Choice with multiple alternatives

Specification of the deterministic part

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Introduction to choice models

Quantitative explanatory variables

Quantitative attributes

Numerical and continuous

- $(z_{in})_k \in \mathbb{R}, \forall i, n, k$
- Associated with a specific unit
- Vary across both i and n.

Examples

- Auto in-vehicle time (in min.)
- Transit in-vehicle time (in min.)
- Auto out-of-pocket cost (in cents)
- Transit fare (in cents)
- Walking time to the bus stop (in min.)

Straightforward modeling

Quantitative attributes

- V_{in} is unitless
- Therefore, β depends on the unit of the associated attribute
- Example: consider two specifications

$$\begin{aligned} V_{in} &= \beta_1 \mathrm{TT}_{in} + \cdots \\ V_{in} &= \beta'_1 \mathrm{TT}'_{in} + \cdots \end{aligned}$$

- If TT_{in} is a number of minutes, the unit of β_1 is $1/\mathrm{min}$
- If TT'_{in} is a number of hours, the unit of β'_1 is $1/\mathrm{hour}$
- Both models are equivalent, but the estimated value of the coefficient will be different

$$\beta_1 \mathrm{TT}_{in} = \beta'_1 \mathrm{TT}'_{in} \Longrightarrow \frac{\mathrm{TT}_{in}}{\mathrm{TT}'_{in}} = \frac{\beta'_1}{\beta_1} = 60$$

Quantitative attributes

Generic vs alternative specific

$$\begin{aligned} V_{in} &= \beta_1 \mathrm{TT}_{in} + \cdots \\ V_{jn} &= \beta_1 \mathrm{TT}_{jn} + \cdots \end{aligned}$$

or

$$\begin{aligned} V_{in} &= \beta_1 \mathrm{TT}_{in} + \cdots \\ V_{jn} &= \beta_2 \mathrm{TT}_{jn} + \cdots \end{aligned}$$

Modeling assumption: a minute has/has not the same marginal utility whether it is incurred on the auto or bus mode

Quantitative socio-eco. characteristics

Numerical and continuous

- $(S_n)_k \in \mathbb{R}$, $\forall n, k$
- Associated with a specific unit
- Vary only across n, not across i.

Examples

- Annual income (in KCHF)
- Age (in years)

Modeling heterogeneity

Behavioral assumption

- Individuals have different taste parameters.
- The difference is explained by one socio-economic characteristic.

$$V_{in} = \beta_{1n} z_{in} + \cdots$$

where

$$\beta_{1n} = \beta_{1n}(\text{income}_n).$$

Modeling heterogeneity

Interaction

Typical definition of β_{1n}:

$$\beta_{1n} = \beta_1 \text{income}_n$$

$$V_{in} = \beta_{1n} z_{in} + \cdots = \beta_1 \text{income}_n z_{in} + \cdots = \beta_1 x_{in} + \cdots$$

where

$$x_{in} = \text{income}_n z_{in}$$

Modeling heterogeneity

Behavioral assumption

- Individuals have different taste parameters.
- The difference is explained by several socio-economic characteristics.

$$V_{in} = \beta_{1n} z_{in} + \cdots$$

where

$$\beta_{1n} = \beta_{1n}(\text{income}_n, \text{age}_n).$$

Modeling heterogeneity

Interaction

Typical definition of β_{1n}:

$$\beta_{1n} = \beta_1 \text{income}_n \text{age}_n$$

$$V_{in} = \beta_{1n} z_{in} + \cdots = \beta_1 \text{income}_n \text{age}_n z_{in} + \cdots = \beta_1 x_{in} + \cdots$$

where

$$x_{in} = \text{income}_n \text{age}_n z_{in}$$

Modeling heterogeneity

Creativity and relevance

- Several functional forms can be investigated.
- For instance, if z_{in} is the cost variables, we write

$$\beta_{cn} = \beta_c / \text{income}_n$$

- Indeed, in this case, the new variable can be interpreted as the share of the income dedicated to this purchase:

$$x_{in} = z_{in} / \text{income}_n$$

Modeling heterogeneity: alternative specific constants

ASCs can also vary across individuals

Base model

$$\begin{aligned} V_{1n} &= \beta_x x_{1n1} + \beta_1 + \cdots \\ V_{2n} &= \beta_x x_{2n1} + \beta_2 + \cdots \\ V_{3n} &= \beta_x x_{3n1} + \cdots \end{aligned}$$

Heterogeneous specification

$$\begin{aligned} V_{1n} &= \beta_x x_{1n1} + \beta_{1n} + \cdots \\ V_{2n} &= \beta_x x_{2n1} + \beta_{2n} + \cdots \\ V_{3n} &= \beta_x x_{3n1} + \cdots \end{aligned}$$

where

$$\beta_{in} = \beta_i \text{income}_n$$

Modeling heterogeneity: alternative specific constants

Heterogeneous specification

$$\begin{aligned} V_{1n} &= \beta_x x_{1n1} + \beta_{1n} + \cdots \\ V_{2n} &= \beta_x x_{2n1} + \beta_{2n} + \cdots \\ V_{3n} &= \beta_x x_{3n1} + \cdots \end{aligned}$$

where

$$\beta_{in} = \beta_i \text{income}_n$$

$$\begin{aligned} V_{1n} &= \beta_x x_{1n1} + \beta_1 \text{income}_n + \cdots \\ V_{2n} &= \beta_x x_{2n1} + \beta_2 \text{income}_n + \cdots \\ V_{3n} &= \beta_x x_{3n1} + \cdots \end{aligned}$$