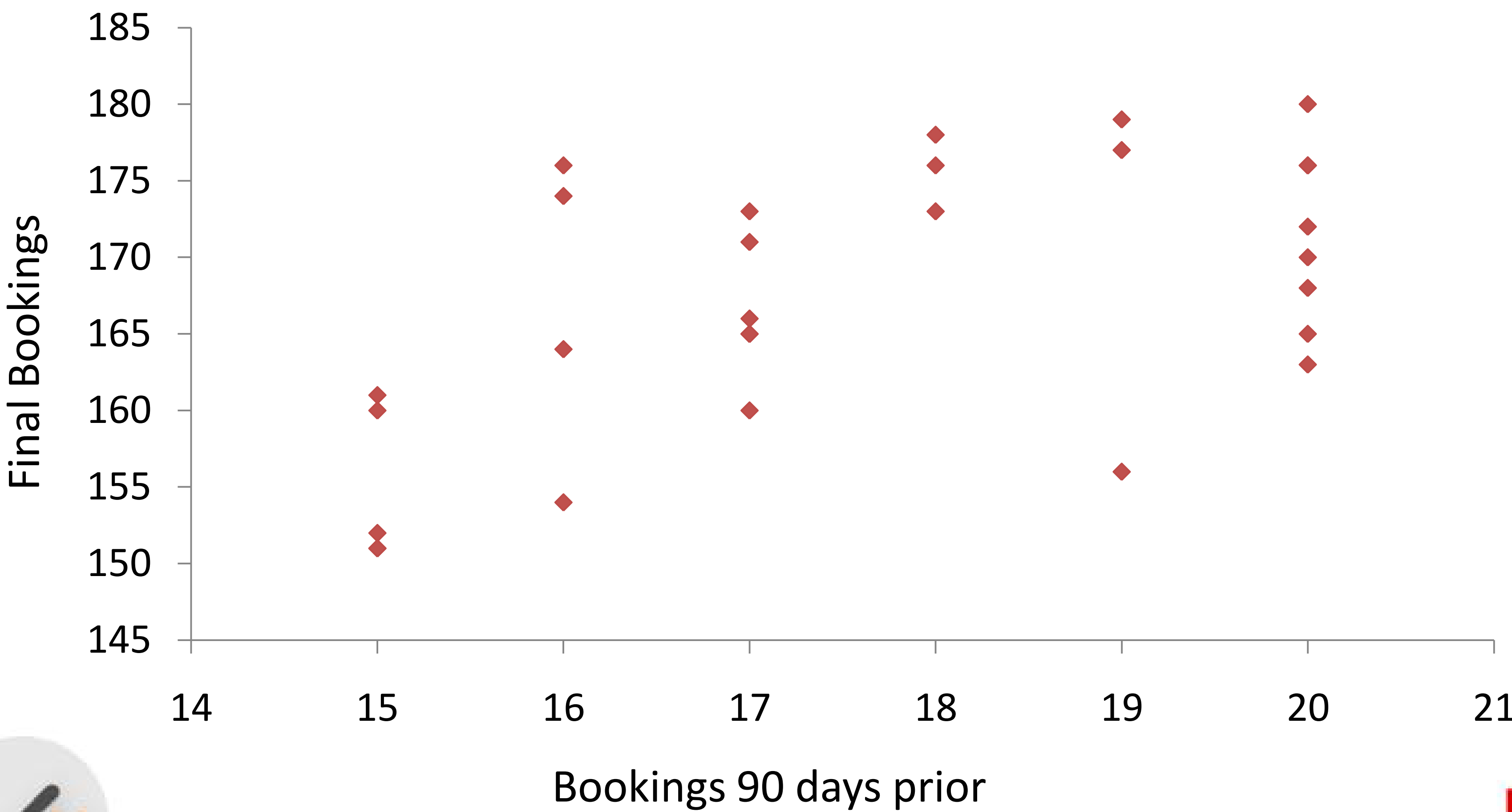
# Forecasting Methods

Exponential Smoothing ($\alpha$=0.7)



