Testing – 6.3 t-tests

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Solution to the practice quiz

The estimation results are summarized in Table 1.

Parameter number	Description	Coeff. estimate	Robust Asympt. std. error	t-stat	p-value
1	One stop–same airline dummy	-1.17	0.278	-4.19	0.00
2	One stop–multiple airlines dummy	-1.45	0.292	-4.98	0.00
3	**Elapsed time (hours) (non stop)**	-0.341	0.0854	-3.99	0.00
4	**Elapsed time (hours) (one stop–same airline)**	-0.291	0.0822	-3.54	0.00
5	**Elapsed time (hours) (one stop–multiple airlines)**	-0.310	0.0802	-3.87	0.00
6	Round trip fare ($100)	-1.78	0.151	-11.84	0.00
7	Leg room (inches), if male	0.108	0.0232	4.65	0.00
8	Leg room (inches), if female	0.132	0.0221	5.99	0.00
9	Being early (hours)	-0.151	0.0188	-8.02	0.00
10	Being late (hours)	-0.0960	0.0167	-5.73	0.00
11	More than 2 air trips per year (one stop–same airline)	-0.307	0.141	-2.18	0.03
12	More than 2 air trips per year (one stop–multiple airlines)	-0.0910	0.157	-0.58	0.56
13	Male dummy (one stop–same airline)	0.199	0.126	1.59	0.11
14	Male dummy (one stop–multiple airlines)	0.293	0.132	2.21	0.03
15	Round trip fare / income ($100/$1000)	-24.0	8.09	-2.97	0.00

Summary statistics
Number of observations = 2544

$$
\begin{aligned}
\mathcal{L}(0) &= -2794.870 \\
\mathcal{L}(c) &= -2203.160 \\
\mathcal{L}(\widehat{\beta}) &= -1641.932 \\
-2[\mathcal{L}(0) - \mathcal{L}(\widehat{\beta})] &= 2305.875 \\
\rho^2 &= 0.413 \\
\bar{\rho}^2 &= 0.407
\end{aligned}
$$

Table 1: Estimation results

1. Testing the null hypothesis that the true value of the coefficient of the variable “being early” is zero requires a t-test. The t statistic of parameter number 9 in Table 1 is -8.02 which is larger in absolute value than 2.56, so the null hypothesis can be rejected at the 1% level. Actually, the fact that the p value is so small that the two first digits

after the decimal point are zero, is a sign that the hypothesis can be safely rejected at any reasonable level. The variable plays a role in the model.

2. The next three questions require a t-test to compare two coefficients β_i and β_j. The null hypothesis is that both parameters are equal ($H_0 : \beta_i = \beta_j$) and the t-statistic is given by

$$\frac{\widehat{\beta}_i - \widehat{\beta}_j}{\sqrt{\mathrm{Var}(\widehat{\beta}_i - \widehat{\beta}_j)}}$$

where

$$\mathrm{Var}(\widehat{\beta}_i - \widehat{\beta}_j) = \mathrm{Var}(\widehat{\beta}_i) + \mathrm{Var}(\widehat{\beta}_j) - 2\mathrm{Cov}(\widehat{\beta}_i, \widehat{\beta}_j).$$

The variance of a parameter is the square of its standard error. The complete variance-covariance matrix can be found in `v634_Boeing_M0.html`. It is reported in Table 2 for the involved coefficients.

	β_3	β_4	β_5
β_3	0.00729	0.00627	0.006
β_4	0.00627	0.00676	0.00553
β_5	0.006	0.00553	0.00643

Table 2: Variance covariance matrix for the involved coefficients

The three t-tests are applied below.
$H_0 : \beta_3 = \beta_4$

$$\frac{\widehat{\beta}_3 - \widehat{\beta}_4}{\sqrt{\mathrm{Var}(\widehat{\beta}_3 - \widehat{\beta}_4)}} = \frac{-0.341 - (-0.291)}{\sqrt{0.00729 + 0.00676 - 2 \times 0.00627}} = -1.287,$$

and the p-value is 0.2. The null hypothesis can be rejected only at the 20% level. It is therefore reasonable not to reject it.

3. $H_0 : \beta_4 = \beta_5$

$$\frac{\widehat{\beta}_4 - \widehat{\beta}_5}{\sqrt{\mathrm{Var}(\widehat{\beta}_4 - \widehat{\beta}_5)}} = \frac{-0.291 - (-0.310)}{\sqrt{0.00676 + 0.00643 - 2 \times 0.00553}} = 0.412,$$

and the p-value is 0.68. The null hypothesis can be rejected only at the 68% level. It is therefore reasonable not to reject it.

4. $H_0 : \beta_3 = \beta_5$

$$\frac{\widehat{\beta}_3 - \widehat{\beta}_5}{\sqrt{\mathrm{Var}(\widehat{\beta}_3 - \widehat{\beta}_5)}} = \frac{-0.341 - (-0.310)}{\sqrt{0.00729 + 0.00643 - 2 \times 0.006}} = -0.747,$$

   and the p-value is 0.46. The null hypothesis can be rejected only at the 46% level. It is therefore reasonable not to reject it.

In conclusion, we have no evidence from the data that suggests that the elapsed time is not generic. Consequently, in such a circumstances, it may be worth investigating a model with a generic elapsed time, that will be more parsimonious.