EPFL
ENAC TRANSP-OR
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Mathematical Modeling of Behavior
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EXERCISE SESSION 7

Exercise 1 We estimated a *binary logit* model for a route choice problem where travelers have to choose between two alternatives, denoted as path 1 and path 2. The deterministic parts of the utility functions of the two alternatives are specified as follows:

$$V_{path_1} = ASC_{path_1} + \beta_{time} * time_{path_1} + \beta_{toll} * highwayToll_{path_1} \quad (1)$$

$$V_{path_2} = \beta_{time} * time_{path_2} \quad (2)$$

where *time* is the travel time in minutes and $highwayToll$ equals 1 if the path passes through a tolled highway and 0 otherwise. The parameters that we obtained are shown in Table 1.

Table 1: Estimated parameters for the binary logit model.

ASC_{path_1}	-0.5
β_{time}	-1.5
β_{toll}	-15

1. Consider one traveler for whom the attributes of the alternatives are given in Table 2 and compute the following:
   (a) utilities of the two alternatives;
   (b) probability of each alternative to be chosen;
   (c) point elasticity of the probability of the chosen alternative with respect to travel time.

Table 2: Attributes of the alternatives for a specific individual in the sample.

alternative	travel time (min)	toll of highway
$path_1$	30	1
$path_2$	40	0

2. How much extra travel time would this traveler be willing to spend in order to avoid the toll?

3. Given that the above traveler actually chose alternative 1, i.e. $path_1$, does the model predict a high probability for the chosen alternative? If not, could you think of a reason why this may happen given the model specification that we considered in this example?

4. Consider now a scenario where due to construction works along $path_2$ its travel time increases by 5 minutes. How would this affect the choice probabilities? Compare the indicators of this scenario with the ones you obtained from the initial scenario in questions 1 and 2.

Exercise 2 Recall the red bus/ blue bus paradox from today's lecture. Travelers initially face a decision between two modes of transportation: car and blue bus. The travel times of both modes, car and bus, are equal. Travel time is also the only variable considered in the utility. Then, we suppose that a third mode, namely the red bus, is introduced and that the travelers consider it to be exactly like the blue bus.

1. In the case of a logit model, what will the choice probability of each of the three modes be? Use the IIA property to derive these probabilities.

2. Are the resulting choice probabilities intuitive? If not, explain why and describe what you would expect them to be?

3. Assume that the error terms for the red and blue bus are correlated and that the correlation is 95%. Derive the scale parameter (μ_m) and calculate the probabilities of choosing car and bus[1].

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mbi/ ek/ afa / mp

[1]Note that μ is normalized to one.