EPFL
ENAC TRANSP-OR
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Mathematical Modeling of Behavior
Fall 2016

EXERCISE SESSION 9

For today's lecture you will have to use material from the book[1] that you have not seen in the lectures. Make sure that you have read and understood section 10.5.3 about the incremental logit before doing the exercises.

Exercise 1 In a case study of transportation mode choice, the parameters of the utility functions have been estimated as follows:

$$\begin{aligned} U_{1n} &= 1 - \frac{3}{100} \cdot tt_{1n} - \frac{6}{100} \cdot c_{1n} + \frac{1}{2} \cdot \text{income}_n + \varepsilon_{1n} \\ U_{2n} &= -\frac{2}{100} \cdot tt_{2n} - \frac{3}{80} \cdot c_{2n} + \frac{1}{2} \cdot \text{university}_n + \varepsilon_{2n} \end{aligned} \tag{1}$$

where tt_{in} is the travel time in minutes and c_{in} is the cost in CHF for individual n, with $i \in \{\text{car}, \text{train}\}$. $income_n$ takes value 1 if the individual's monthly income is larger than 6000CHF and 0 otherwise, and $university_n$ takes value 1 if the individual went to university and 0 otherwise. $\varepsilon_{1n}, \varepsilon_{2n} \overset{iid}{\sim} \text{EV}(0, 1)$. Perform the following tasks:

1. Calculate the direct and cross point elasticities of travel time for an individual with $tt_1 = 20, tt_2 = 15, c_1 = 3, c_2 = 5$, a monthly income of 9000CHF and who did not go to university. *Hint: you need to calculate* $\frac{\partial P_n(i)}{\partial tt_j} \cdot \frac{tt_j}{P_n(i)} \, \forall i, j \in \{1, 2\}$.

2. Calculate the direct and cross arc elasticities of the same individual described above in a scenario where travel time of the car alternative increases by 30%.

3. Interpret and compare the point and arc elasticities.

4. Calculate the change in consumer surplus corresponding to a 30% decrease in the travel time by public transportation for the individual described above. How do you interpret this value?

[1] Ben-Akiva, M., Bierlaire, M., McFadden, D. & Walker, J. Discrete Choice Analysis.

Exercise 2

1. In an Stated Preferences (SP) mode choice experiment, respondents are exposed to the following three alternatives: (1) biking, (2) walking and (3) taking the metro. You have estimated a model with the following utility functions:

$$\begin{aligned} U_{1n} &= \beta_1 \cdot t_{1n} + \varepsilon_{1n} \\ U_{2n} &= ASC_2 + \beta_2 \cdot t_{2n} + \varepsilon_{2n} \\ U_{3n} &= ASC_3 + \beta_3 \cdot t_{3n} + \varepsilon_{3n} \end{aligned}$$

where t_{in} is the travel time of respondent n using mode i, $\varepsilon_{1n}, \varepsilon_{2n}, \varepsilon_{3n} \overset{iid}{\sim} \text{EV}(0,1)$ and β_i, ASC_i are the parameters that have been estimated, with $i \in \{1,2,3\}$. The population of interest consists of exactly *four* individuals: one with travel time t_{11}, two with equal travel times $t_{12} = t_{13}$ and one with travel time t_{14}. Fig. 1 illustrates the probability of choosing alternative 1, i.e. bicycle, as predicted by this model, for each level of travel time.

Compute the fraction of the population choosing alternative 1.

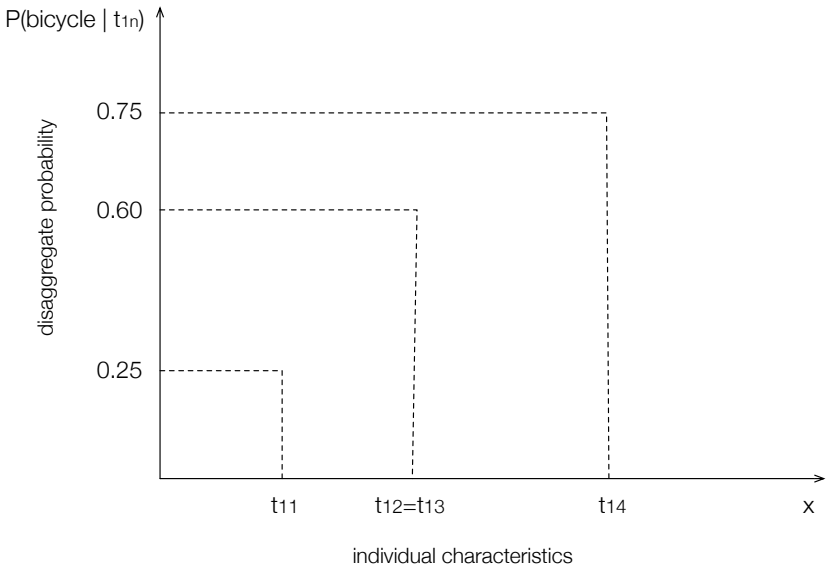

Figure 1: $P(\text{bicycle}|t_{1n})$

2. Consider now the first individual of the population with travel time by bicycle t_{11}. Your

linear-in-parameters logit model from the previous sub-question has predicted the probability with which this individual will choose each alternative as shown in column $P(i|t_{11})$ of Table 1. Due to an increase in the number of bike lanes in the city, there is a change in travel time from t_{11} to t'_{11}. The utilities of the three alternatives for this individual change as indicated in column ΔV_{in}.

Mode	$P(i\|t_{11})$	ΔV_{in}	$P(i\|t'_{11})$
Bicycle	0.25	0.5	
Walk	0.45	0	
Metro	0.30	0	

Table 1: Choice probabilities before and after an increase in number of bike lanes

Complete Table 1 by computing the new choice probabilities $P'(i|t_{11})$ for the three alternatives. *Hint: the incremental logit model may be useful here.*

3. Given the utilities of the alternatives before the change in t_{11} (column V_{before} of Table 2) and the change in utilities as in the previous subquestion, compute the change in consumer surplus for individual 1 between the two scenarios, before and after the change.

Mode	V_{before}	ΔV_{in}	V_{after}
Bicycle	0.4	0.5	0.9
Walk	1.0	0	1.0
Metro	0.6	0	0.6

Table 2: Utilities before and after increase in number of bike lanes

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