Forecasting – 7.3 Indicators

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Consumer surplus

The consumer surplus is the difference between what a consumer is willing to pay for a good and what she actually pays for the good. If the reader is not familiar with the concept of consumer surplus, we suggest to read a primer on the topic in a text book on microeconomics, such as Nicholson and Snyder (2007). The change in consumer surplus is often used to evaluate public policy decisions. For example, the impact on consumers of changing emissions regulations or increasing investments in the public transit system.

It is equal to the area under the demand curve and above the market price. In classical microeconomics, the demand curve gives the price of a good as a function of the quantity consumed. In discrete choice, the demand for individual n is characterized by the choice probability. Also, the role of price is taken by the utility of the good. In Figure 1 the choice probability is represented on the x-axis, and the y-axis represents the negative utility of alternative i, $-V_i$. The minus sign helps in obtaining the same interpretation as in classical microeconomic: going up the axis corresponds to a deterioration.

Simulating a future increase of the utility of item i (that is, a decrease of the quantity $-V_i$), while the utility of other alternatives is constant, the change in consumer surplus is represented by the filled area. For binary logit, this area can be calculated by the following integral, where the index n has been dropped to simplify the notations:

$$\int_{V_i^1}^{V_i^2} P(i|V_i, V_j) dV_i = \int_{V_i^1}^{V_i^2} \frac{e^{\mu V_i}}{e^{\mu V_i} + e^{\mu V_j}} dV_i \tag{1}$$

which is

$$\frac{1}{\mu} \ln(e^{\mu V_i^2} + e^{\mu V_j}) - \frac{1}{\mu} \ln(e^{\mu V_i^1} + e^{\mu V_j}). \tag{2}$$

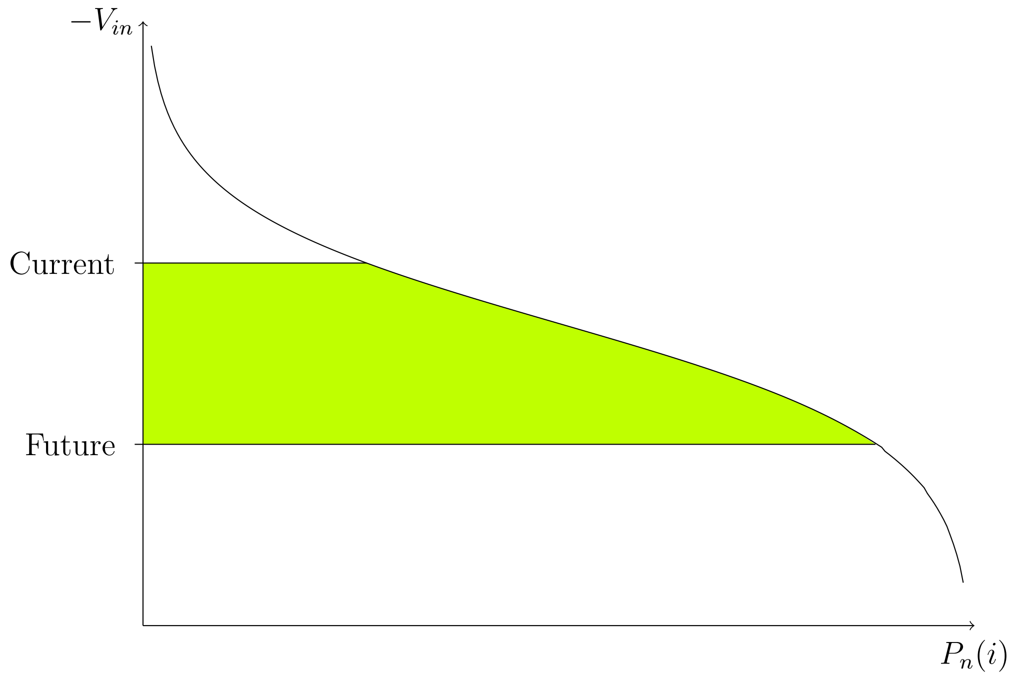

Figure 1: Illustration of the consumer surplus for binary logit

To generalize this result, we calculate the difference in an individual's consumer surplus between two situations corresponding to vectors of systematic utilities V^1 and V^2 as follows:

$$\sum_{i \in \mathcal{C}} \int_{V^1}^{V^2} P(i|V) dV_i, \tag{3}$$

where the choice probability is denoted as conditional on the vector V of systematic utilities in order to make the dependency explicit. Note that the term $P(i|V)dV_i$ corresponds to the filled area in Figure 1 where the change in the utility of i is infinitesimal. The difficulty here is that the utility of **all** alternatives are modified. Therefore, the integral in (3) is a line integral. And there are infinitely many ways to move from utility vector V^1 to utility vector V^2 in the J-dimensional space. This is illustrated for an example with two alternatives in Figures 2 and 3. In order to simplify the calculation of the integral, we consider paths that are updating each coordinate at a time. In Figure 2, the path moves first from $V^1 = (V_i^1, V_j^1)$ to (V_i^2, V_j^1), and then from (V_i^2, V_j^1) to $(V_i^2, V_j^2) = V^2$. The path in Figure 3 moves first from $V^1 = (V_i^1, V_j^1)$ to (V_i^1, V_j^2), and then from (V_i^1, V_j^2) to $(V_i^2, V_j^2) = V^2$.