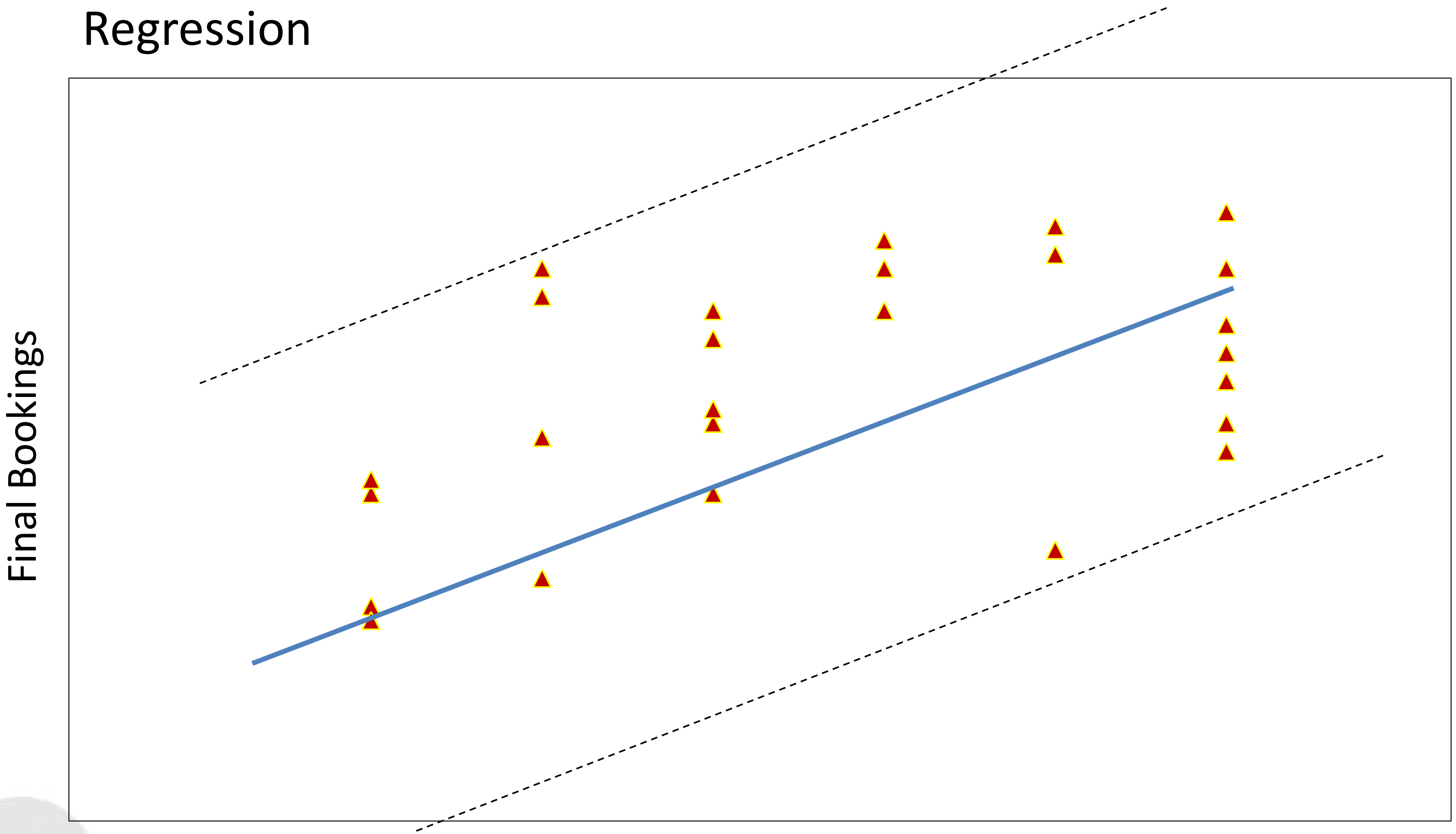
# Forecasting Methods

Regression



# Forecasting Methods

Regression



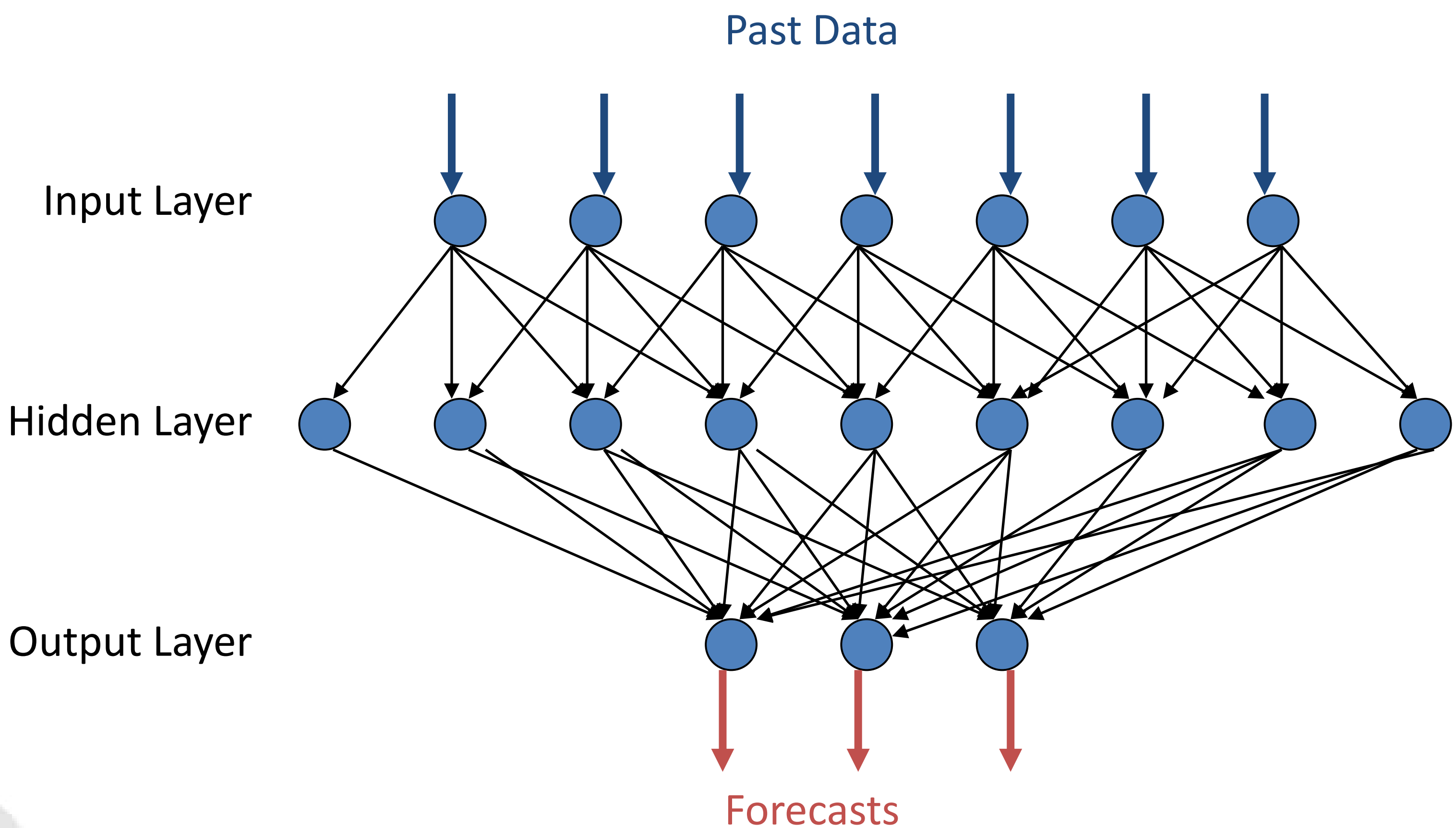
# Forecasting Methods

Pick-Up Forecasting

| Days Prior to Usage | | | | | | | | | Usage Date |
|---|---|---|---|---|---|---|---|---|---|
| -8 | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 | |
| 6 | 3 | 11 | 4 | 9 | 8 | 13 | 3 | 13 | 9-Apr |
| 8 | 6 | 6 | 3 | 16 | 11 | 5 | 4 | 2 | 10-Apr |
| 1 | 2 | 0 | 0 | 3 | 6 | 2 | 6 | 8 | 11-Apr |
| 6 | 0 | 4 | 1 | 2 | 6 | 3 | 2 | ? | 12-Apr |
| 3 | 8 | 8 | 7 | 5 | 1 | 2 | ? | | 13-Apr |
| 1 | 0 | 2 | 6 | 6 | 4 | ? | | | 14-Apr |
| 0 | 1 | 1 | 6 | 5 | ? | | | | 15-Apr |
| 1 | 11 | 12 | 6 | ? | | | | | 16-Apr |

# Forecasting Methods

## Neural Networks



# Forecasting Methods: Unconstraining

The Problem

| True Demand | Booking Limits | Observed Demand |
|---|---|---|
| 22 | 24 | 22 |
| 15 | 20 | 15 |
| 24 | 17 | 17 |
| 33 | 35 | 33 |
| 16 | 16 | 16 |
| 26 | 22 | 22 |
| 22 | 22 | 22 |
| 23 | 15 | 15 |
| 22 | 22 | 22 |
| 17 | 17 | 17 |

Unconstraining

# Forecasting Methods: Unconstraining

The Method (The EM Algorithm)

| Observed Demand |
| --- |
| 22 |
| 15 |
| 17 |
| 33 |
| 16 |
| 22 |
| 22 |
| 15 |
| 22 |
| 17 |

Find the mean and the Standard deviation of the non-truncated demand:

Mean (m) = (22+15+33+…+17)/7
= 21

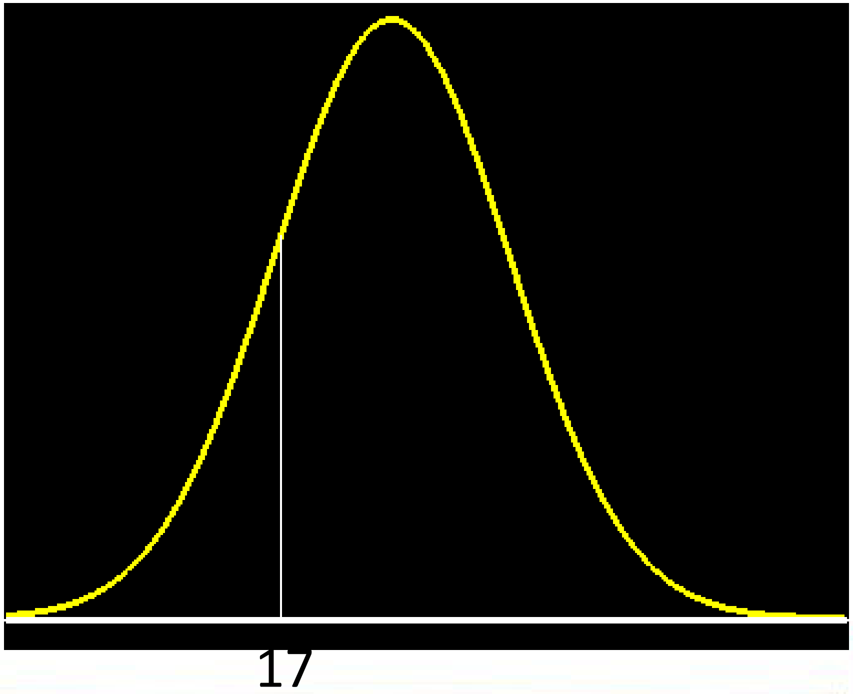
Std. Dev. (s) = 6.11

# Forecasting Methods: Unconstraining

The Method (The EM Algorithm)

| Observed Demand |
|---|
| 22 |
| 15 |
| 17 |
| 33 |
| 16 |
| 22 |
| 22 |
| 15 |
| 22 |
| 17 |

Unconstraining 17:

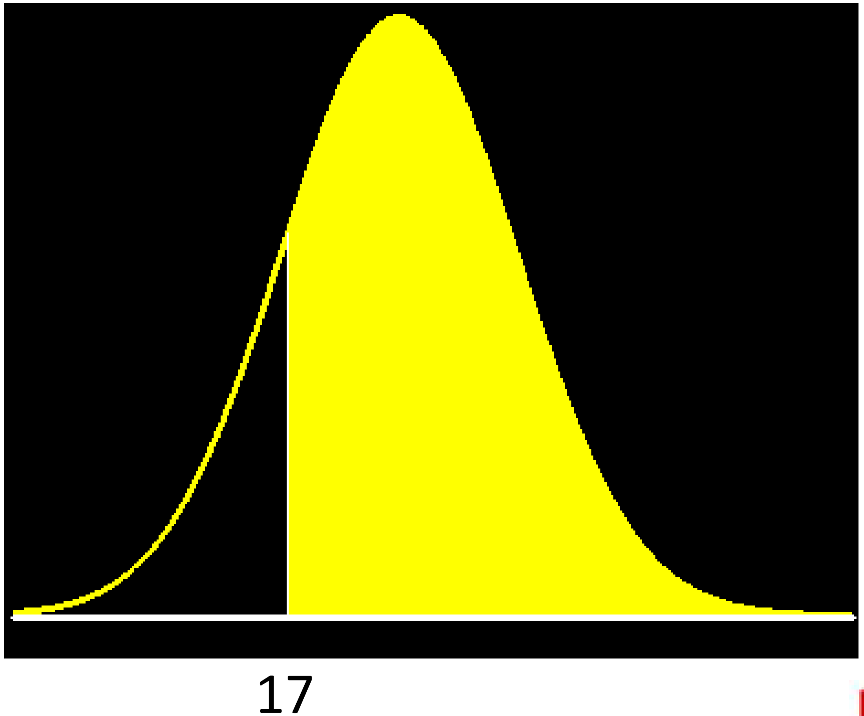


# Forecasting Methods: Unconstraining

The Method (The EM Algorithm)

| Observed Demand |
|---|
| 22 |
| 15 |
| 17 |
| 33 |
| 16 |
| 22 |
| 22 |
| 15 |
| 22 |
| 17 |

Unconstraining 17:



# Forecasting Methods: Unconstraining

The Method (The EM Algorithm)

| Observed Demand |
|---|
| 22 |
| 15 |
| 23.64 |
| 33 |
| 16 |
| 22 |
| 22 |
| 15 |
| 22 |
| 17 |

In a similar manner, handle the unconstraining of 22 and 15.

