Introduction to choice models

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Modeling behavior

- Individual behavior (vs. aggregate behavior)
- Theory of behavior which is
  - descriptive: how people behave and not how they should
  - abstract: not too specific
  - operational: can be used in practice for forecasting
- Type of behavior: choice

Motivations

Field :

- Marketing
- Transportation
- Politics
- Management
- New technologies

Type of behavior:

- Choice of a brand
- Choice of a transportation mode
- Choice of a president
- Choice of a management policy
- Choice of investments

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Importance

Daniel

L.

McFadden

1937–

- UC Berkeley 1963, MIT 1977, UC Berkeley 1991
- Laureate of *The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel 2000*
- Owns a farm and vineyard in Napa Valley
- “Farm work clears the mind, and the vineyard is a great place to prove theorems”

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Example

Travel Information System

- What is the market penetration?
- How will the penetration change in the future?
- Assumption: level of education is an important explanatory factor

Data collection

- sample of 600 persons, randomly selected
- Two questions:
  1. Do you subscribe to a travel information system? (yes/no)
  2. How many years of education have you had? (low/medium/high)

Example

- Contingency table

	Education			
TIS	Low	Medium	High	
Yes	10	100	120	230
No	140	200	30	370
	150	300	150	600

- Penetration in the sample: $230/600 = 38.3\%$
- Forecasting: need for a model

Example: a model

- Dependent variable:

$$y = \begin{cases} 1 & \text{if subscriber} \\ 2 & \text{if not subscriber} \end{cases}$$

Discrete dependent variable

- Independent or explanatory variable

$$x = \begin{cases} 1 & \text{if level of education is low} \\ 2 & \text{if level of education is medium} \\ 3 & \text{if level of education is high} \end{cases}$$

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Example: a model

- Market penetration in the sample: $\hat{p}(y=1)$
- Market penetration in the population: $p(y=1)$ estimated by $\hat{p}(y=1)$
- Joint probabilities: $\hat{p}(y=1,x=2) = 100/600 = 0.1667$
- Marginal probabilities: $\hat{p}(y=1) = \sum_{k=1}^{3} \hat{p}(1,k) = 10/600 + 100/600 + 120/600 = 0.383$
- Conditional probabilities: $\hat{p}(y=1|x=2)$

$$
\begin{aligned}
\hat{p}(y=1,x=2) &= \hat{p}(y=1|x=2)\hat{p}(x=2) \\
\hat{p}(y=1|x=2) &= \hat{p}(y=1,x=2)/\hat{p}(x=2) \\
&= 0.1667/0.5 = 0.333
\end{aligned}
$$

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Example: a model

Similarly, we obtain

$$\begin{aligned}\hat{p}(y=1|x=1) &= 0.067\\ \hat{p}(y=1|x=2) &= 0.333\\ \hat{p}(y=1|x=3) &= 0.8\end{aligned}$$

We obtain a causal relationship.

- Behavioral model: $\hat{p}(y=i|x=j)$
- Forecasting assumption: stable over time

Example: forecasting

- Model:

$$\begin{array}{rcccl} p(y=1|x=1) & = & \pi_1 & = & 0.067 \\ p(y=1|x=2) & = & \pi_2 & = & 0.333 \\ p(y=1|x=3) & = & \pi_3 & = & 0.8 \end{array}$$

  where π_1, π_2, π_3 are estimated parameters
- Assumption: future level of education: 10%-60%-30%

$$\begin{array}{rcl} p(y=1) & = & \sum_{i=1}^{3} p(y=1|x=i)p(x=i) \\ & = & 0.1\pi_1 + 0.6\pi_2 + 0.3\pi_3 \\ & = & 44.67\% \end{array}$$

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Example: forecasting

- If the level of education increases
- from 25%-50%-25% to 10%-60%-30%
- Market penetration of TIS will increase
- from 38.33 % to 44.67%

In summary

- $p(x = j)$ can be easily obtained and forecasted
- $p(y = i|x)$ is the behavioral model to be developed

Bibliography

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