EPFL
ENAC TRANSP-OR
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Mathematical Modeling of Behavior
Fall 2017

EXERCISE SESSION 2

Exercise 1 We want to build a model that predicts the market penetration of electric vehicles (EV) as a function of the income level. We have a sample of 1000 individuals. The data is summarized in Table 1.

	Income			
EV	low	medium	high	
yes	15	50	135	200
no	200	450	150	800
	215	500	285	1000

Table 1: Contingency table of EV ownership conditional on income level

1. Estimate the parameters π_1, π_2 and π_3 using maximum likelihood estimation, where:

$$\begin{aligned} \text{Prob}(\text{EV} = \text{yes} \quad | \quad \text{income} = \text{low}) &= \pi_1 \\ \text{Prob}(\text{EV} = \text{yes} \quad | \quad \text{income} = \text{medium}) &= \pi_2 \\ \text{Prob}(\text{EV} = \text{yes} \quad | \quad \text{income} = \text{high}) &= \pi_3 \end{aligned} \tag{1}$$

   Hint: write the likelihood function and find its maximum.

2. Do the values of the parameters make sense?

3. Calculate the final log likelihood of the model.

4. Call the model described above M1. Consider a model with only two income categories: (i) low and medium income and (ii) high income. Call this M2. Now, consider another model with only one income category. Call this M0. Among the three models which one would you choose as the best model? Why?
   Hint: Calculate the final log likelihood associated with each model and perform likelihood ratio tests.

5. Suppose now that after some economical growth, the income distribution is as follows: 75 individuals with low income, 400 individuals with medium income and 525 with high income. Use the best model to predict the market penetration of EV for this scenario.

6. What is the main motivation for using discrete choice models in the context above? Could we have used linear regression instead? If linear regression was used, what would be the dependent variable?

Exercise 2 Derive the demand function for the following utility function:

$$\widetilde{U}(q; \lambda) = \frac{1}{\lambda} \prod_{\ell=1}^{L} q_\ell^\lambda, \tag{2}$$

where $q \in \mathbb{R}^L$ is a vector of L quantities, and $\lambda > 0$ is a parameter. To do so, complete the following steps:

1. Express consumer behavior as an optimization problem.

2. Write down the necessary first order optimality conditions, based on the Lagrangian function, that are verified by the optimal solution of the optimization problem from step 1. Note that the Lagrangian transforms a constrained optimization problem into an unconstrained optimization problem.

3. Apply the necessary optimality conditions for unconstrained optimization to the problem from step 2.

4. Multiply each of the optimality conditions associated with q_l obtained in step 3 by q_ℓ.

5. Sum all the conditions from step 4.

6. Use the optimality condition associated with μ (the Lagrange multiplier) obtained in step 3 in the sum of step 5.

7. Use the resulting statement of step 6 in the resulting statement of step 4.

8. Solve the resulting statement of step 7 for q_ℓ.

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