# Forecasting Methods: Unconstraining

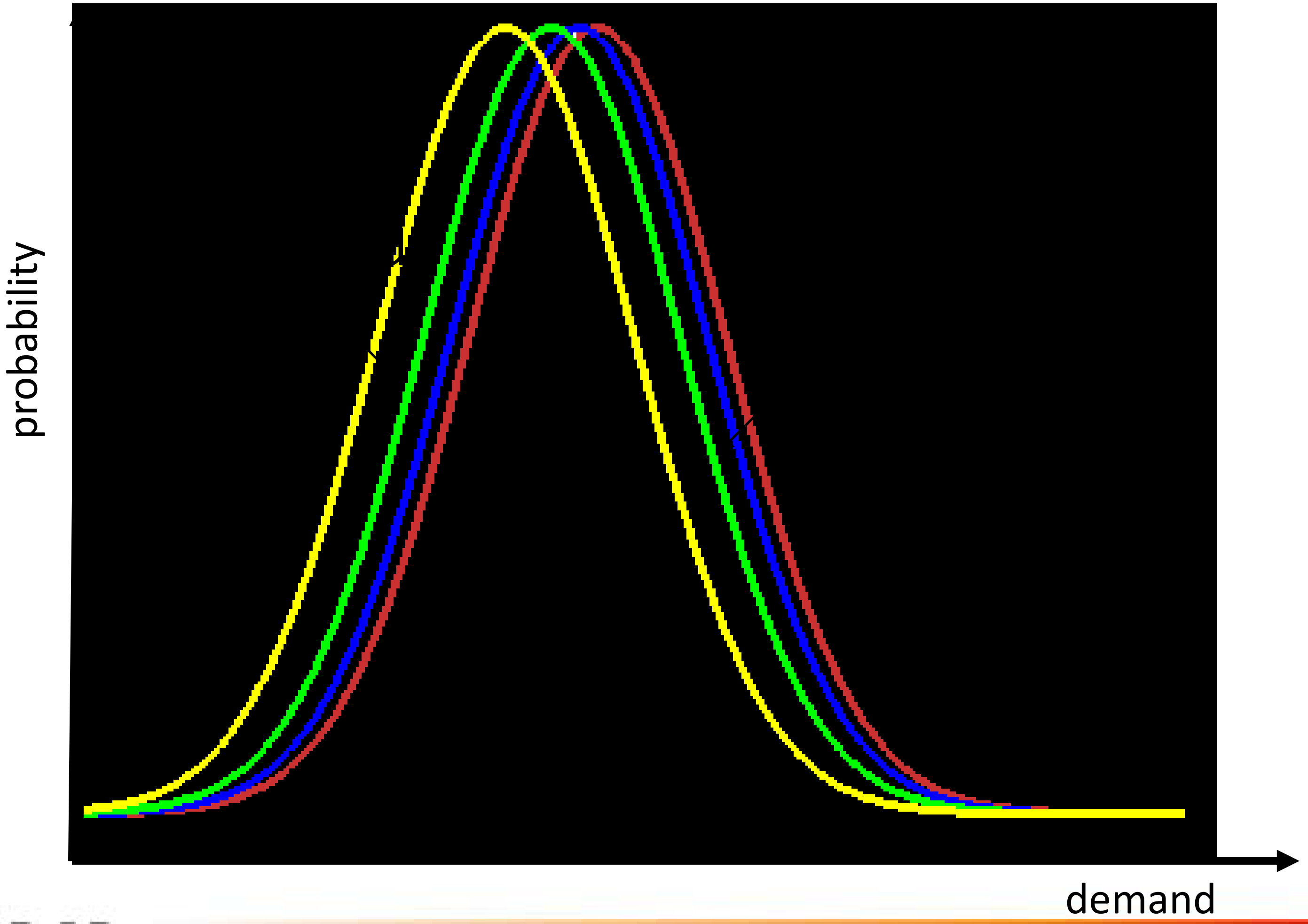
The Method (The EM Algorithm)

| Observed Demand |
|---|
| 22 |
| 15 |
| 23.64 |
| 33 |
| 16 |
| 26.53 |
| 22 |
| 22.79 |
| 22 |
| 17 |

| True Demand |
|---|
| 22 |
| 15 |
| 24 |
| 33 |
| 16 |
| 26 |
| 22 |
| 23 |
| 22 |
| 17 |

# Forecasting Methods: Unconstraining

The Method (The EM Algorithm)



# Revenue Management: Inventory Allocation

- Airlines have fixed capacity in the short run
- Airline seats are perishable inventory
- The problem - How should seats on a flight be allocated to different fare classes
- Booking for flights open long before the departure date - typically an year in advance
- Typically low yield passengers book early

# Revenue Management: Inventory Allocation

- Leisure passengers are price sensitive and book early
- Business passengers value time and flexibility and usually book late
- The Dilemma - How many seats should be reserved for high yield demand expected to arrive late?
  - Too much ⟹ spoilage - the aircraft departs which empty seats which could have been filled
  - Too little ⟹ spillage - turning away of high yield passengers resulting in loss of revenue opportunity

# Load Factor versus Yield Emphasis

400 Seat Aircraft - Two Fare Classes
(Example from Daudel and Vialle)

| | LOAD FACTOR EMPHASIS | YIELD EMPHASIS | REVENUE EMPHASIS |
|---|---|---|---|
| Seats sold For $ 1000 | 80 | 248 | 192 |
| Seats sold For $ 750 | 280 | 40 | 132 |
| TOTAL | 360 | 288 | 324 |
| LOAD FACTOR | 90% | 72% | 81% |
| REVENUE | 290,000 | 278,000 | 291,000 |
| YIELD | 805 | 965 | 898 |

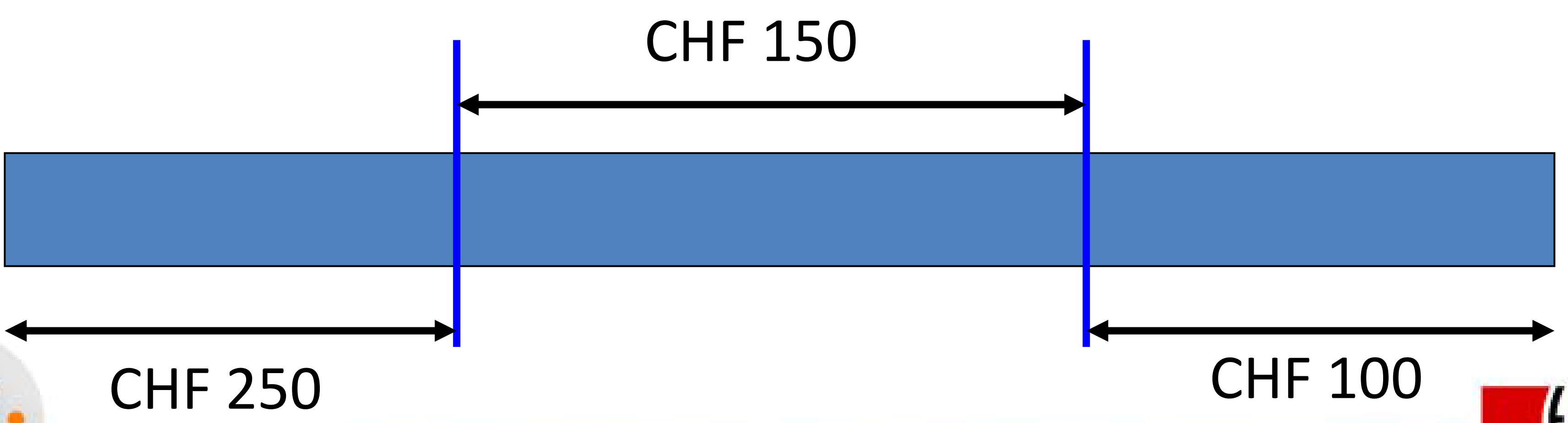
Need a Revenue Management System to balance load factor and yield

# Inventory Allocation

Geneva-Paris-Geneva case study for Baboo


Three fare classes, CHF 250, CHF 150, & CHF 100

Partitioned Booking Limits:

