Binary choice – 3.2 Apply the model on data

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Solution of the practice quiz.

In order to complete the table, we have to remember that only the difference between the constants can be identified. This difference should be the same for any normalization. As $ASC_{\text{bicycle}} - ASC_{\text{metro}} = -3$ with the first normalization, it has to be the same for the second one. Therefore, $ASC_{\text{bicycle}} = -3$. The normalization of the constants has no impact on the coefficients of the attributes. Therefore, the β parameters remain unchanged. The result is:

Parameters	Normalization 1	Normalization 2
ASC_{bicycle}	0	-3
ASC_{metro}	3	0
β_{distance}	-0.8	-0.8
β_{time}	-0.5	-0.5
β_{cost}	-1	-1

Now, in order to calculate the choice probabilities with various models, we need first to calculate the utility functions for the scenario that is considered. We have, for the first normalization,

$$\begin{aligned} V_{\text{bicycle}} &= 0 - 0.8 \cdot 10 = -8, \\ V_{\text{metro}} &= 3 - 0.5 \cdot 20 - 1 \cdot 2.2 = -9.2. \end{aligned}$$

And for the second one,

$$\begin{aligned} V_{\text{bicycle}} &= -3 - 0.8 \cdot 10 = -11, \\ V_{\text{metro}} &= 0 - 0.5 \cdot 20 - 1 \cdot 2.2 = -12.2. \end{aligned}$$

1. The logit model with the parameters of normalization 1:

$$P(\text{bicycle}) = \frac{\exp(V_{\text{bicycle}})}{\exp(V_{\text{bicycle}}) + \exp(V_{\text{metro}})} = \frac{\exp(-8)}{\exp(-8) + \exp(-9.2)} = 0.77$$

and

$$P(\text{metro}) = \frac{\exp(V_{\text{metro}})}{\exp(V_{\text{bicycle}}) + \exp(V_{\text{metro}})} = \frac{\exp(-9.2)}{\exp(-8) + \exp(-9.2)} = 0.23.$$

2. The probit model with the parameters of normalization 1:

$$P(\text{bicycle}) = \Phi(V_{\text{bicycle}} - V_{\text{metro}}) = \Phi(-8 + 9.2) = \Phi(1.2) = 0.88$$

and

$$P(\text{metro}) = \Phi(V_{\text{metro}} - V_{\text{bicycle}}) = \Phi(-9.2 + 8) = \Phi(-1.2) = 0.12$$

3. The logit model with the parameters of normalization 2:

$$P(\text{bicycle}) = \frac{\exp(V_{\text{bicycle}})}{\exp(V_{\text{bicycle}}) + \exp(V_{\text{metro}})} = \frac{\exp(-11)}{\exp(-11) + \exp(-12.2)} = 0.77$$

and

$$P(\text{metro}) = \frac{\exp(V_{\text{metro}})}{\exp(V_{\text{bicycle}}) + \exp(V_{\text{metro}})} = \frac{\exp(-12.2)}{\exp(-11) + \exp(-12.2)} = 0.23.$$

4. The probit model with the parameters of normalization 2:

$$P(\text{bicycle}) = \Phi(V_{\text{bicycle}} - V_{\text{metro}}) = \Phi(-11 + 12.2) = \Phi(1.2) = 0.88$$

and

$$P(\text{metro}) = \Phi(V_{\text{metro}} - V_{\text{bicycle}}) = \Phi(-12.2 + 11) = \Phi(-1.2) = 0.12$$

It can be seen that the choice probability does not depend on the normalization of the constants.