

# Choice theory

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# Outline

## 1 Theoretical foundations

- Choice theory
- Decision maker
- Characteristics
- Choice set
- Alternative attributes
- Decision rule

## 2 Microeconomic consumer theory

- Preferences
  - Utility maximization
  - Indirect utility
  - Microeconomic results
- ### 3 Discrete goods
- Utility maximization
- ### 4 Simple example
- ### 5 Probabilistic choice theory
- The random utility model

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# Choice theory

Choice: outcome of a sequential decision-making process

- defining the choice problem
- generating alternatives
- evaluating alternatives
- making a choice,
- executing the choice.

Theory of behavior that is

- **descriptive**: how people behave and not how they should
- **abstract**: not too specific
- **operational**: can be used in practice for forecasting

# Building the theory

## Define

- 1 who (or what) is the decision maker,
- 2 what are the characteristics of the decision maker,
- 3 what are the alternatives available for the choice,
- 4 what are the attributes of the alternatives, and
- 5 what is the decision rule that the decision maker uses to make a choice.

# Decision maker

## Individual

- a person
- a group of persons (internal interactions are ignored)
  - household, family
  - firm
  - government agency
- notation:  $n$

# Characteristics of the decision maker

## Disaggregate models

### Individuals

- face different choice situations
- have different tastes

## Characteristics

- income
- sex
- age
- level of education
- household/firm size
- etc.

# Alternatives

## Choice set

- Non empty finite and countable set of alternatives
- Universal:  $\mathcal{C}$
- Individual specific:  $\mathcal{C}_n \subseteq \mathcal{C}$
- Availability, awareness

## Example

Choice of a transportation model

- $\mathcal{C} = \{\text{car, bus, metro, walking}\}$
- If the decision maker has no driver license, and the trip is 12km long

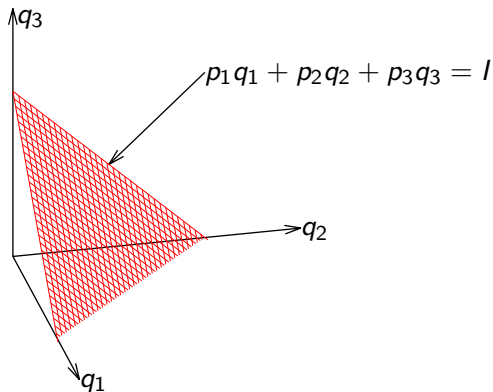
$$\mathcal{C}_n = \{\text{bus, metro}\}$$

# Continuous choice set

## Microeconomic demand analysis

### Commodity bundle

- $q_1$ : quantity of milk
- $q_2$ : quantity of bread
- $q_3$ : quantity of butter
- Unit price:  $p_i$
- Budget:  $I$

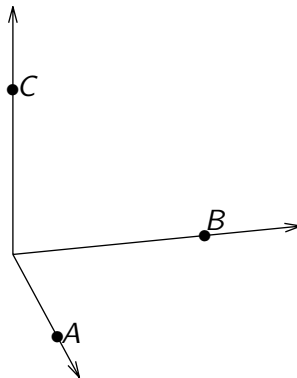


# Discrete choice set

## Discrete choice analysis

### List of alternatives

- Brand  $A$
- Brand  $B$
- Brand  $C$



# Alternative attributes

Characterize each alternative  $i$   
for each individual  $n$

- price
- travel time
- frequency
- comfort
- color
- size
- etc.

Nature of the variables

- Discrete and continuous
- Generic and specific
- Measured or perceived

# Decision rule

## Homo economicus

Rational and narrowly self-interested economic actor who is optimizing her outcome

## Utility

$$U_n : \mathcal{C}_n \longrightarrow \mathbb{R} : a \rightsquigarrow U_n(a)$$

- captures the attractiveness of an alternative
- measure that the decision maker wants to optimize

## Behavioral assumption

- the decision maker associates a utility with each alternative
- the decision maker is a perfect optimizer
- the alternative with the highest utility is chosen

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# Microeconomic consumer theory

## Continuous choice set

- Consumption bundle

$$Q = \begin{pmatrix} q_1 \\ \vdots \\ q_L \end{pmatrix}; p = \begin{pmatrix} p_1 \\ \vdots \\ p_L \end{pmatrix}$$

- Budget constraint

$$p^T Q = \sum_{\ell=1}^L p_{\ell} q_{\ell} \leq I.$$

- No attributes, just quantities

# Preferences

## Operators $\succ$ , $\sim$ , and $\succsim$

- $Q_a \succ Q_b$ :  $Q_a$  is preferred to  $Q_b$ ,
- $Q_a \sim Q_b$ : indifference between  $Q_a$  and  $Q_b$ ,
- $Q_a \succsim Q_b$ :  $Q_a$  is at least as preferred as  $Q_b$ .