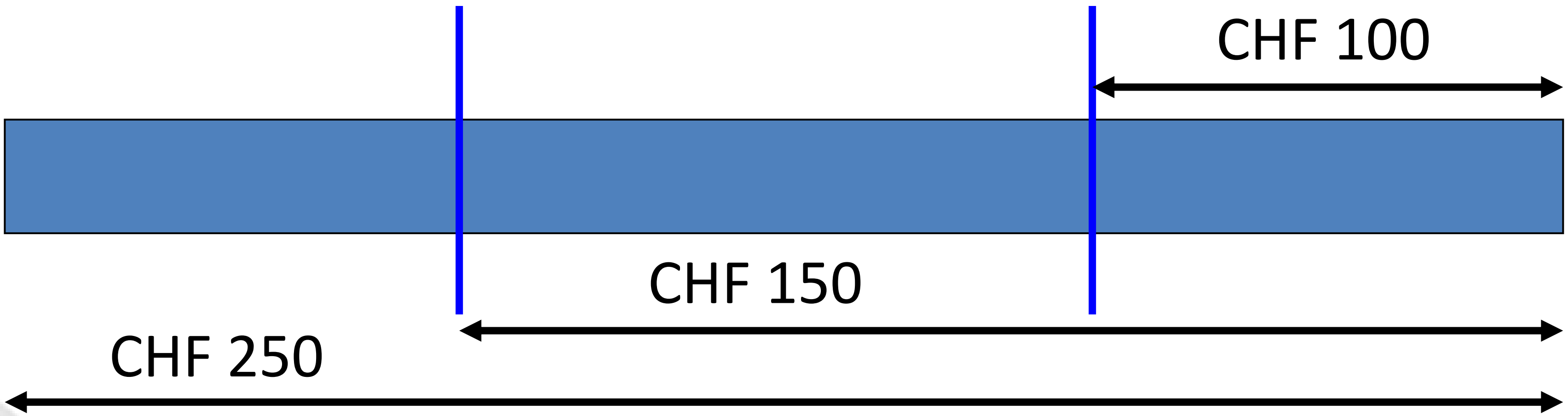


# Inventory Allocation: Nesting

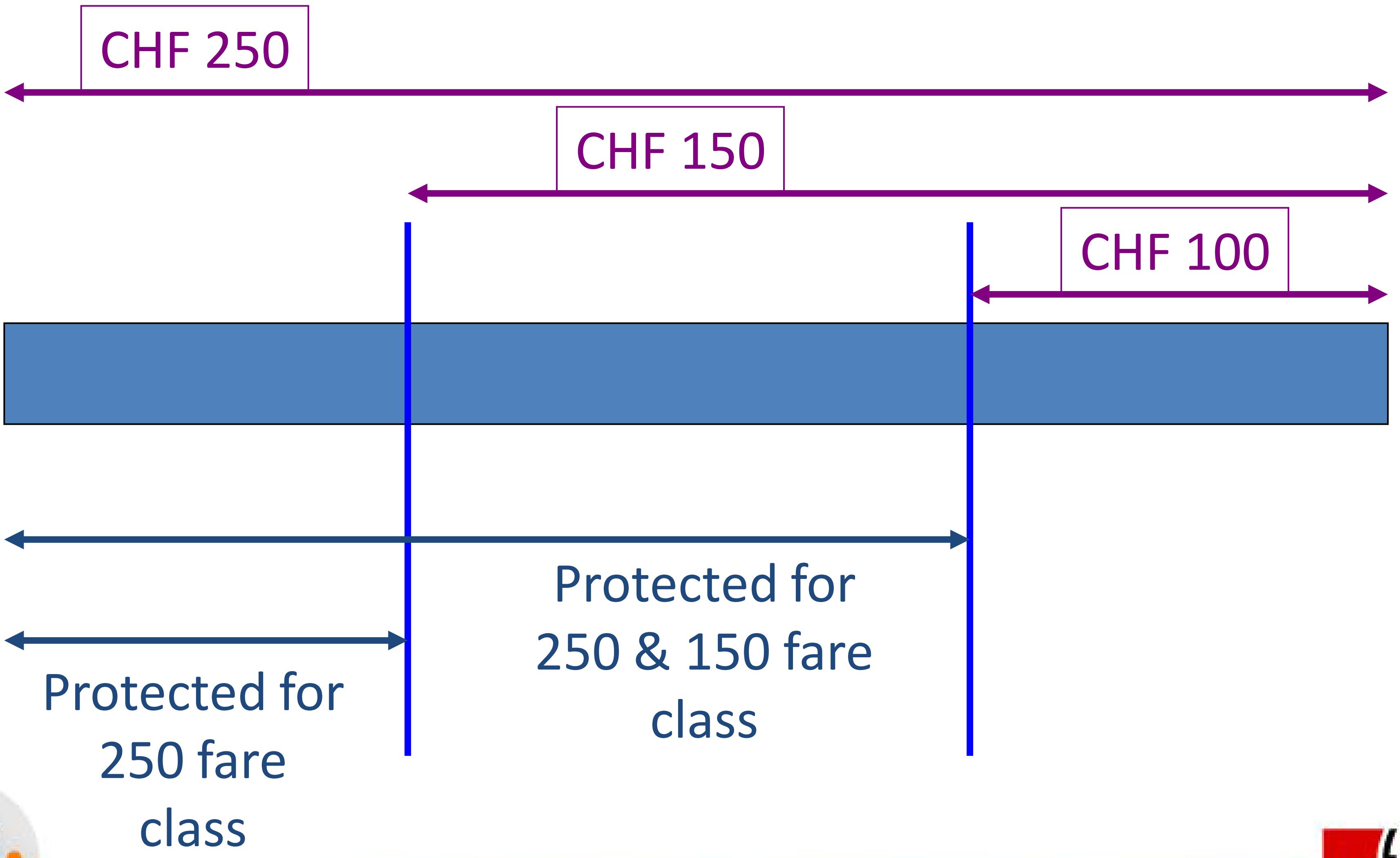
120 seats

Three fare classes, CHF 250, CHF 150, & CHF 100

Nested Booking Limits:



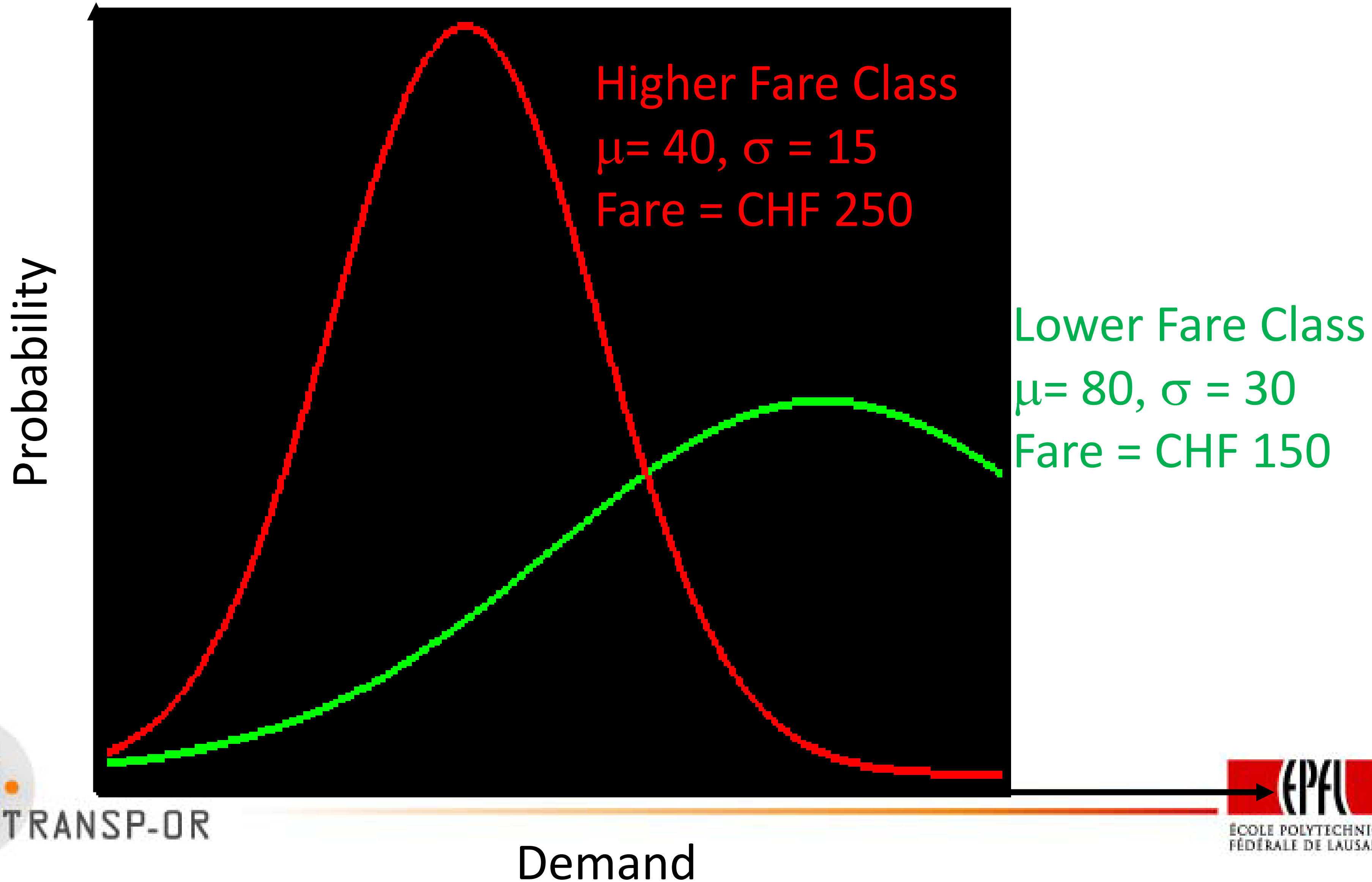
# Inventory Allocation: Protection levels



# Inventory Allocation: Two-class model

- Total number of seats: 120
- Seats divided into two classes based on fare: CHF 250 and CHF 150.
- Demands are distinct.
- Low fare class demand occurs earlier than the high fare class demand.

# Inventory Allocation: Two-class model



# Inventory Allocation: Two-class model

45 seats have already been booked in the lower fare class. Should we allow the 46th booking in the same class?

# Inventory Allocation: Two-class model

Revenue from the lower fare class:
$R_L$ = CHF150

Revenue from the higher fare class:
$R_H$ = CHF 0 if the higher fare demand < 74,
CHF 250 otherwise.

Expected Revenue from the higher fare class:
$E(R_H)$ = CHF 0  P(higher fare demand < 74)
+ CHF250  P(higher fare demand $\geq$ 74)

# Inventory Allocation: Two-class model

Revenue from the lower fare class:

$R_L$ = CHF150

Revenue from the higher fare class:

$R_H$ = CHF 0 if the higher fare demand < 74,

CHF 250 otherwise.

