



EXERCISE SESSION 6

Exercise 1 Consider a logit model for mode choice with three alternatives: car (c), public transportation (p) and slow modes (s). The utility specifications are the following:

$$\begin{aligned}U_{c,n} &= \beta_{c,1} + \beta_{c,2} \cdot \text{cost}_{c,n} + \beta_{c,3} \cdot \text{tt}_{c,n} + \varepsilon_{c,n}, \\U_{p,n} &= \beta_{p,1} + \beta_{p,2} \cdot \text{cost}_{p,n} + \beta_{p,3} \cdot \text{tt}_{p,n} + \varepsilon_{p,n}, \\U_{s,n} &= \beta_{s,1} + \beta_{s,3} \cdot \text{tt}_{s,n} + \varepsilon_{s,n},\end{aligned}$$

where $\text{cost}_{i,n}$ is the cost associated by customer n with alternative $i \in \{c, p\}$, and $\text{tt}_{i,n}$ is the travel time associated with alternative $i \in \{c, p, s\}$ by customer n . We denote by $\mathbb{E}[\varepsilon_{i,n}] = \alpha_i$ the mean of the distribution of the error terms $\varepsilon_{i,n} \forall i \in \{c, p, s\}, n$.

Show that it is possible to rewrite the utility functions in order to have $\mathbb{E}[\varepsilon_{i,n}] = 0 \forall i \in \{c, p, s\}, n$ and the same probabilities.

Exercise 2 Consider the logit model from the previous exercise and define the following additional specifications (use the model from exercise 1 as a base model for each new specification). For each specification, indicate if it is linear-in-parameters and if it is linear in the involved variables.

1. Propose a linear-in-parameters specification and a non linear-in-parameters specification that captures that the marginal effect of travel time in the utility varies with time.
2. Propose a specification that captures a variation in the sensitivity towards travel time for trips made by car when its length is classified as short ($\text{tt}_{c,n} \leq 20$ minutes), medium ($20 < \text{tt}_{c,n} \leq 60$ minutes) and long ($\text{tt}_{c,n} > 60$ minutes).
3. Define a dummy variable for individuals owning a travel card in order to propose a specification that assumes that owning a travel card might have an impact on the choice.
4. Assume that the variable age_n represents the age of individual n . Propose a specification that captures the fact that the travel time of slow modes varies continuously with the age.

Exercise 3 Answer to the following questions.

1. Describe the Independence from Irrelevant Alternatives (IIA) property. Under which circumstance is the IIA property violated? (There are different circumstances, please describe here the one discussed in the lecture.)
2. Recall the red bus/ blue bus paradox that has been seen in the lecture. Travelers initially face a decision between two modes of transportation: car and blue bus. The travel times of both modes, car and blue bus, are equal. Travel time is also the only variable considered in the utility. Then, we suppose that a third mode, namely the red bus, is introduced and that the travelers consider it to be exactly the same as the blue bus. Assume that the error terms for the red and blue bus are correlated and that the correlation is 95%. Derive the scale parameter (μ_m) and calculate the probabilities of choosing car and bus¹.

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¹Note that μ is normalized to one.