## Rationality

- Completeness: for all bundles  $a$  and  $b$ ,

$$Q_a \succ Q_b \text{ or } Q_a \prec Q_b \text{ or } Q_a \sim Q_b.$$

- Transitivity: for all bundles  $a$ ,  $b$  and  $c$ ,

$$\text{if } Q_a \succsim Q_b \text{ and } Q_b \succsim Q_c \text{ then } Q_a \succsim Q_c.$$

- “Continuity”: if  $Q_a$  is preferred to  $Q_b$  and  $Q_c$  is arbitrarily “close” to  $Q_a$ , then  $Q_c$  is preferred to  $Q_b$ .

# Utility

## Utility function

- Parametrized function:

$$\tilde{U} = \tilde{U}(q_1, \dots, q_L; \theta) = \tilde{U}(Q; \theta)$$

- Consistent with the preference indicator:

$$\tilde{U}(Q_a; \theta) \geq \tilde{U}(Q_b; \theta)$$

is equivalent to

$$Q_a \succsim Q_b.$$

- Unique up to an order-preserving transformation

# Optimization

## Optimization problem

$$\max_Q \tilde{U}(Q; \theta)$$

subject to

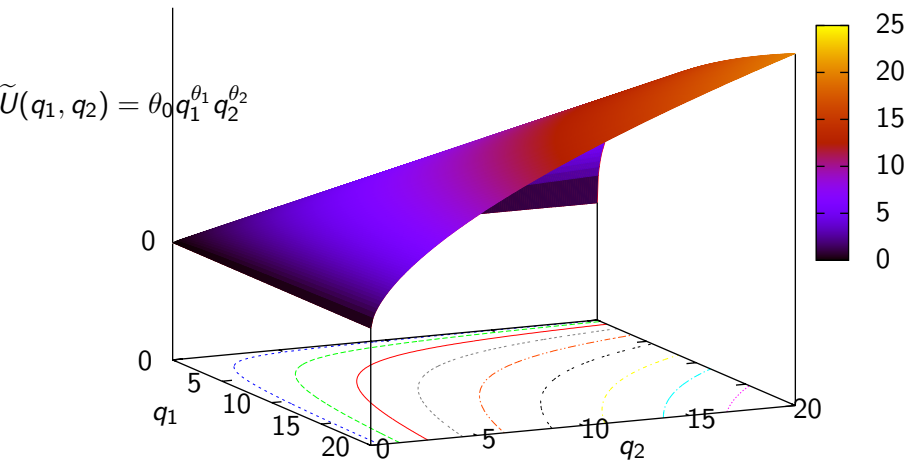
$$p^T Q \leq I, \quad Q \geq 0.$$

## Demand function

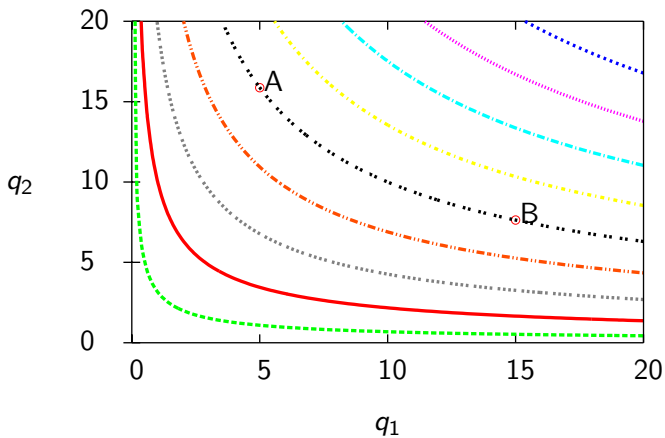
- Solution of the optimization problem
- Quantity as a function of prices and budget