Expected Revenue from the higher fare class:

$E(R_H)$ = CHF 0  0.9883 *(Normal tables)*

+ CHF250  0.0117 *(Normal tables)*

≈ **CHF 3**

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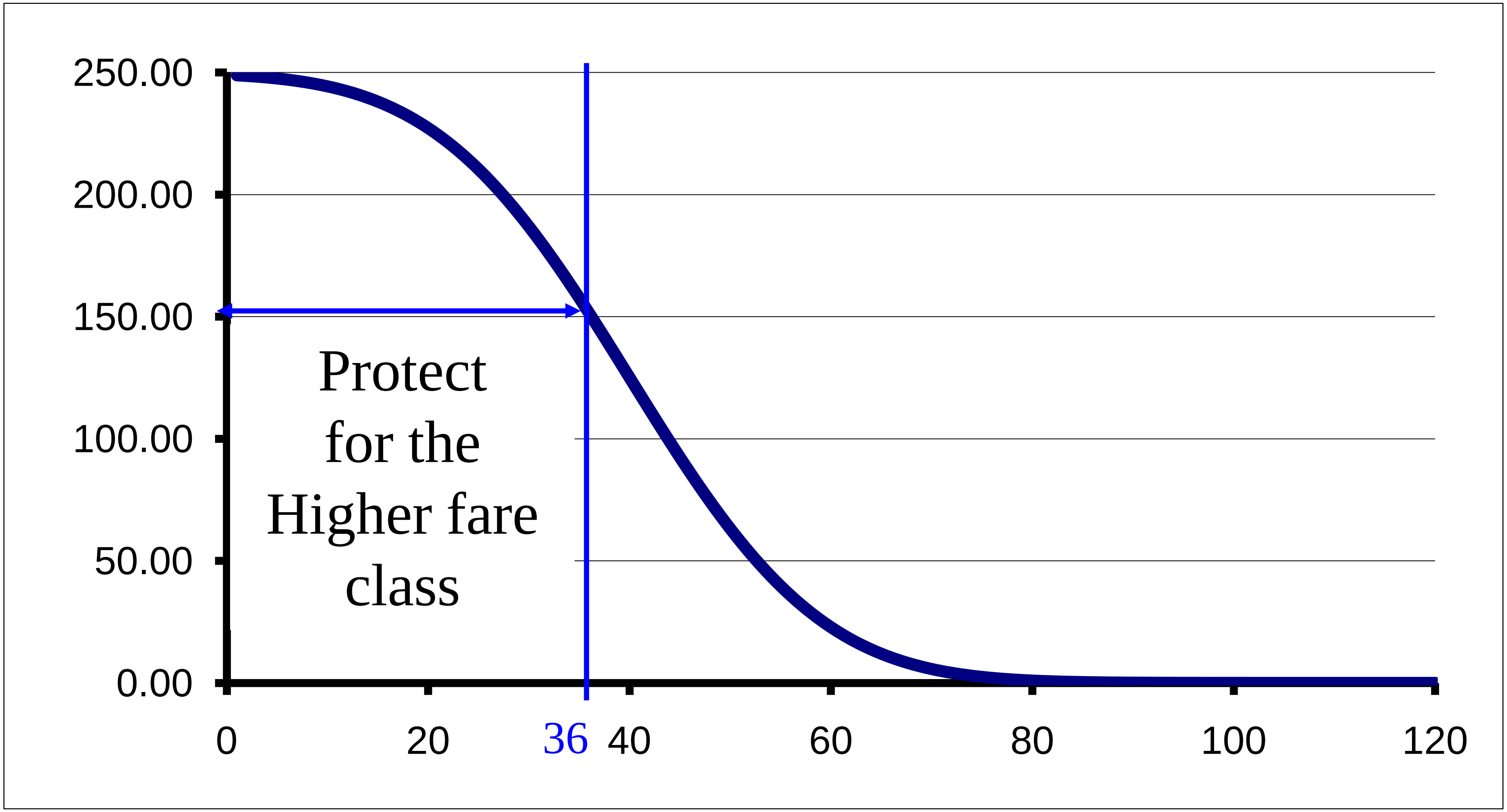
# Inventory Allocation: Two-class model




Expected Revenue from the Higher Class

# Inventory Allocation: Two-class model

## Decision Rule

- Accept up to 86 reservations from the lower fare class and then reject further reservations from this class.

Littlewood's rule

# Inventory Allocation: Exercise

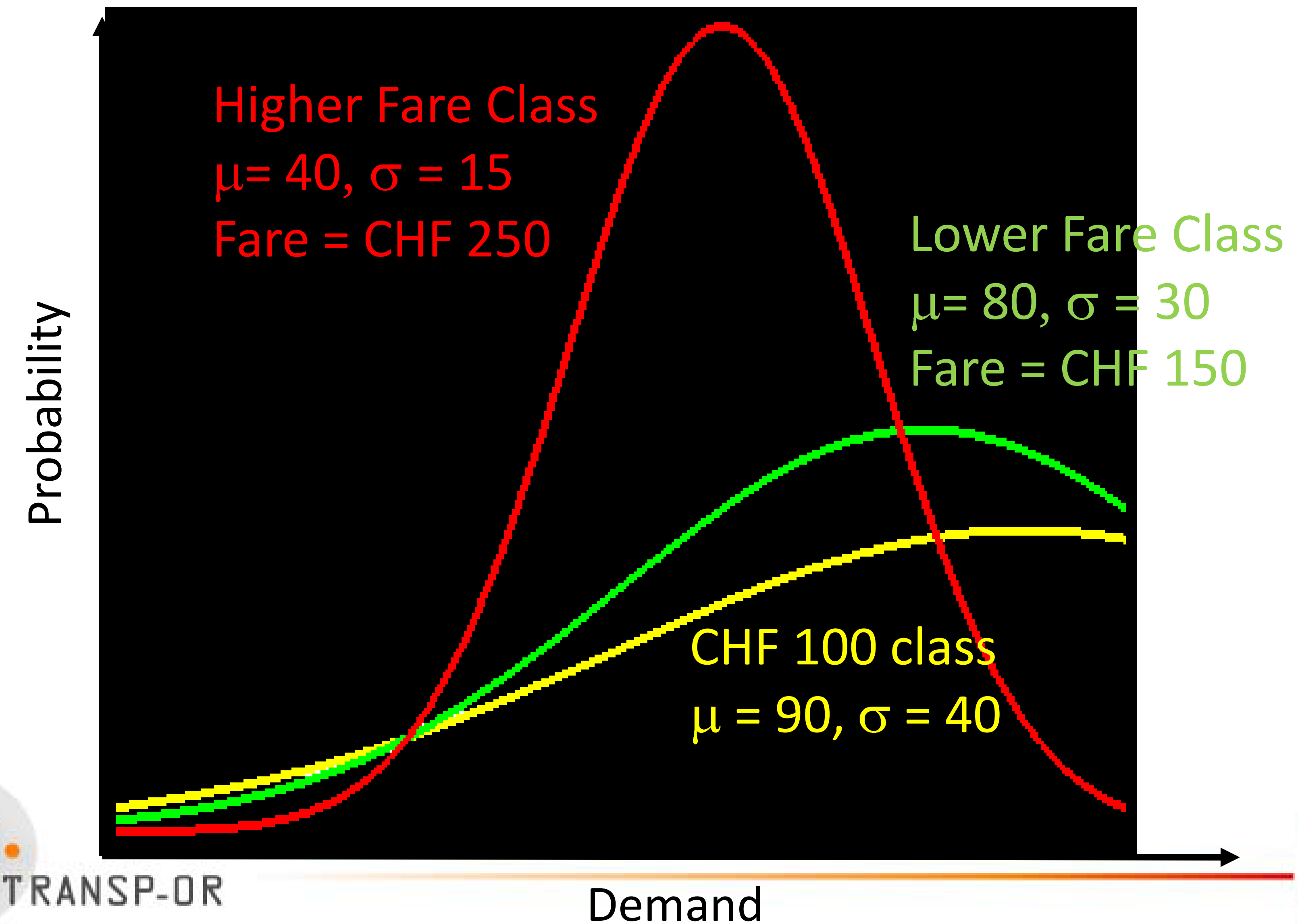
What happens if

- Our forecast improves?
- If the fare for the lower fare class drops?

# Inventory Allocation: Three-class model

- Total number of seats: 120
- Seats divided into three classes:
  CHF 250, CHF 150, and CHF 100.
- Demands are distinct.
- Low fare class demand occurs earlier than the high fare class demand.

# Inventory Allocation: Three-class model



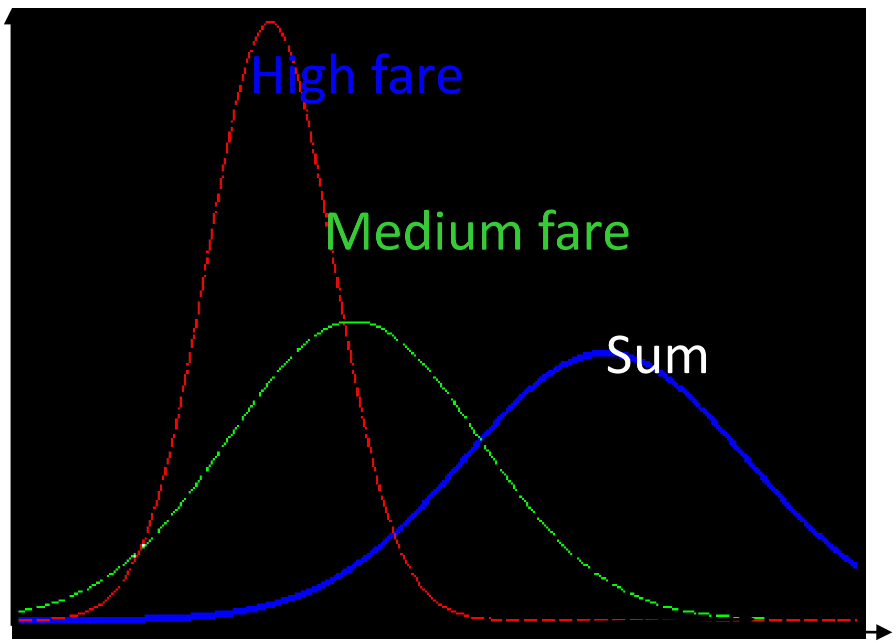
# Inventory Allocation: Three-class model

The EMSR-b Method

- Step 1: Aggregate the demand and fares for the higher classes.
- Step 2: Apply Littlewood's formula for two class model to obtain protection levels.

