Testing – 6.5 Non nested hypotheses

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Davidson and McKinnon J test

A disadvantage of the Cox test is the need to estimate a model with a potentially very large number of parameters. We now describe the J test developed by Davidson and MacKinnon (1981) which is a general solution to the selection between two non-nested models. The J test is in general preferred to the Cox test. As we will see, it is also subject to the same four outcomes.

This is a general treatment based on generating artificial regressions that embed two competing non nested model formulations to explain a given dependent variable. Consider two specifications:

$$M_1 : U_{in} = V_{in}^{(1)}(x_{in}; \beta) + \varepsilon_{in}^{(1)}, \tag{1}$$

$$M_2 : U_{in} = V_{in}^{(2)}(x_{in}; \gamma) + \varepsilon_{in}^{(2)}. \tag{2}$$

To choose between model 1 in Equation (1) and model 2 in Equation (2), we consider the following composite specification:

$$M_C : U_{in} = (1-\alpha)V_{in}^{(1)}(x_{in}; \beta) + \alpha V_{in}^{(2)}(x_{in}; \gamma) + \varepsilon_{in}. \tag{3}$$

Intuitively, the idea is to test the competing models against the composite model in equation (3). Note that if $\alpha = 0$, the model collapses to the model M_1 while with $\alpha = 1$, the composite model collapses to the model M_2. The major problem is that very often, the composite model cannot be estimated. Namely, there may be exact multicollinearity among the explanatory variables. Moreover, the α coefficient may not be identified.

The J test solution to this problem is to replace the unknown parameters not being tested by consistent estimates. In order to test M_1, one could consider the following composite model:

$$M_C : U_{in} = (1-\alpha)V_{in}^{(1)}(x_{in}; \beta) + \alpha V_{in}^{(2)}(x_{in}; \widehat{\gamma}) + \varepsilon_{in}, \tag{4}$$

where model 2 in Equation (2) has been previously estimated, and $\widehat{\gamma}$ is the vector of estimates. Thus, $V_{in}^{(2)}(x_{in}; \widehat{\gamma})$ corresponds to the fitted systematic utility of model 2 and represents in this artificial model a single variable associated with the parameter α. Under the null hypothesis that model 1 is correct, the true value of α in the composite model is 0. The objective is then to test if $\alpha = 0$ using a t test. This would involve estimating model 1 with the additional variable computed as $V_{in}^{(2)}(x_{in}; \widehat{\gamma})$.

In order to test M_2, one could instead consider the following composite model:

$$M_C : U_{in} = (1-\alpha) V_{in}^{(1)}(x_{in}; \widehat{\beta}) + \alpha V_{in}^{(2)}(x_{in}; \gamma) + \varepsilon_{in}. \tag{5}$$

where model 1 in Equation (1) has been previously estimated. Thus, $V_{in}^{(1)}(x_{in}; \widehat{\beta})$ corresponds to the fitted systematic utility of model 1 and represents in this artificial model a single variable associated with the parameter $(1-\alpha)$. Under the null hypothesis that model 2 is correct, the true value of α is 1. The objective is then to test if $\alpha = 1$ using a t test.

The J test has the same four outcomes presented for the Cox test, that is

- M_1 is rejected and M_2 is not rejected. Then, it is reasonable to prefer model 2.
- M_1 is not rejected and M_2 is rejected. Then it is reasonable to prefer model 1.
- M_1 and M_2 are rejected. This indicates that better models should be developed.
- Neither M_1 nor M_2 can be rejected. Then, in this case, the data does not seem to be informative enough to distinguish between the two competing models, and the $\bar{\rho}^2$ should be used.

It should now be clear that one of the two models does not have to represent the truth. Both models could be unsatisfactory.

References

Davidson, R. and MacKinnon, J. (1981). Several tests for model specification in the presence of alternative hypotheses, *Econometrica: Journal of the Econometric Society* pp. 781–793.