$$Q^* = f(I, p; \theta)$$

# Example: Cobb-Douglas



# Example



# Example

## Optimization problem

$$\max_{q_1, q_2} \tilde{U}(q_1, q_2; \theta_0, \theta_1, \theta_2) = \theta_0 q_1^{\theta_1} q_2^{\theta_2}$$

subject to

$$p_1 q_1 + p_2 q_2 = I.$$

Lagrangian of the problem:

$$L(q_1, q_2, \lambda) = \theta_0 q_1^{\theta_1} q_2^{\theta_2} + \lambda(I - p_1 q_1 - p_2 q_2).$$

Necessary optimality condition

$$\nabla L(q_1, q_2, \lambda) = 0$$

# Example

## Necessary optimality conditions

$$\begin{aligned}\theta_0 \theta_1 q_1^{\theta_1-1} q_2^{\theta_2} &- \lambda p_1 = 0 & (\times q_1) \\ \theta_0 \theta_2 q_1^{\theta_1} q_2^{\theta_2-1} &- \lambda p_2 = 0 & (\times q_2) \\ p_1 q_1 + p_2 q_2 &- I = 0.\end{aligned}$$

We have

$$\begin{aligned}\theta_0 \theta_1 q_1^{\theta_1} q_2^{\theta_2} &- \lambda p_1 q_1 = 0 \\ \theta_0 \theta_2 q_1^{\theta_1} q_2^{\theta_2} &- \lambda p_2 q_2 = 0.\end{aligned}$$

Adding the two and using the third condition, we obtain

$$\lambda I = \theta_0 q_1^{\theta_1} q_2^{\theta_2} (\theta_1 + \theta_2)$$

or, equivalently,

$$\theta_0 q_1^{\theta_1} q_2^{\theta_2} = \frac{\lambda I}{(\theta_1 + \theta_2)}$$

# Solution

From the previous derivation

$$\theta_0 q_1^{\theta_1} q_2^{\theta_2} = \frac{\lambda I}{(\theta_1 + \theta_2)}$$

