TRANSP-OR

EXERCISE SESSION 13

1 Get familiar with latent variable models

Download the folder called `Latent Variables`. There you will find:

1. the latent variables model `05latentChoiceFull.py` [1], and

2. the data file `optima.dat`.

Table 1 presents the estimation results of the `05latentChoiceFull.py` model. Go through the `05latentChoiceFull.py` in detail and:

1. Identify the attitude that is modeled as a latent variable.

2. Identify the indicators that are considered.

3. Identify the structural equations for both the choice and the latent variable models.

4. Identify the measurement equation for both the choice and the latent variable models.

5. Interpret the parameters of the model.

2 Latent variables exercise

An analyst is interested in modeling a mode choice problem between car and public transportation (PT) in a large city taking into account the effect of attitudes such as discomfort of using public transportation (*AttAntiPT*) and environmental concern (*AttProEnv*). In this context, an integrated choice and latent variable model is appropriate in order to account for the effects of the attitudes on mode choice.

The analyst is designing the survey for the data collection. She will collect information about the characteristics of the individuals and the observed attributes of the alternative modes, which are exogenous explanatory variables, as well as *attitudinal indicators* for the latent variables model. To obtain these indicators, she proposes statements and asks the respondent to position himself about

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[1] It is one of the examples of hybrid choice models involving latent variables provided in http://biogeme.epfl.ch/examples_latent.html

Parameter number	Description	Coeff. estimate	Robust Asympt. std. error	t-stat	p-value
1	ASC_CAR	0.703	0.118	5.96	0.00
2	ASC_SM	0.261	0.345	0.76	0.45
3	BETA_COST_HWH	-1.43	0.341	-4.19	0.00
4	BETA_COST_OTHER	-0.526	0.161	-3.27	0.00
5	BETA_DIST	-1.41	0.386	-3.66	0.00
6	BETA_TIME_CAR_CL	-0.955	0.169	-5.65	0.00
7	BETA_TIME_CAR_REF	-9.50	1.94	-4.90	0.00
8	BETA_TIME_PT_CL	-0.456	0.143	-3.19	0.00
9	BETA_TIME_PT_REF	-3.23	0.839	-3.84	0.00
10	BETA_WAITING_TIME	-0.0205	0.00963	-2.13	0.03
11	B_Envir02_F1	-0.459	0.0308	-14.88	0.00
12	B_Envir03_F1	0.484	0.0316	15.32	0.00
13	B_Mobil11_F1	0.572	0.0419	13.65	0.00
14	B_Mobil14_F1	0.575	0.0350	16.42	0.00
15	B_Mobil16_F1	0.525	0.0425	12.36	0.00
16	B_Mobil17_F1	0.514	0.0420	12.25	0.00
17	INTER_Envir02	0.460	0.0308	14.92	0.00
18	INTER_Envir03	-0.367	0.0289	-12.69	0.00
19	INTER_Mobil11	0.418	0.0373	11.22	0.00
20	INTER_Mobil14	-0.173	0.0278	-6.21	0.00
21	INTER_Mobil16	0.147	0.0336	4.39	0.00
22	INTER_Mobil17	0.140	0.0329	4.24	0.00
23	SIGMA_STAR_Envir02	0.918	0.0344	26.64	0.00
24	SIGMA_STAR_Envir03	0.857	0.0352	24.34	0.00
25	SIGMA_STAR_Mobil11	0.895	0.0409	21.89	0.00
26	SIGMA_STAR_Mobil14	0.759	0.0333	22.81	0.00
27	SIGMA_STAR_Mobil16	0.873	0.0397	21.97	0.00
28	SIGMA_STAR_Mobil17	0.876	0.0392	22.36	0.00
29	coef_ContIncome_0_4000	0.146	0.0606	2.41	0.02
30	coef_ContIncome_10000_more	0.119	0.0365	3.25	0.00
31	coef_ContIncome_4000_6000	-0.279	0.114	-2.45	0.01
32	coef_ContIncome_6000_8000	0.321	0.137	2.34	0.02
33	coef_ContIncome_8000_10000	-0.666	0.157	-4.25	0.00
34	coef_age_65_more	0.0403	0.0748	0.54	0.59
35	coef_haveChildren	-0.0276	0.0563	-0.49	0.62
36	coef_haveGA	-0.745	0.0999	-7.46	0.00
37	coef_highEducation	-0.265	0.0670	-3.96	0.00
38	coef_individualHouse	-0.116	0.0560	-2.08	0.04
39	coef_intercept	0.373	0.169	2.21	0.03
40	coef_male	0.0776	0.0534	1.45	0.15
41	coef_moreThanOneBike	-0.365	0.0686	-5.32	0.00
42	coef_moreThanOneCar	0.711	0.0667	10.66	0.00
43	delta_1	0.328	0.0127	25.81	0.00
44	delta_2	0.989	0.0358	27.64	0.00
45	sigma_s	0.855	0.0549	15.57	0.00

