Testing

t-tests

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Introduction to choice models

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Usage of the t-tests

t-test

Question

Is the parameter θ significantly different from a given value θ^*?

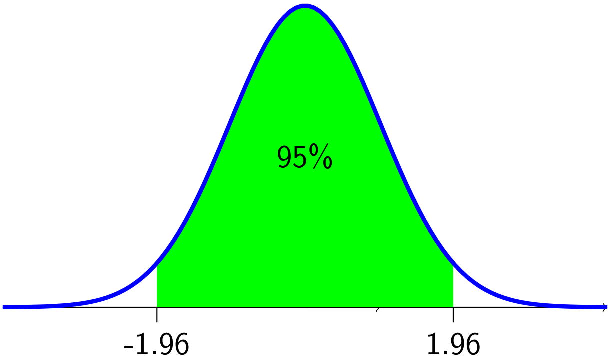
- $H_0 : \theta = \theta^*$
- $H_1 : \theta \neq \theta^*$

Statistic (assuming maximum likelihood estimator)

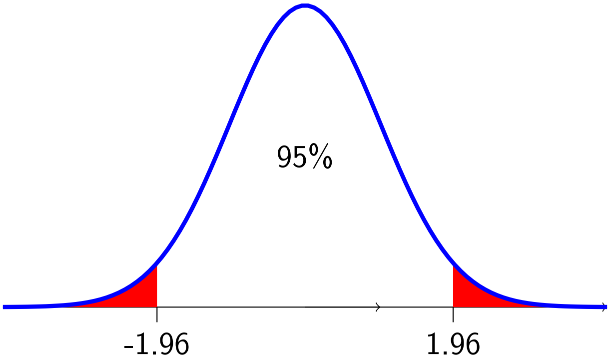
Under H_0, if $\hat{\theta}$ is normally distributed with known variance σ^2

$$\frac{\hat{\theta} - \theta^*}{\sigma} \sim N(0, 1).$$

t-test: under H_0



t-test: if the statistic lies outside



H_0 is rejected at the 5% level.

Applying the test

Statistic

$$P(-1.96 \leq \frac{\hat{\theta} - \theta^*}{\sigma} \leq 1.96) = 0.95 = 1 - 0.05$$

Decision

H_0 can be rejected at the 5% level ($\alpha = 0.05$) if

$$\left| \frac{\hat{\theta} - \theta^*}{\sigma} \right| \geq 1.96.$$

Comments

- If $\hat{\theta}$ asymptotically normal
- If variance unknown
- A t test should be used with N degrees of freedom.
- When $N \geq 30$, the Student t distribution is well approximated by a $N(0,1)$

p value

- probability to get a t statistic at least as large (in absolute value) as the one reported, under the null hypothesis
- it is calculated as
$$p = 2(1 - \Phi(t))$$
where $\Phi(\cdot)$ is the CDF of the standard normal.
- the null hypothesis is rejected when the p-value is lower than the significance level (typically 0.05)

Comparing two coefficients

Hypothesis

$$H_0 : \beta_1 = \beta_2.$$

Statistic

$$\frac{\widehat{\beta}_1 - \widehat{\beta}_2}{\sqrt{\mathrm{Var}(\widehat{\beta}_1 - \widehat{\beta}_2)}}$$

where

$$\mathrm{Var}(\widehat{\beta}_1 - \widehat{\beta}_2) = \mathrm{Var}(\widehat{\beta}_1) + \mathrm{Var}(\widehat{\beta}_2) - 2\,\mathrm{Cov}(\widehat{\beta}_1, \widehat{\beta}_2)$$

Distribution

Under H_0, distributed as $N(0, 1)$.