First condition

$$\theta_0 \theta_1 q_1^{\theta_1} q_2^{\theta_2} = \lambda p_1 q_1.$$

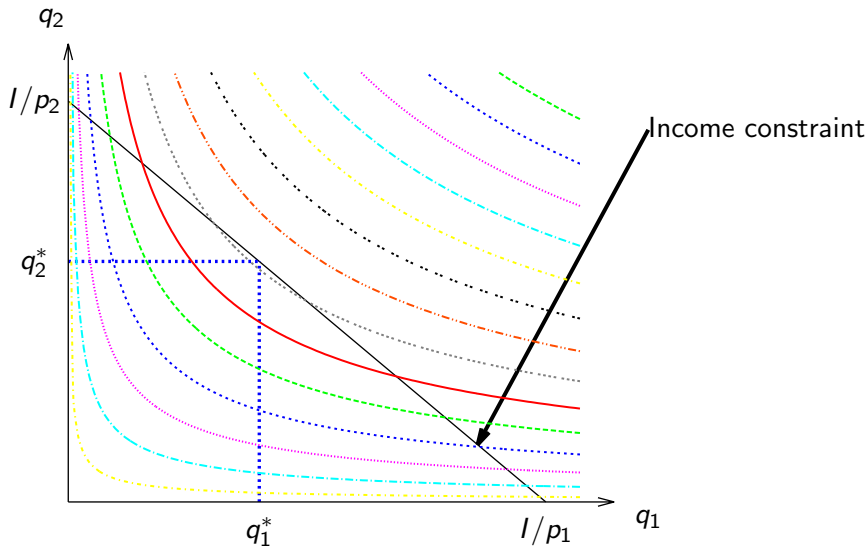
Solve for  $q_1$

$$q_1^* = \frac{I \theta_1}{p_1 (\theta_1 + \theta_2)}$$

Similarly, we obtain

$$q_2^* = \frac{I \theta_2}{p_2 (\theta_1 + \theta_2)}$$

# Optimization problem



# Demand functions

## Product 1

$$q_1^* = \frac{I}{p_1} \frac{\theta_1}{\theta_1 + \theta_2}$$

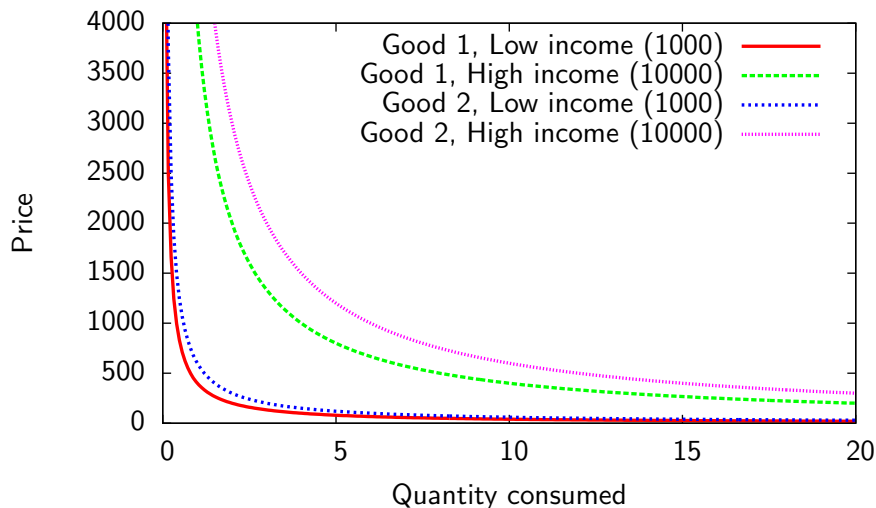
## Product 2

$$q_2^* = \frac{I}{p_2} \frac{\theta_2}{\theta_1 + \theta_2}$$

## Comments

- Demand decreases with price
- Demand increases with budget
- Demand independent of  $\theta_0$ , which does not affect the ranking
- Property of Cobb Douglas: the demand for a good is only dependent on its own price and independent of the price of any other good.

# Demand curve (inverse of demand function)



# Indirect utility

Substitute the demand function into the utility

$$U(I, p; \theta) = \theta_0 \left( \frac{I}{p_1} \frac{\theta_1}{\theta_1 + \theta_2} \right)^{\theta_1} \left( \frac{I}{p_2} \frac{\theta_2}{\theta_1 + \theta_2} \right)^{\theta_2}$$

## Indirect utility

Maximum utility that is achievable for a given set of prices and income

In discrete choice...

- only the indirect utility is used
- therefore, it is simply referred to as “utility”
- we review some results from microeconomics useful for discrete choice

# Roy's identity

Derive the demand function from the indirect utility

$$q_\ell = - \frac{\partial U(I, p; \theta) / \partial p_\ell}{\partial U(I, p; \theta) / \partial I}$$

# Elasticities

## Direct price elasticity

Percent change in demand resulting from a 1% change in price

$$E_{p_\ell}^{q_\ell} = \frac{\% \text{ change in } q_\ell}{\% \text{ change in } p_\ell} = \frac{\Delta q_\ell / q_\ell}{\Delta p_\ell / p_\ell} = \frac{p_\ell}{q_\ell} \frac{\Delta q_\ell}{\Delta p_\ell}.$$

## Asymptotically

$$E_{p_\ell}^{q_\ell} = \frac{p_\ell}{q_\ell(I, p; \theta)} \frac{\partial q_\ell(I, p; \theta)}{\partial p_\ell}.$$

## Cross price elasticity

$$E_{p_m}^{q_\ell} = \frac{p_m}{q_\ell(I, p; \theta)} \frac{\partial q_\ell(I, p; \theta)}{\partial p_m}.$$

# Consumer surplus

## Definition

Difference between what a consumer is willing to pay for a good and what she actually pays for that good.

