Choice with multiple alternatives

Specification of the deterministic part

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Introduction to choice models

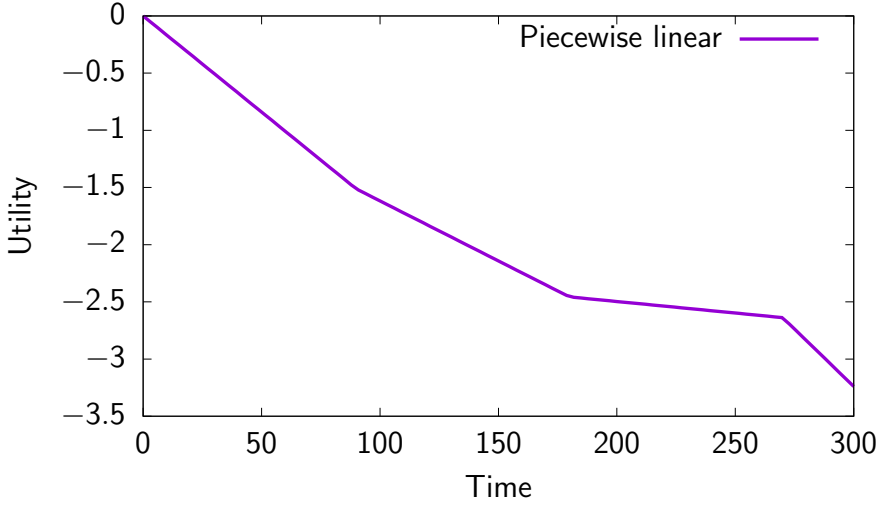
Nonlinear specifications: piecewise linear specification

Piecewise linear specification

Again: sensitivity to travel time varies with travel time

- Log transform is not the only specification
- Another possibility: split the range of values of the variable
  - Short trips: 0–90 min.
  - Medium strips: 90–180 min.
  - Long trips: 180–270 min.
  - Very long trips: 270 min. and more
- Each category is associated with a different coefficient.

Piecewise linear specification

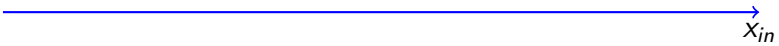


Piecewise linear specification

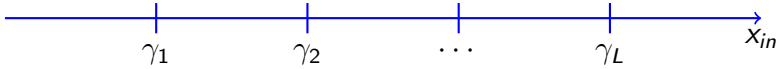
Procedure

- Select breakpoints $\gamma_1 < \gamma_2 < \ldots < \gamma_L$
- Define new variables

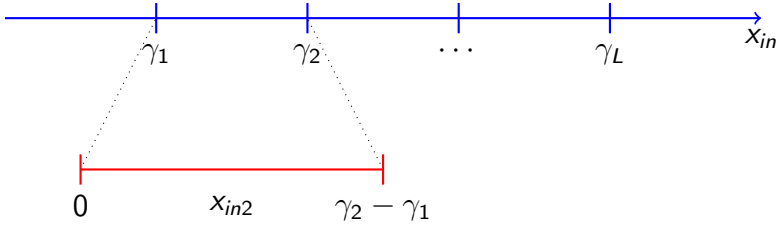
Piecewise linear specification



Piecewise linear specification



Piecewise linear specification



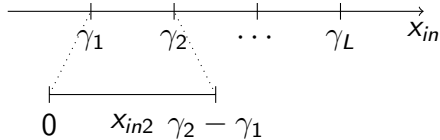
Piecewise linear specification

Formulation

$$x_{in1} = \begin{cases} x_{in} & \text{if } x_{in} < \gamma_1 \\ \gamma_1 & \text{otherwise} \end{cases}$$

$$x_{in\ell} = \begin{cases} 0 & \text{if } x_{in} < \gamma_{\ell-1} \\ x_{in} - \gamma_{\ell-1} & \text{if } \gamma_{\ell-1} \leq x_{in} < \gamma_{\ell} \\ \gamma_{\ell} - \gamma_{\ell-1} & \text{otherwise} \end{cases}$$

$$x_{inL} = \begin{cases} 0 & \text{if } x_{in} < \gamma_L \\ x_{in} - \gamma_L & \text{otherwise} \end{cases}$$



Piecewise linear specification

Equivalent formulations

$$x_{in1} = \begin{cases} x_{in} & \text{if } x_{in} < \gamma_1 \\ \gamma_1 & \text{otherwise} \end{cases} \qquad x_{in1} = \min(x_{in}, \gamma_1)$$

$$x_{in\ell} = \begin{cases} 0 & \text{if } x_{in} < \gamma_{\ell-1} \\ x_{in} - \gamma_{\ell-1} & \text{if } \gamma_{\ell-1} \leq x_{in} < \gamma_\ell \\ \gamma_\ell - \gamma_{\ell-1} & \text{otherwise} \end{cases} \qquad x_{in\ell} = \max(0, \min(x_{in} - \gamma_{\ell-1}, \gamma_\ell - \gamma_{\ell-1}))$$

$$x_{inL} = \begin{cases} 0 & \text{if } x_{in} < \gamma_L \\ x_{in} - \gamma_L & \text{otherwise} \end{cases} \qquad x_{inL} = \max(0, x_{in} - \gamma_L)$$

Piecewise linear specification

Examples

$\gamma_1 = 90$, $\gamma_2 = 180$, $\gamma_3 = 270$.

x_{in}	50	100	200	300
x_{in1}	50	90	90	90
x_{in2}	0	10	90	90
x_{in3}	0	0	20	90
x_{in4}	0	0	0	30

$\gamma_1 = 1$, $\gamma_2 = 5$, $\gamma_3 = 10$.

x_{in}	0.5	4	8	12
x_{in1}	0.5	1	1	1
x_{in2}	0	3	4	4
x_{in3}	0	0	3	5
x_{in4}	0	0	0	2

Utility function

$$V_{in} = \sum_{\ell=1}^{L} \beta_\ell x_{in\ell}$$

Box-Cox transforms

Box and Cox (1964)

$$V_{in} = \beta x_{in}(\lambda) + \cdots$$

where

$$x_{in}(\lambda) = \begin{cases} \dfrac{x_{in}^{\lambda} - 1}{\lambda} & \text{if } \lambda \neq 0 \\ \ln x_{in} & \text{if } \lambda = 0. \end{cases}$$

and $x_{in} > 0$.

Box-Cox transforms