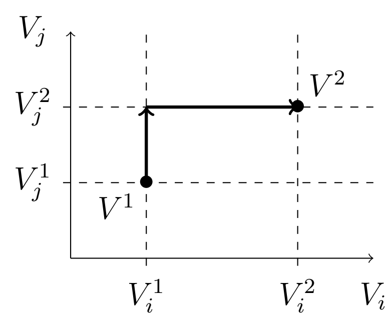

Figure 2: Moving from utility vector V_1 to utility vector V_2: first path

For the example with two alternatives, where the path in Figure 2 is followed, the integral (3) is

$$\int_{V_i^1}^{V_i^2} P(i|V_i, V_j^1) dV_i + \int_{V_j^1}^{V_j^2} P(j|V_i^2, V_j) dV_j. \tag{4}$$

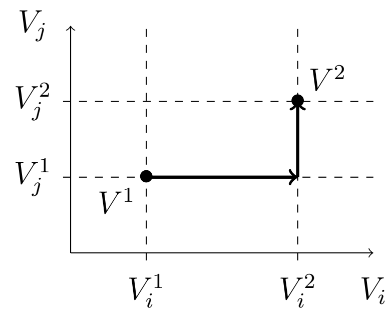

Figure 3: Moving from utility vector V_1 to utility vector V_2: second path

Assuming now a binary logit model, the first integral is

$$\int_{V_i^1}^{V_i^2} \frac{e^{\mu V_i}}{e^{\mu V_i} + e^{\mu V_j^1}} dV_i. \tag{5}$$

Let $t = e^{\mu V_i} + e^{\mu V_j^1}$ so that $dt = \mu e^{\mu V_i} dV_i$, we obtain

$$\frac{1}{\mu} \int_{e^{\mu V_i^1} + e^{\mu V_j^1}}^{e^{\mu V_i^2} + e^{\mu V_j^1}} \frac{dt}{t} = \frac{1}{\mu} \ln(e^{\mu V_i^2} + e^{\mu V_j^1}) - \frac{1}{\mu} \ln(e^{\mu V_i^1} + e^{\mu V_j^1}). \tag{6}$$

The second integral is

$$\int_{V_j^1}^{V_j^2} \frac{e^{\mu V_j}}{e^{\mu V_i^2} + e^{\mu V_j}} dV_j. \tag{7}$$

Let $t = e^{\mu V_i^2} + e^{\mu V_j}$ so that $dt = \mu e^{\mu V_j} dV_j$, we obtain

$$\frac{1}{\mu} \int_{e^{\mu V_i^2} + e^{\mu V_j^1}}^{e^{\mu V_i^2} + e^{\mu V_j^2}} \frac{dt}{t} = \frac{1}{\mu} \ln(e^{\mu V_i^2} + e^{\mu V_j^2}) - \ln(e^{\mu V_i^2} + e^{\mu V_j^1}). \tag{8}$$

Adding (6) and (8), we obtain the difference of logsum, that is

$$\frac{1}{\mu} \ln(e^{\mu V_i^2} + e^{\mu V_j^2}) - \frac{1}{\mu} \ln(e^{\mu V_i^1} + e^{\mu V_j^1}). \tag{9}$$

Calculating the integral following the path described in Figure 3 leads to the exact same result. We say that the calculation of the integral is *path* *independent*. Not all line integrals are path independent. If the choice model happens to have equal cross-derivatives, that is

$$\frac{\partial P(i|V,\mathcal{C})}{\partial V_j} = \frac{\partial P(j|V,\mathcal{C})}{\partial V_i}, \ \forall i, j \in \mathcal{C}, \tag{10}$$

the calculation of the integral in (3) is path independent.

The logit model, as well as several choice models used in practice, have this property. Therefore, for logit, we can select the path of integration. We calculate (3) using a path that first updates the utility of alternative 1, then of alternative 2, and so on until J. The kth term integrates over V_k, with all utilities of alternatives 1 to $k-1$ set at the level V^2, while all utilities of alternatives $k+1$ to J are set at the level V^1. This term writes

$$\begin{aligned}&\int_{V_k^1}^{V_k^2} \frac{e^{\mu V_k}}{\sum_{j=1}^{k-1} e^{\mu V_j^2} + e^{\mu V_k} + \sum_{j=k+1}^{J} e^{\mu V_j^1}} dV_k = \\ &\frac{1}{\mu}\ln\left(\sum_{j=1}^{k-1} e^{\mu V_j^2} + e^{\mu V_k^2} + \sum_{j=k+1}^{J} e^{\mu V_j^1}\right) - \frac{1}{\mu}\ln\left(\sum_{j=1}^{k-1} e^{\mu V_j^2} + e^{\mu V_k^1} + \sum_{j=k+1}^{J} e^{\mu V_j^1}\right).\end{aligned} \tag{11}$$

When summing up over k, most terms of the sum over alternatives cancel out, and the difference of consumer surplus for the logit model is

$$\frac{1}{\mu}\ln\sum_{j\in\mathcal{C}} e^{\mu V_j^2} - \frac{1}{\mu}\ln\sum_{j\in\mathcal{C}} e^{\mu V_j^1}. \tag{12}$$

which is the difference among expected maximum utilities in the two situations. When the choice set changes from $\mathcal{C}^1$ to $\mathcal{C}^2$, the result of (3) is

$$\frac{1}{\mu}\ln\sum_{j\in\mathcal{C}^2} e^{\mu V_j^2} - \frac{1}{\mu}\ln\sum_{j\in\mathcal{C}^1} e^{\mu V_j^1}. \tag{13}$$

We refer the reader to Neuburger (1971), Small and Rosen (1981), Hanemann (1984), McConnell (1995), Dagsvik and Karlström (2005) for more detailed discussions about consumer surplus.

References

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