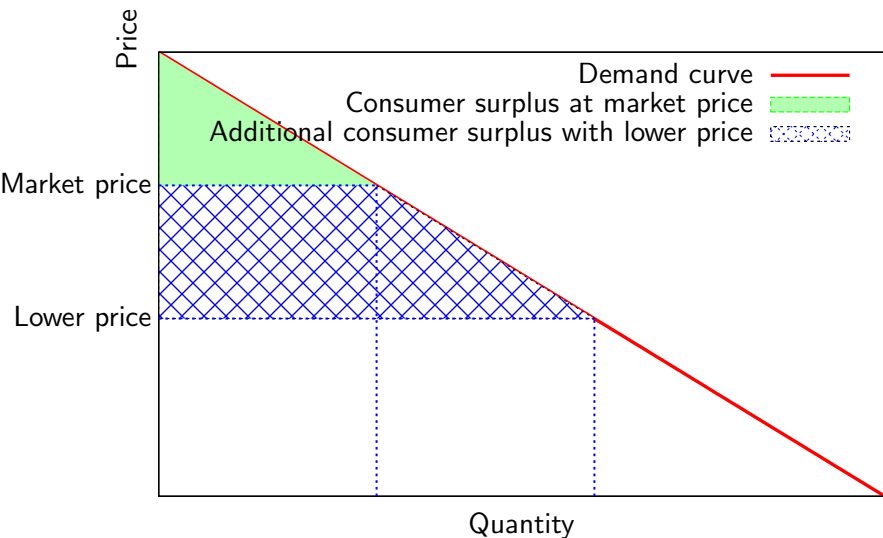
## Calculation

Area under the demand curve and above the market price

## Demand curve

- Plot of the inverse demand function
- Price as a function of quantity

# Consumer surplus



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# Microeconomic theory of discrete goods

## Expanding the microeconomic framework

- Continuous goods
- and discrete goods

## The consumer

- selects the quantities of continuous goods:  $Q = (q_1, \dots, q_L)$
- chooses an alternative in a discrete choice set  $i = 1, \dots, j, \dots, J$
- discrete decision vector:  $(y_1, \dots, y_J)$ ,  $y_j \in \{0, 1\}$ ,  $\sum_j y_j = 1$ .

## Note

- In theory, one alternative of the discrete choice combines all possible choices made by an individual.
- In practice, the choice set will be more restricted for tractability

# Utility maximization

## Utility

$$\tilde{U}(Q, y, \tilde{z}^T y; \theta)$$

- $Q$ : quantities of the continuous good
- $y$ : discrete choice
- $\tilde{z}^T = (\tilde{z}_1, \dots, \tilde{z}_i, \dots, \tilde{z}_J) \in \mathbb{R}^{K \times J}$ :  $K$  attributes of the  $J$  alternatives
- $\tilde{z}^T y \in \mathbb{R}^K$ : attributes of the chosen alternative
- $\theta$ : vector of parameters

# Utility maximization

## Optimization problem

$$\max_{Q,y} \tilde{U}(Q, y, \tilde{z}^T y; \theta)$$

subject to

$$p^T Q + c^T y \leq I$$

$$\sum_j y_j = 1$$

$$y_j \in \{0, 1\}, \forall j.$$

where  $c^T = (c_1, \dots, c_i, \dots, c_J)$  contains the cost of each alternative.

## Solving the problem

- Mixed integer optimization problem
- No optimality condition
- Impossible to derive demand functions directly

# Solving the problem

## Step 1: condition on the choice of the discrete good

- Fix the discrete good, that is select a feasible  $y$ .
- The problem becomes a continuous problem in  $Q$ .
- Conditional demand functions can be derived:

$$q_{\ell|y} = f(I - c^T y, p, \tilde{z}^T y; \theta),$$

or, equivalently, for each alternative  $i$ ,

$$q_{\ell|i} = f(I - c_i, p, \tilde{z}_i; \theta).$$

- $I - c_i$  is the income left for the continuous goods, if alternative  $i$  is chosen.
- If  $I - c_i < 0$ , alternative  $i$  is declared unavailable and removed from the choice set.

# Solving the problem

## Conditional indirect utility functions

Substitute the demand functions into the utility:

$$U_i = U(I - c_i, p, \tilde{z}_i; \theta) \text{ for all } i \in \mathcal{C}.$$

## Step 2: Choice of the discrete good

$$\max_y U(I - c^T y, p, \tilde{z}^T y; \theta)$$

- Enumerate all alternatives.
- Compute the conditional indirect utility function  $U_i$ .
- Select the alternative with the highest  $U_i$ .
- Note: no income constraint anymore.