# Inventory Allocation: Three-class model

Computing Protection Levels for the High & Medium Fare Classes: Aggregating Demand
($m_H$ = 40, $s_H$ = 15; $m_M$ = 80, $s_M$ = 30; $m_L$ = 90, $s_L$ = 40)




Distribution of demand sum:
Normal with
Mean = 40+80 = 120
Std. Dev. = $\sqrt{(225+900)}$
= 33.54

# Inventory Allocation: Three-class model

Computing Protection Levels for the High & Medium Fare Classes: Aggregating Fares
($\mu_H = 40$, $F_H = 250$; $\mu_M = 80$, $F_M = 150$; $\mu_L = 90$, $F_L = 100$)

$$F_{Agg} = (40 \quad 250 + 80 \quad 150)/(40+80)$$
$$= 183.33$$

# Inventory Allocation: Three-class model

Computing Protection Levels for the High & Medium Fare Classes: Applying Littlewood's Formula

$m_{Agg} = 120$, $s_{Agg} = 33.54$, $F_H = 183.33$;
$m_L = 90$, $s_L = 40$, $F_L = 100$

Littlewood's Formula:
Find x such that
$183.33 \ \text{Prob}(\text{Demand}_{Agg} \geq x) = 100$

# Inventory Allocation: Three-class model

Computing Protection Levels for the High & Medium Fare Classes: Applying Littlewood's Formula

$m_{Agg} = 120$, $s_{Agg} = 33.54$, $F_H = 183.33$;

$m_L = 90$, $s_L = 40$, $F_L = 100$

Applying Littlewood's Formula: $x = 116$

So 116 seats are reserved for the CHF 250 and CHF 150 fare classes.

# Inventory Allocation: Three-class model

Computing Protection Levels for the High Fare Class: Applying Littlewood's Formula

$m_H = 40$, $s_H = 15$, $F_H = 250$;
$m_M = 90$, $s_M = 30$, $F_L = 150$.

Littlewood's Formula:
Find x such that
250 Prob($\text{Demand}_H \geq x$) = 150

# Inventory Allocation: Three-class model

Computing Protection Levels for the High Fare Class: Applying Littlewood's Formula

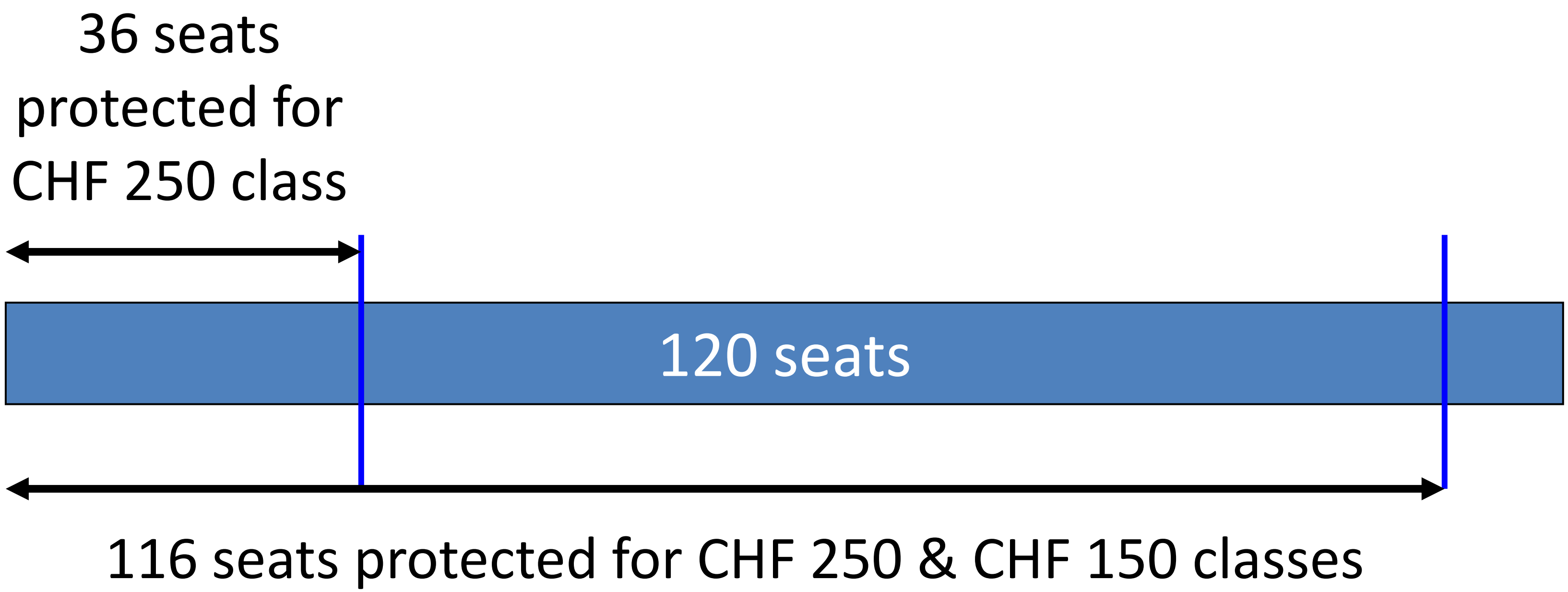
$m_H = 40$, $s_H = 15$, $F_H = 250$;

$m_M = 90$, $s_M = 30$, $F_L = 150$.

Applying Littlewood's Formula: $x = 36$

So 36 seats are reserved for the CHF 250 fare classes.

# Inventory Allocation: Three-class model



# Inventory Allocation: Four-class model

Capacity: 200 Seats

| Room Type | Demand Mean | Demand Std. Dev. | Fares |
|---|---|---|---|
| Executive | 30 | 10 | 7000 |
| Deluxe | 50 | 20 | 6000 |
| Special | 80 | 25 | 4000 |
| Normal | 150 | 100 | 2500 |

# Inventory Allocation: Willingness to pay

- Consider a booking request that comes for the CHF 100 fare class
- Suppose that 25% of the people demanding bookings in the CHF 100 fare class are willing to jump to the CHF 150 fare class if necessary (up-sell probability)
- Also suppose 2 seats are already booked for the CHF 100 fare class

# Inventory Allocation: Willingness to pay

If we turn her away, then

- She may pay for higher class
- She may refuse and higher class demand $< 118$
- She may refuse and higher class demand $\geq 118$

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# Inventory Allocation: Willingness to pay

If we turn her away, then expected value $E = 0.25 \times 150$

- She may refuse and higher class demand $< 118$

- She may refuse and higher class demand $\geq 118$

# Inventory Allocation: Willingness to pay

If we turn her away, then expected value

E =

$0.25 \times 150$

+

0

- She may refuse and higher class demand $\geq 118$

# Inventory Allocation: Willingness to pay

If we turn her away, then expected value

E =

$0.25 \times 150$

+

0

+

$(1\text{-}0.25) \times 1833.33 \times \text{Prob}(\text{Demand}_{\text{Agg}} \geq 118)$

# Inventory Allocation: Willingness to pay

If E > 100, then

we refuse the seat at CHF 100 but remain open for booking it at 150;

Else

we book the seat at CHF 100.

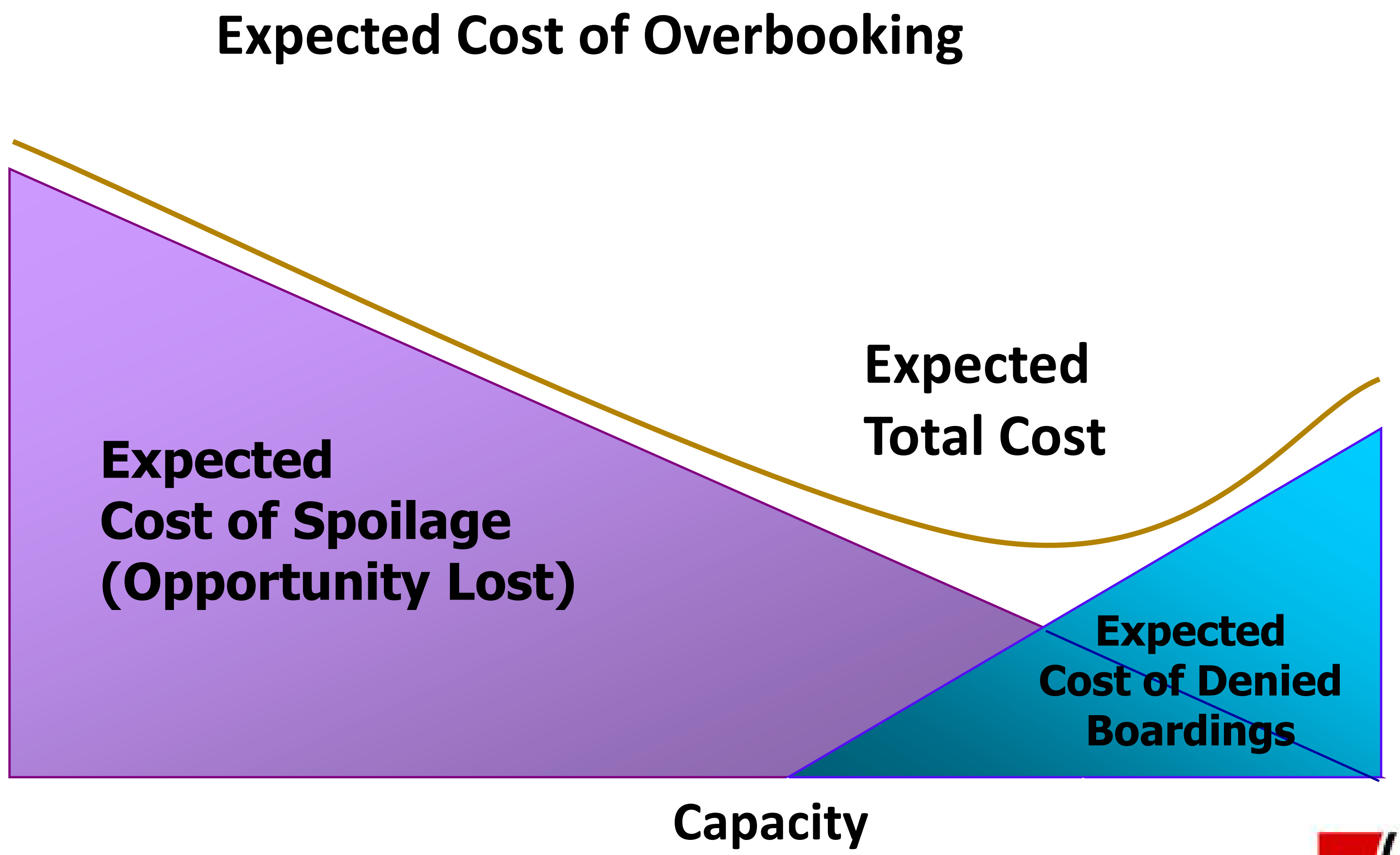
# Capacity Management

- All service industries, airlines in particular, need to manage limited capacity optimally
- Transferring capacity between compartments
  - Upgrades
  - Moving Curtains
- Changing aircraft capacity
  - Upgrade/downgrade aircraft configuration
  - Swapping aircraft

