Forecasting – 7.3 Indicators

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Solution of the practice quiz

Consider the sample enumeration estimator of the market share of alternative i in the population

$$\widehat{W}(i) = \frac{1}{S}\sum_{n=1}^{S}\omega_n P_n(i|x_n;\theta). \tag{1}$$

For continuous variables, we assume that the relative change of the variable is the same for every individual in the population. For arc elasticities, we have

$$\frac{\Delta x_{ink}}{x_{ink}} = \frac{\Delta x_{ipk}}{x_{ipk}} = \frac{\Delta x_{ik}}{x_{ik}}, \tag{2}$$

and for point elasticities, we have an infinitesimal change, that is

$$\frac{\partial x_{ink}}{x_{ink}} = \frac{\partial x_{ipk}}{x_{ipk}} = \frac{\partial x_{ik}}{x_{ik}}, \tag{3}$$

where

$$x_{ik} = \frac{1}{S}\sum_{n=1}^{S} x_{ink}. \tag{4}$$

1. The aggregate direct arc elasticity of the model with respect to the average value x_{ik} is defined as

$$E_{x_{ik}}^{\widehat{W}(i)} = \frac{\Delta\widehat{W}(i)}{\Delta x_{ik}}\frac{x_{ik}}{\widehat{W}(i)}. \tag{5}$$

Using (1), we can calculate $\Delta\widehat{W}(i)$ and we obtain

$$E_{x_{ik}}^{\widehat{W}(i)} = \frac{1}{S}\sum_{n=1}^{S} w_n \frac{\Delta P_n(i|x_n,\mathcal{C}_n)}{\Delta x_{ik}}\frac{x_{ik}}{\widehat{W}(i)}. \tag{6}$$

We can now replace the term $\frac{\Delta x_{ik}}{x_{ik}}$ by $\frac{\Delta x_{ink}}{x_{ink}}$ (see (2))

$$E_{x_{ik}}^{\widehat{W}(i)} = \frac{1}{S}\sum_{n=1}^{S} w_n \frac{\Delta P_n(i|x_n, \mathcal{C}_n)}{\Delta x_{ink}} \frac{x_{ink}}{\widehat{W}(i)}. \tag{7}$$

By introducing $\frac{P_n(i|x_n,\mathcal{C}_n)}{P_n(i|x_n,\mathcal{C}_n)}$, we obtain the definition of the disaggregate direct arc elasticity:

$$\begin{aligned} E_{x_{ik}}^{\widehat{W}(i)} &= \frac{1}{S}\sum_{n=1}^{S} w_n \frac{\Delta P_n(i|x_n, \mathcal{C}_n)}{\Delta x_{ink}} \frac{x_{ink}}{P_n(i|x_n, \mathcal{C}_n)} \frac{P_n(i|x_n, \mathcal{C}_n)}{\widehat{W}(i)} && (8) \\ &= \frac{1}{S}\sum_{n=1}^{S} w_n E_{x_{ink}}^{P_n(i)} \frac{P_n(i|x_n, \mathcal{C}_n)}{\widehat{W}(i)}. && (9) \end{aligned}$$

Finally, applying (1) again we obtain

$$E_{x_{ik}}^{\widehat{W}(i)} = \sum_{n=1}^{S} E_{x_{ink}}^{P_n(i)} \frac{w_n P_n(i|x_n, \mathcal{C}_n)}{\sum_{n=1}^{S} w_n P_n(i|x_n, \mathcal{C}_n)}. \tag{10}$$

This equation shows that the calculation of aggregate elasticities involves a weighted sum of disaggregate elasticities. However, the weight is not w_n as for the market share, but a normalized version of

$$w_n P_n(i|x_n, \mathcal{C}_n).$$

2. The derivation follows the same logic. The aggregate cross point elasticity of the model with respect to the average value x_{jk} is defined as

$$E_{x_{jk}}^{\widehat{W}(i)} = \frac{\partial \widehat{W}(i)}{\partial x_{jk}} \frac{x_{jk}}{\widehat{W}(i)}. \tag{11}$$

Using (1), we can calculate $\partial \widehat{W}(i)$ and we obtain

$$E_{x_{jk}}^{\widehat{W}(i)} = \frac{1}{S}\sum_{n=1}^{S} w_n \frac{\partial P_n(i|x_n, \mathcal{C}_n)}{\partial x_{jk}} \frac{x_{jk}}{\widehat{W}(i)}. \tag{12}$$

We can now replace the term $\frac{\partial x_{jk}}{x_{jk}}$ by $\frac{\partial x_{jnk}}{x_{jnk}}$ (see (3))

$$E_{x_{jk}}^{\widehat{W}(i)} = \frac{1}{S}\sum_{n=1}^{S} w_n \frac{\partial P_n(i|x_n, \mathcal{C}_n)}{\partial x_{jnk}} \frac{x_{jnk}}{\widehat{W}(i)}. \tag{13}$$

By introducing $\frac{P_n(i|x_n,\mathcal{C}_n)}{P_n(i|x_n,\mathcal{C}_n)}$, we obtain the definition of the disaggregate cross point elasticity:

$$E_{x_{jk}}^{\widehat{W}(i)} = \frac{1}{S}\sum_{n=1}^{S} w_n \frac{\partial P_n(i|x_n,\mathcal{C}_n)}{\partial x_{jnk}} \frac{x_{jnk}}{P_n(i|x_n,\mathcal{C}_n)} \frac{P_n(i|x_n,\mathcal{C}_n)}{\widehat{W}(i)} \tag{14}$$

$$= \frac{1}{S}\sum_{n=1}^{S} w_n E_{x_{jnk}}^{P_n(i)} \frac{P_n(i|x_n,\mathcal{C}_n)}{\widehat{W}(i)} \tag{15}$$

Finally, applying (1) again we obtain

$$E_{x_{jk}}^{\widehat{W}(i)} = \sum_{n=1}^{S} E_{x_{jnk}}^{P_n(i)} \frac{w_n P_n(i|x_n,\mathcal{C}_n)}{\sum_{n=1}^{S} w_n P_n(i|x_n,\mathcal{C}_n)}. \tag{16}$$