## Model for individual $n$

$$\max_y U(I_n - c_n^T y, p_n, \tilde{z}_n^T y; \theta_n)$$

### Simplifications

- We cannot estimate a set of parameters for each individual  $n$
- Therefore, population level parameters are interacted with characteristics  $S_n$  of the decision-maker
- Prices of the continuous goods are neglected  $p_n$
- Income is considered as another characteristic and merged into  $S_n$
- $c_i$  is considered as another attribute and merged into  $\tilde{z}$

$$z_n = \{\tilde{z}_n, c_n\}$$

$$\max_i U_{in} = U(z_{in}, S_n; \theta)$$

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# Simple example: mode choice

## Attributes

Alternatives	Attributes	
	Travel time ( $t$ )	Travel cost ( $c$ )
Car (1)	$t_1$	$c_1$
Bus (2)	$t_2$	$c_2$

## Utility

$$\tilde{U} = \tilde{U}(y_1, y_2),$$

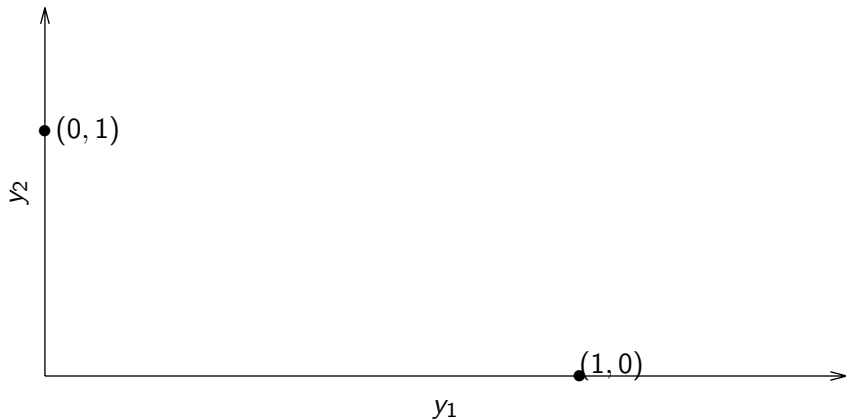
where we impose the restrictions that, for  $i = 1, 2$ ,

$$y_i = \begin{cases} 1 & \text{if travel alternative } i \text{ is chosen,} \\ 0 & \text{otherwise;} \end{cases}$$

and that only one alternative is chosen:  $y_1 + y_2 = 1$ .

# Simple example: mode choice

Choice set



# Simple example: mode choice

## Utility functions

$$U_1 = -\beta_t t_1 - \beta_c c_1,$$

$$U_2 = -\beta_t t_2 - \beta_c c_2,$$

where  $\beta_t > 0$  and  $\beta_c > 0$  are parameters.

## Equivalent specification

$$U_1 = -(\beta_t/\beta_c)t_1 - c_1 = -\beta t_1 - c_1$$

$$U_2 = -(\beta_t/\beta_c)t_2 - c_2 = -\beta t_2 - c_2$$

where  $\beta > 0$  is a parameter.

## Choice

- Alternative 1 is chosen if  $U_1 \geq U_2$ .
- Ties are ignored.

# Simple example: mode choice

## Choice

Alternative 1 is chosen if

$$-\beta t_1 - c_1 \geq -\beta t_2 - c_2$$

or

$$-\beta(t_1 - t_2) \geq c_1 - c_2$$

Alternative 2 is chosen if

$$-\beta t_1 - c_1 \leq -\beta t_2 - c_2$$

or

$$-\beta(t_1 - t_2) \leq c_1 - c_2$$

## Dominated alternative

- If  $c_2 > c_1$  and  $t_2 > t_1$ ,  $U_1 > U_2$  for any  $\beta > 0$
- If  $c_1 > c_2$  and  $t_1 > t_2$ ,  $U_2 > U_1$  for any  $\beta > 0$

# Simple example: mode choice

## Trade-off

- Assume  $c_2 > c_1$  and  $t_1 > t_2$ .
- Is the traveler willing to pay the extra cost  $c_2 - c_1$  to save the extra time  $t_1 - t_2$ ?
- Alternative 2 is chosen if