# Flight Overbooking

- Airlines overbook to compensate for pre-departure cancellation and day of departure no-shows
- Spoilage cost - incurred due to insufficient OB
  - Lost revenue from empty seat which could have been filled
- Denied Boarding Cost (DBC) - incurred due to too much OB
  - Cash compensation
  - Travel vouchers
  - Meal and accommodation costs
  - Seats on other airlines
  - Cost of lost goodwill

# Flight Overbooking



# Overbooking: Illustration

- Consider a fare class (with 120 seats) in a airline where booking starts 10 days in advance.
- Each day a certain (random) number of reservation requests come in.
- Each day a certain number of bookings get cancelled (cancellation fraction = 0.1).

# Overbooking: Illustration

| Day | No Limits | | | | | | | | | Bookings |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 14 | | | | | | | | | **14** |
| 2 | -1 | 23 | | | | | | | | **36** |
| 3 | -1 | -2 | 46 | | | | | | | **79** |
| 4 | -1 | -2 | -5 | 17 | | | | | | **88** |
| 5 | -1 | -2 | -4 | -2 | 50 | | | | | **129** |
| 6 | -1 | -2 | -4 | -2 | -5 | 27 | | | | **142** |
| 7 | -1 | -2 | -3 | -1 | -5 | -3 | 27 | | | **154** |
| 8 | -1 | -1 | -3 | -1 | -4 | -2 | -3 | 33 | | **172** |
| 9 | -1 | -1 | -3 | -1 | -4 | -2 | -2 | -3 | 14 | **169** |
| 10 | -1 | -1 | -2 | -1 | -3 | -2 | -2 | -3 | -1 | **153** |

# Overbooking: Illustration

| Day | No Overbooking | | | | | | | | | Bookings |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 14 | | | | | | | | | **14** |
| 2 | -1 | 23 | | | | | | | | **36** |
| 3 | -1 | -2 | 46 | | | | | | | **79** |
| 4 | -1 | -2 | -5 | 17 | | | | | | **88** |
| 5 | -1 | -2 | -4 | -2 | **41** | | | | | **120** |
| 6 | -1 | -2 | -4 | -2 | -4 | **13** | | | | **120** |
| 7 | -1 | -2 | -3 | -1 | -4 | -1 | **12** | | | **120** |
| 8 | -1 | -1 | -3 | -1 | -3 | -1 | -1 | **11** | | **120** |
| 9 | -1 | -1 | -3 | -1 | -3 | -1 | -1 | -1 | **12** | **120** |
| 10 | -1 | -1 | -2 | -1 | -3 | -1 | -1 | -1 | -1 | **108** |

# Overbooking: Illustration

| Day | Overbooking 10 seats | | | | | | | | | Bookings |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 14 | | | | | | | | | **14** |
| 2 | -1 | 23 | | | | | | | | **36** |
| 3 | -1 | -2 | 46 | | | | | | | **79** |
| 4 | -1 | -2 | -5 | 17 | | | | | | **88** |
| 5 | -1 | -2 | -4 | -2 | 50 | | | | | **129** |
| 6 | -1 | -2 | -4 | -2 | -5 | **15** | | | | **130** |
| 7 | -1 | -2 | -3 | -1 | -5 | -2 | **14** | | | **130** |
| 8 | -1 | -1 | -3 | -1 | -4 | -1 | -1 | **12** | | **130** |
| 9 | -1 | -1 | -3 | -1 | -4 | -1 | -1 | -1 | **13** | **130** |
| 10 | -1 | -1 | -2 | -1 | -3 | -1 | -1 | -1 | -1 | **118** |

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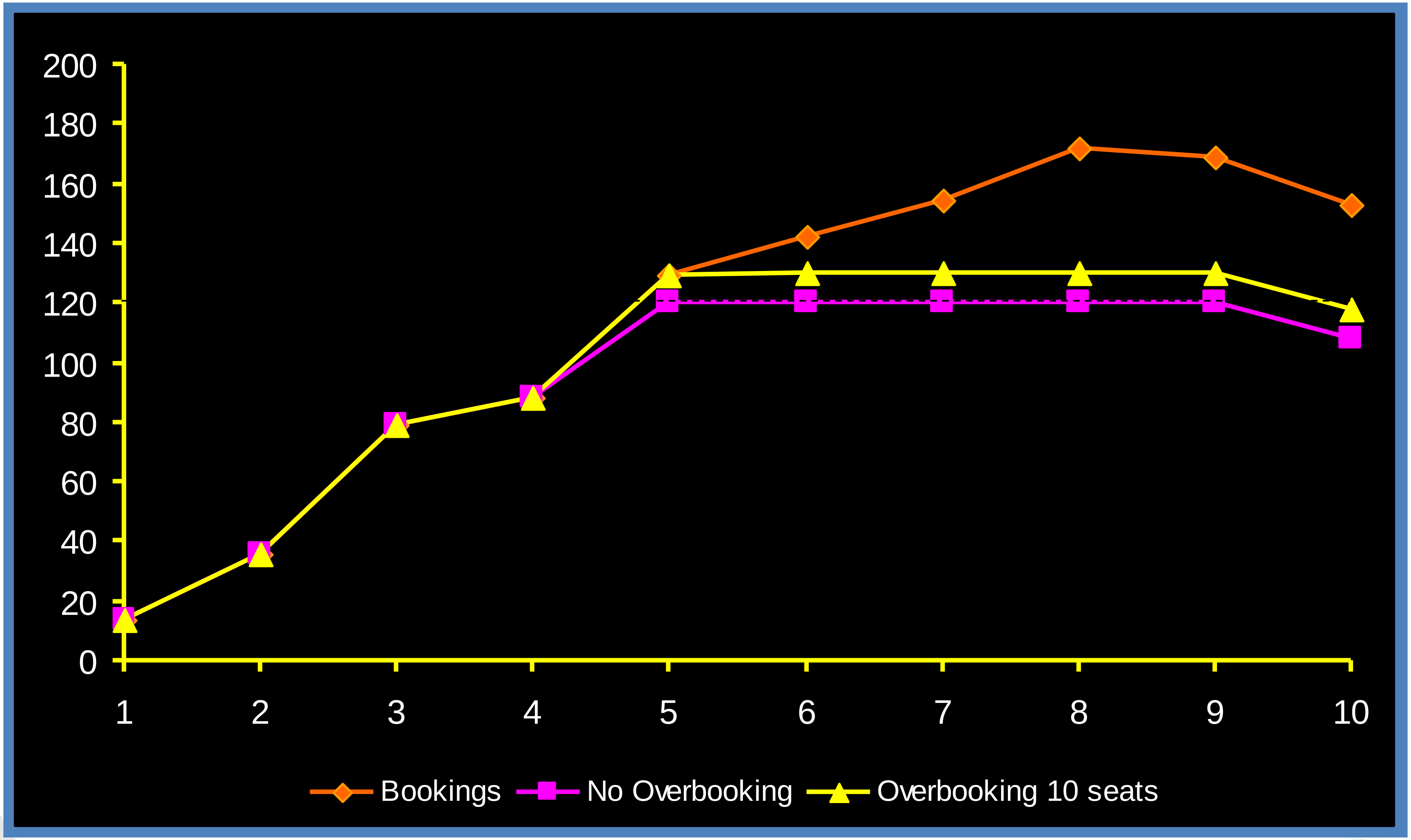
# Overbooking: Illustration

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# Overbooking: Concept

## Cancellations

- Customers cancel independently of each other.
- Each customer has the same probability of cancelling.
- The cancellation probability depends only on the time remaining.