Summary statistics

Number of observations = 1906

Number of excluded observations = 359

Number of estimated parameters = 45

$$\mathcal{L}(\beta\_0) = -28534.376$$
$$\mathcal{L}(\hat{\beta}) = -18383.063$$
$$-2[\mathcal{L}(\beta\_0) - \mathcal{L}(\hat{\beta})] = 20302.626$$
$$\rho^2 = 0.356$$
$$\bar{\rho}^2 = 0.354$$

Table 1: Estimation results of the latent variable model.

it, using a scale ranging from -2 (totally disagree) to 2 (totally agree). Two indicators related to each attitude are collected. The two indicators related to the attitude *AttAntiPT* are noted I_{1n} and I_{2n} and the two indicators related to *AttProEnv* are noted I_{3n} and I_{4n}.

1. In the specification of the choice model the analyst has considered the effect of: i) cost, ii) time, iii) number of children, iv) frequency of public transportation (Figure 1). Complete the diagram in Figure 1 in order to integrate the effect of the two latent variables in the choice model. Make sure to use appropriate shapes and arrows and explain the drawing convention, i.e. what do the different shapes and arrows represent.

2. Update your diagram once more (use Figure 2), assuming that people with high education level and people who own a bike have a pro-environmental attitude, while people with low education level and owning more than one car have an attitude against public transportation.

3. Write down the structural equations corresponding to the a) *choice* and b) *latent variable* models, accordingly.

4. Write down the measurement equations corresponding to the a) *choice* and b) *latent variable* models, accordingly. Consider the specification of the updated diagram obtained in sub-question 2. Take into account that the collected indicators are discrete.
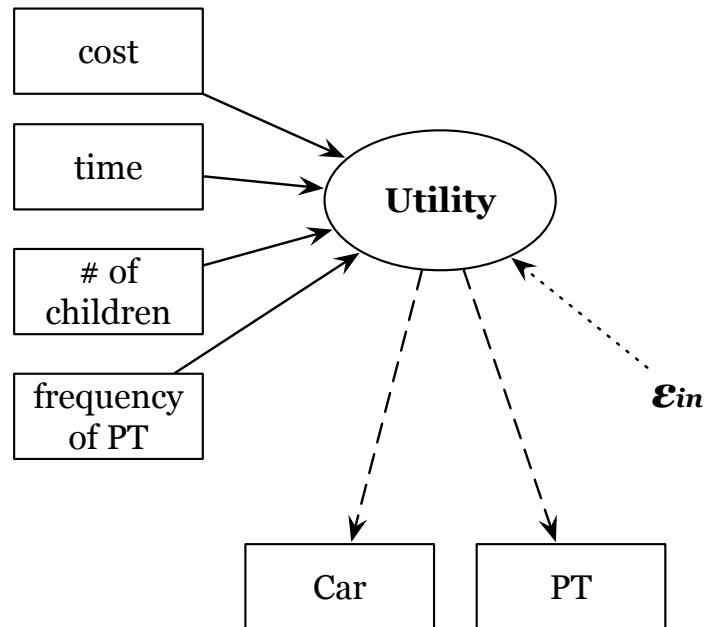

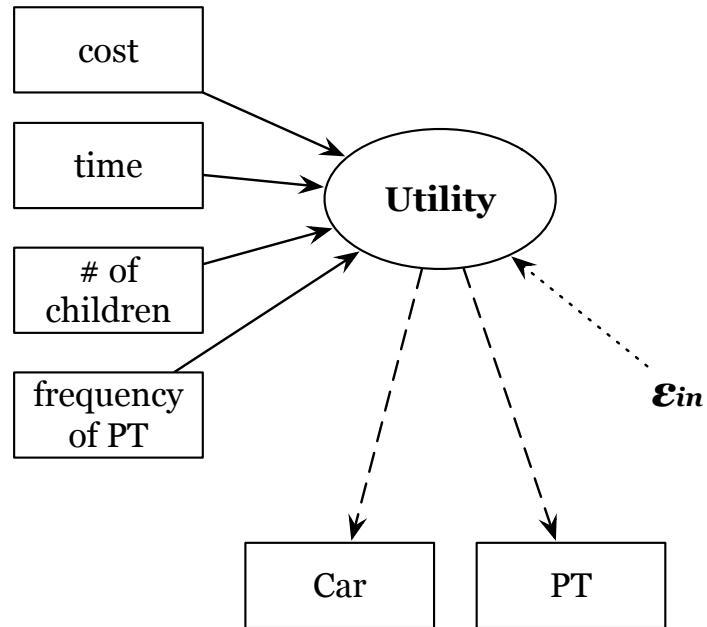Figure 1: Specification diagram of the standard mode choice model.

Figure 2: Specification diagram of the standard mode choice model.