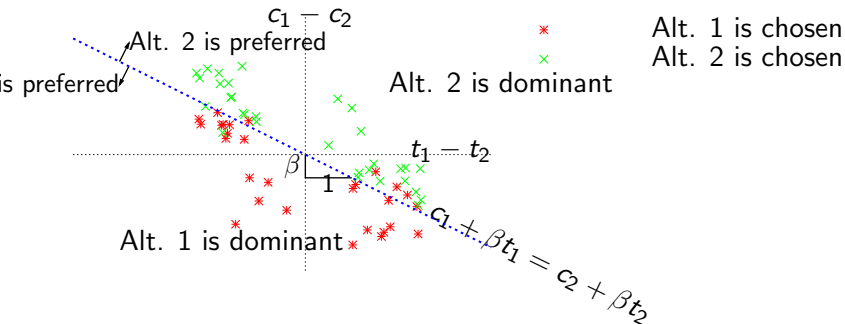
$$-\beta(t_1 - t_2) \leq c_1 - c_2$$

or

$$\beta \geq \frac{c_2 - c_1}{t_1 - t_2}$$

- $\beta$  is called the *willingness to pay* or *value of time*

## Simple example: mode choice



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# Behavioral validity of the utility maximization?

## Assumptions

### Decision-makers

- are able to process information
- have perfect discrimination power
- have transitive preferences
- are perfect maximizer
- are always consistent

## Relax the assumptions

Use a probabilistic approach: what is the probability that alternative  $i$  is chosen?

# Introducing probability

## Constant utility

- Human behavior is inherently random
- Utility is deterministic
- Consumer does not maximize utility
- Probability to use inferior alternative is non zero

## Random utility

- Decision-maker are rational maximizers
- Analysts have no access to the utility used by the decision-maker
- Utility becomes a random variable

## Niels Bohr

*Nature is stochastic*

## Albert Einstein

*God does not throw dice*

# Random utility model

## Probability model

$$P(i|\mathcal{C}_n) = \Pr(U_{in} \geq U_{jn}, \text{ all } j \in \mathcal{C}_n),$$

## Random utility

$$U_{in} = V_{in} + \varepsilon_{in}.$$

## Random utility model

$$P(i|\mathcal{C}_n) = \Pr(V_{in} + \varepsilon_{in} \geq V_{jn} + \varepsilon_{jn}, \text{ all } j \in \mathcal{C}_n),$$

or

$$P(i|\mathcal{C}_n) = \Pr(\varepsilon_{jn} - \varepsilon_{in} \leq V_{in} - V_{jn}, \text{ all } j \in \mathcal{C}_n).$$

# Derivation

## Joint distributions of $\varepsilon_n$

- Assume that  $\varepsilon_n = (\varepsilon_{1n}, \dots, \varepsilon_{J_n n})$  is a multivariate random variable
- with CDF

$$F_{\varepsilon_n}(\varepsilon_1, \dots, \varepsilon_{J_n})$$

- and pdf

$$f_{\varepsilon_n}(\varepsilon_1, \dots, \varepsilon_{J_n}) = \frac{\partial^{J_n} F}{\partial \varepsilon_1 \dots \partial \varepsilon_{J_n}}(\varepsilon_1, \dots, \varepsilon_{J_n}).$$

## Derive the model for the first alternative (wlog)

$$P_n(1|C_n) = \Pr(V_{2n} + \varepsilon_{2n} \leq V_{1n} + \varepsilon_{1n}, \dots, V_{J_n} + \varepsilon_{J_n} \leq V_{1n} + \varepsilon_{1n}),$$

or

$$P_n(1|C_n) = \Pr(\varepsilon_{2n} - \varepsilon_{1n} \leq V_{1n} - V_{2n}, \dots, \varepsilon_{J_n} - \varepsilon_{1n} \leq V_{1n} - V_{J_n}).$$

# Derivation

## Model

$$P_n(1|C_n) = \Pr(\varepsilon_{2n} - \varepsilon_{1n} \leq V_{1n} - V_{2n}, \dots, \varepsilon_{Jn} - \varepsilon_{1n} \leq V_{1n} - V_{Jn}).$$