# Overbooking: Concept

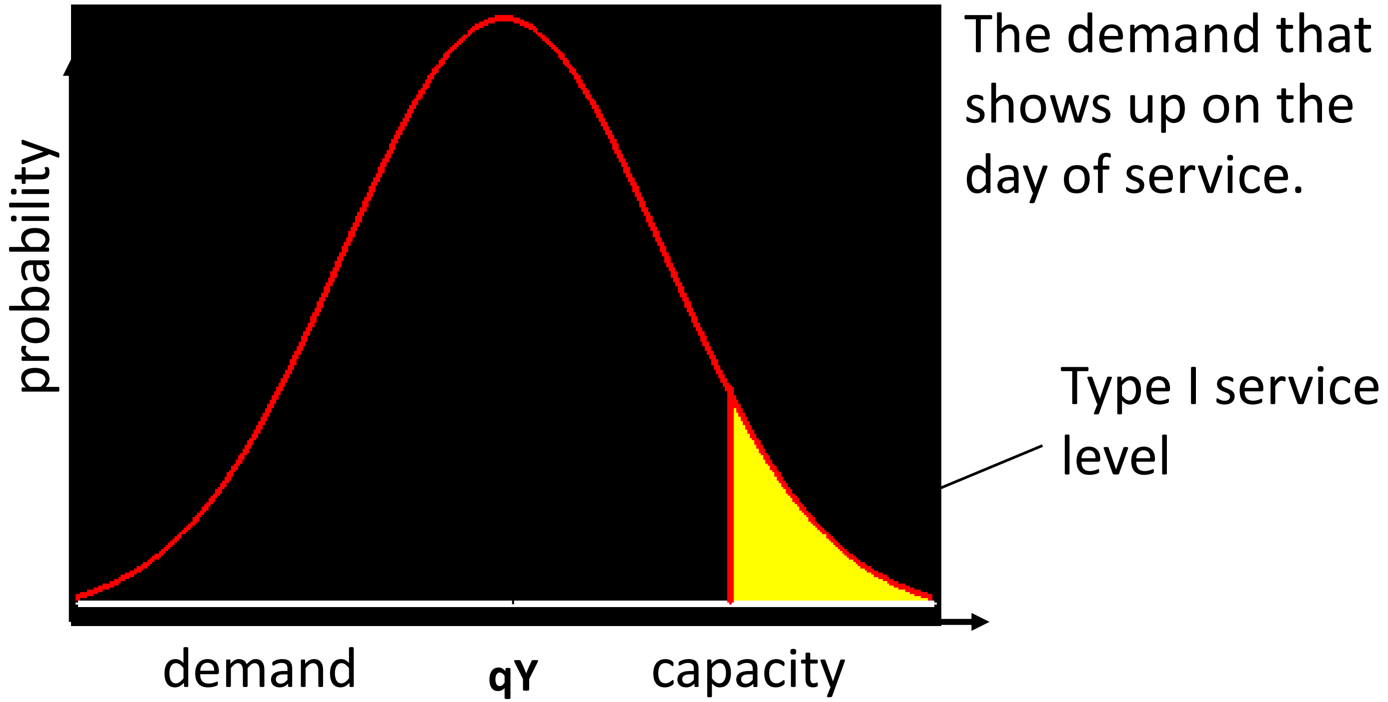
Let
$Y$ : number of reservations at hand, and
$q$ : probability of showing up for each reservation.

Then
the number of reservations that show up
$\approx$ Binomial with mean $qY$, and variance $q(1-q)Y$.

*We can approximate this with*
*Normal with mean $qY$, and variance $q(1-q)Y$.*

# Overbooking: Concept

Criterion – Type I service level: The probability that the demand that shows up exceeds the capacity.



# Overbooking: Concept

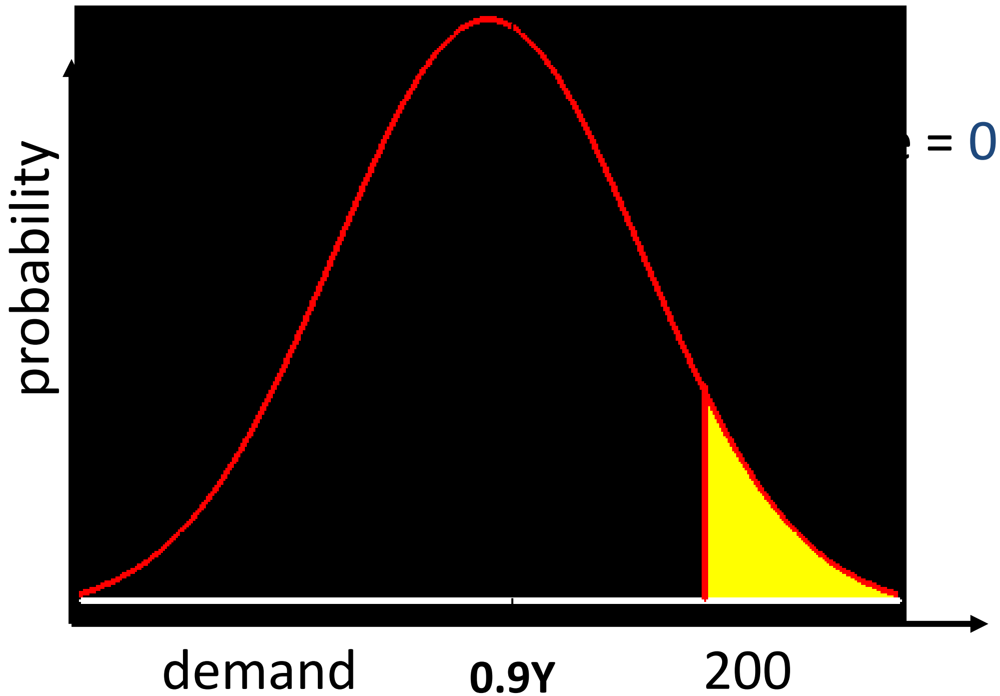
Criterion – Type I service level:

| | |
|---|---|
| Capacity: | 200 seats |
| Showing up probability: | 0.9 |
| Reqd. Type I service level: | 0.5% |

Overbooking limit?

# Overbooking: Concept

Let the limit be Y.




[redacted] = 0.9×0.1×Y

Y turns out to be 219.

# Overbooking: Concept

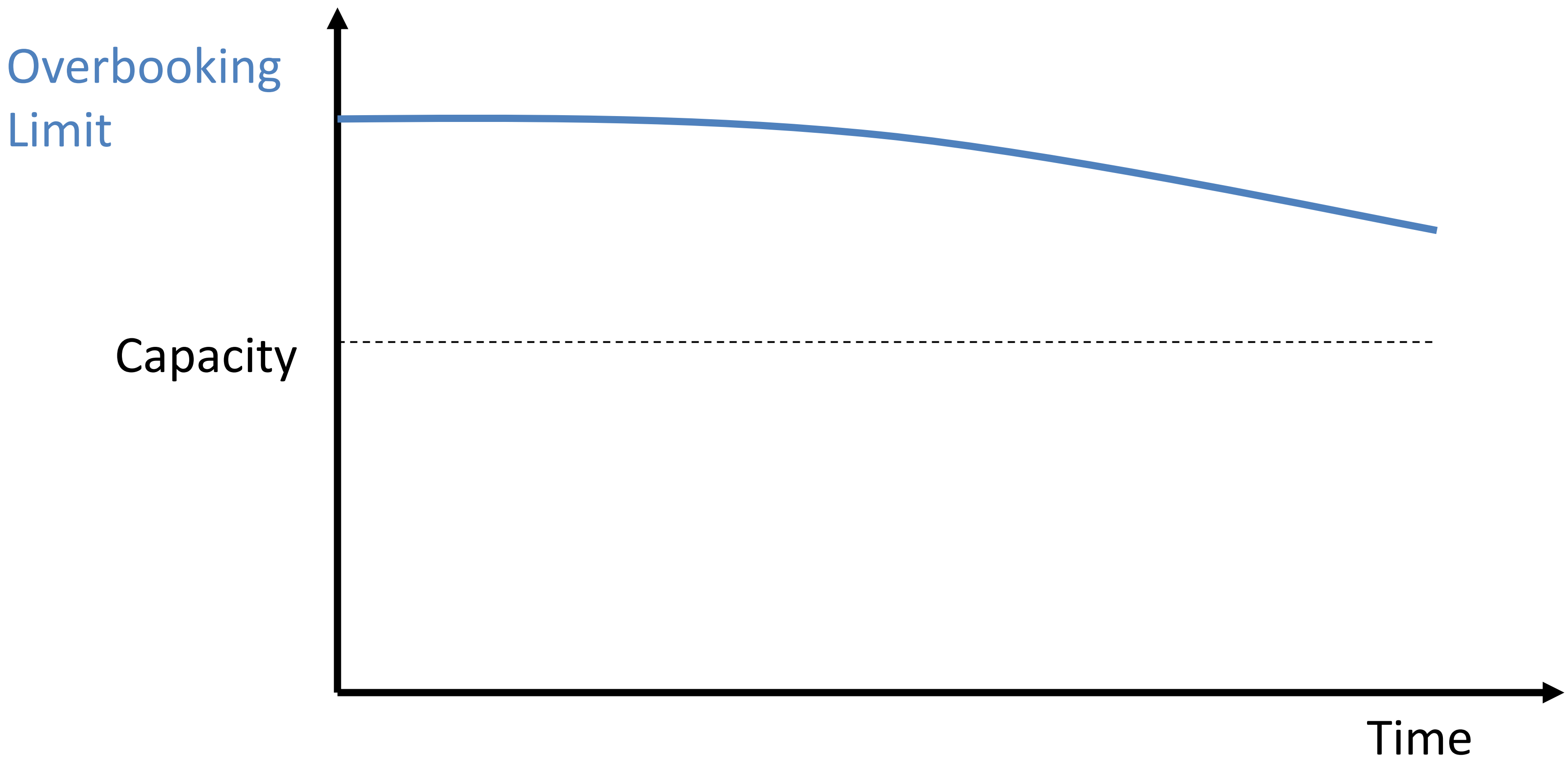
- Criterion – Type II service level: The fraction of customers denied service in the long run i.e. (Expected number of customers denied service / Expected number of customers )

- Criterion – Minimize Spillage and Spoilage costs

# Overbooking: Cancellation probabilities


Cancellation Probabilities remain constant over time

# Overbooking: Cancellation Probabilities




Cancellation Probabilities decreasing with time