EPFL
ENAC TRANSP-OR
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Mathematical Modeling of Behavior
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EXERCISE SESSION 4

Exercise 1 In a case study of transportation mode choice, the parameters of the utility functions have been estimated as follows:

$$
\begin{aligned}
U_{1n} &= 1 - \frac{3}{100} \cdot tt_{1n} - \frac{6}{100} \cdot c_{1n} + 0.5 \cdot \text{income}_n + \varepsilon_{1n} \\
U_{2n} &= -\frac{2}{100} \cdot tt_{2n} - \frac{3.75}{100} \cdot c_{2n} + 0.5 \cdot \text{university}_n + \varepsilon_{2n}
\end{aligned} \tag{1}
$$

where tt_{in} is the travel time in minutes and c_{in} is the cost in CHF for respondent n, with $i \in \{\text{car}, \text{train}\}$. $income_n$ takes value 1 if the respondent's monthly income is larger than 6000CHF and 0 otherwise, and $university_n$ takes value 1 if the respondent went to the university and 0 otherwise. $\varepsilon_{1n}, \varepsilon_{2n} \overset{iid}{\sim} \text{EV}(0, 1)$.

1. Compute the probability to choose each mode for the following individuals:

Name	tt_1	tt_2	c_1	c_2	monthly income	university
Eva	22	18	2	2.1	7000	yes
Matthieu	120	100	10	15	3000	yes
Michel	10	50	3	5	10000	no
Meri	25	9	7	2.1	5000	no

2. What does the alternative specific constant in alternative 1 represent? Interpret one by one all the parameters.

Exercise 2 In a route choice case study, the utility functions are defined as follows:

$$
\begin{aligned}
U_1 &= ASC_1 + \beta_{length} \cdot \text{length}_1 + \varepsilon_1 \\
U_2 &= ASC_2 + \beta_{length} \cdot \text{length}_2 + \varepsilon_2
\end{aligned} \tag{2}
$$

where alternatives 1 and 2 represent different routes, ASC_1, ASC_2 and β_{length} are parameters to be estimated and $\text{length}_i, i \in \{1, 2\}$ is the length of each route in kilometers.

The estimation results of a binary logit model, where ASC_1 has been normalized to zero, are shown in the first column of the following table. The second column corresponds to the same specification where ASC_2 has been normalized to zero:

	Logit 1	Logit 2
ASC_1	0	x
ASC_2	-2	0
β_{length}	10	x

Perform the following tasks:

1. Replace the x in the table by the value of the corresponding parameter.
2. What are the distributions of ε_1, ε_2 and $\varepsilon_1 - \varepsilon_2$?

Exercise 3

1. Define the Box-Cox transformation. What modeling assumption are you testing when specifying a Box-Cox transformation of the travel cost in a model of transportation mode choice? Let λ be the parameter of the Box-Cox transformation. What particular cases do you obtain when $\lambda = 1$ or $\lambda = 0$?
2. In a model developed for the transportation mode choice in the Netherlands case study, the deterministic parts of the utilities for the car and rail alternatives are specified as follows:

$$\begin{aligned} V_{Car,n} &= ASC_{CAR} + \beta_{TIME\_CAR} \cdot time_{car,n} + \beta_{COST} \cdot cost_{car,n} \\ V_{Rail,n} &= ASC_{RAIL} + \beta_{TIME\_RAIL} \cdot time_{rail,n} + \beta_{COST} \cdot cost_{rail,n} + \beta_{MALE} \cdot male_n \end{aligned} \quad (3)$$

   where $time_{car,n}$ and $time_{rail,n}$ are the travel times for car and rail respectively for individual n, $cost_{car,n}$ and $cost_{rail,n}$ are the travel costs for car and rail for individual n, and $male_n$ takes value 1 if the individual is a male, and 0 if she is a female. The estimation results for this model is shown in Figure 1.

Model:	Multinomial Logit
Number of estimated parameters:	5
Number of observations:	228
Number of individuals:	228
Null log-likelihood:	-158.038
Cte log-likelihood:	-148.347
Init log-likelihood:	-158.038
Final log-likelihood:	-115.880
Likelihood ratio test:	84.314
Rho-square:	0.267
Adjusted rho-square:	0.235
Final gradient norm:	+2.460e-04
Diagnostic:	Convergence reached...
Iterations:	9
Run time:	00:01
Variance-covariance:	from analytical hessian
Sample file:	netherlands.dat

Utility parameters

Name	Value	Std err	t-test	p-value		Robust Std err	Robust t-test	p-value	
ASC_CAR	2.85	1.09	2.62	0.01		1.02	2.80	0.01	
ASC_RAIL	0.00	fixed							
BETA_COST	-0.130	0.0251	-5.17	0.00		0.0265	-4.89	0.00	
BETA_GENDER	0.675	0.330	2.05	0.04		0.329	2.05	0.04	
BETA_TIME_CAR	-2.34	0.489	-4.78	0.00		0.495	-4.73	0.00	
BETA_TIME_RAIL	-0.529	0.418	-1.27	0.20	*	0.414	-1.28	0.20	*

Utility functions

Id	Name	Availability	Specification
0	Car	one	ASC_CAR * one + BETA_TIME_CAR * car_time + BETA_COST * car_cost_euro
1	Rail	one	ASC_RAIL * one + BETA_TIME_RAIL * rail_time + BETA_COST * rail_cost_euro + BETA_GENDER * gender

Figure 1: Estimation results of the base model

In addition to Model 1 we now estimate a model with a Box-Cox transformation of the cost variables. A snapshot of the estimation results is presented in Figure 2.

Model: Multinomial Logit
Number of estimated parameters: 6
Number of observations: 228
Number of individuals: 228
Null log-likelihood: -158.038
Cte log-likelihood: -148.347
Init log-likelihood: -158.038
Final log-likelihood: -113.265
Likelihood ratio test: 89.546
Rho-square: 0.283
Adjusted rho-square: 0.245
Final gradient norm: +5.044e-04
Diagnostic: Convergence reached...
Iterations: 14
Run time: 00:00
Variance-covariance: from finite difference hessian
Sample file: netherlands.dat

Utility parameters

Name	Value	Std err	t-test	p-value		Robust Std err	Robust t-test	p-value	
ASC_CAR	2.64	1.09	2.41	0.02		1.03	2.56	0.01	
ASC_RAIL	0.00	fixed							
BETA_COST	-0.544	0.266	-2.05	0.04		0.249	-2.19	0.03	
BETA_GENDER	0.735	0.338	2.18	0.03		0.334	2.20	0.03	
BETA_TIME_CAR	-2.42	0.500	-4.84	0.00		0.509	-4.76	0.00	
BETA_TIME_RAIL	-0.616	0.427	-1.44	0.15	*	0.423	-1.46	0.15	*
LAMBDA	0.400	0.224	1.78	0.07	*	0.211	1.90	0.06	*

Utility functions

Id	Name	Availability	Specification
0	Car	one	ASC_CAR * one + BETA_TIME_CAR * car_time + BETA_COST * (((car_cost_euro ** LAMBDA) - 1) / LAMBDA)
1	Rail	one	ASC_RAIL * one + BETA_TIME_RAIL * rail_time + BETA_GENDER * gender + BETA_COST * (((rail_cost_euro ** LAMBDA) - 1) / LAMBDA)

Figure 2: Estimation results of model with a Box-Cox transformation

(a) Comment and interpret the estimation results of both models using formal and informal tests.

(b) Propose two different statistical tests to illustrate if model 2 provides an improvement compared to model 1.

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