## Change of variables

$$\xi_{1n} = \varepsilon_{1n}, \quad \xi_{in} = \varepsilon_{in} - \varepsilon_{1n}, \quad i = 2, \dots, J_n,$$

that is

$$\begin{pmatrix} \xi_{1n} \\ \xi_{2n} \\ \vdots \\ \xi_{(J_n-1)n} \\ \xi_{J_n n} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ -1 & 1 & \cdots & 0 & 0 \\ & & \vdots & & \\ -1 & 0 & \cdots & 1 & 0 \\ -1 & 0 & \cdots & 0 & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{1n} \\ \varepsilon_{2n} \\ \vdots \\ \varepsilon_{(J_n-1)n} \\ \varepsilon_{J_n n} \end{pmatrix}.$$

# Derivation

## Model in $\varepsilon$

$$P_n(1|C_n) = \Pr(\varepsilon_{2n} - \varepsilon_{1n} \leq V_{1n} - V_{2n}, \dots, \varepsilon_{Jn} - \varepsilon_{1n} \leq V_{1n} - V_{Jn}).$$

## Change of variables

$$\xi_{1n} = \varepsilon_{1n}, \quad \xi_{in} = \varepsilon_{in} - \varepsilon_{1n}, \quad i = 2, \dots, J_n,$$

## Model in $\xi$

$$P_n(1|C_n) = \Pr(\xi_{2n} \leq V_{1n} - V_{2n}, \dots, \xi_{Jnn} \leq V_{1n} - V_{Jnn}).$$

## Note

The determinant of the change of variable matrix is 1, so that  $\varepsilon$  and  $\xi$  have the same pdf

# Derivation

$$\begin{aligned}
 & P_n(1|\mathcal{C}_n) \\
 &= \Pr(\xi_{2n} \leq V_{1n} - V_{2n}, \dots, \xi_{J_n n} \leq V_{1n} - V_{J_n n}) \\
 &= F_{\xi_{1n}, \xi_{2n}, \dots, \xi_{J_n}}(+\infty, V_{1n} - V_{2n}, \dots, V_{1n} - V_{J_n n}) \\
 &= \int_{\xi_1 = -\infty}^{+\infty} \int_{\xi_2 = -\infty}^{V_{1n} - V_{2n}} \dots \int_{\xi_{J_n} = -\infty}^{V_{1n} - V_{J_n n}} f_{\xi_{1n}, \xi_{2n}, \dots, \xi_{J_n}}(\xi_1, \xi_2, \dots, \xi_{J_n}) d\xi, \\
 &= \int_{\varepsilon_1 = -\infty}^{+\infty} \int_{\varepsilon_2 = -\infty}^{V_{1n} - V_{2n} + \varepsilon_1} \dots \int_{\varepsilon_{J_n} = -\infty}^{V_{1n} - V_{J_n n} + \varepsilon_1} f_{\varepsilon_{1n}, \varepsilon_{2n}, \dots, \varepsilon_{J_n}}(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_{J_n}) d\varepsilon,
 \end{aligned}$$

# Derivation

$$P_n(1|C_n) = \int_{\varepsilon_1=-\infty}^{+\infty} \int_{\varepsilon_2=-\infty}^{V_{1n}-V_{2n}+\varepsilon_1} \cdots \int_{\varepsilon_{J_n}=-\infty}^{V_{1n}-V_{J_n}+\varepsilon_1} f_{\varepsilon_{1n}, \varepsilon_{2n}, \dots, \varepsilon_{J_n}}(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_{J_n})$$

$$P_n(1|C_n) = \int_{\varepsilon_1=-\infty}^{+\infty} \frac{\partial F_{\varepsilon_{1n}, \varepsilon_{2n}, \dots, \varepsilon_{J_n}}}{\partial \varepsilon_1}(\varepsilon_1, V_{1n}-V_{2n}+\varepsilon_1, \dots, V_{1n}-V_{J_n}+\varepsilon_1) d\varepsilon_1.$$

The random utility model:  $P_n(i|C_n) =$

$$\int_{\varepsilon=-\infty}^{+\infty} \frac{\partial F_{\varepsilon_{1n}, \varepsilon_{2n}, \dots, \varepsilon_{J_n}}}{\partial \varepsilon_i}(\dots, V_{in} - V_{(i-1)n} + \varepsilon, \varepsilon, V_{in} - V_{(i+1)n} + \varepsilon, \dots) d\varepsilon$$

# Random utility model

- The general formulation is complex.
- We will derive specific models based on simple assumptions.
- We will then relax some of these assumptions to propose more advanced models.

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