Aggregation and forecasting

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Aggregation

- So far, prediction of individual behavior
- In practice, not useful
- Need for forecast of aggregate demand:
  - number of trips
  - number of passengers
  - etc.

Aggregation

Linear models

$$h_n = \alpha + \beta y_n$$

where

- h_n: quantity of energy n consumed
- y_n: price of energy n

- If $\bar{y}$ is the average price
- $\bar{h} = \alpha + \beta \bar{y}$ is the average consumption

It does not work with choice models, because they are nonlinear

Aggregation

- "Travel/no travel" model, y_n income

$$\begin{array}{lrcl} \text{No travel} & V_1 & = & 0 \\ \text{Travel} & V_2 & = & -3+3y_n \end{array}$$

	Income	V1	V2	P1	P2
Household 1	1	0	0	50%	50%
Household 2	10	0	27	0%	100%
Avg. income	5.5	0	13.5	0%	100%
Avg. probabilities				25%	75%

Aggregation

- Choice model

$$P(i|x_n)$$

  where x_n gathers attributes of all alternatives and socio-economic characteristics of n

- If the population is composed of N individuals, the total expected number of individuals choosing i is

$$N(i) = \sum_{n=1}^{N} P(i|x_n)$$

- Hopeless to know x_n for every and each individual
- The sum would involve a lot of terms.
- The distribution of x could be used.

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Aggregation

- Assume that the distribution of x is continuous with PDF $p(x)$
- Then the share of the population choosing i is given by

$$\widehat{W}(i) = \int_x P(i|x)p(x)dx$$

- In practice, $p(x)$ is also unknown
- The integral may be cumbersome to compute

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Most practical method: sample enumeration

- Population is assumed to be segmented into homogenous segments
- Let n be an observation in the sample belonging to segment s
- Let W_s be the weight of segment s, that is

$$W_s = \frac{N_s}{S_s} = \frac{\text{\# persons in segment } s \text{ in population}}{\text{\# persons in segment } s \text{ in sample}}$$

- The number of persons choosing alt. i is estimated by

$$\widehat{N}(i) = \sum_{n \in \text{sample}} \sum_{s} W_s P(i|x_n) I_{ns}$$

where $I_{ns} = 1$ if individual n belongs to segment s, 0 otherwise

Aggregation

We can write

$$\begin{aligned}\widehat{N}(i) &= \sum_{n\in\text{sample}} \sum_s W_s P(i|x_n) I_{ns} \\ &= \sum_{n\in\text{sample}} P(i|x_n) \sum_s W_s I_{ns}\end{aligned}$$

The term $\sum_s W_s I_{ns}$ is the weight W_n of individuals n belonging to segment s.

The share of alt. i is estimated by $W(i) =$

$$\frac{1}{N} \sum_{n\in\text{sample}} P(i|x_n) W_n = \frac{1}{N} \sum_{n\in\text{sample}} P(i|x_n) W_n$$

Illustration

The travel model:

- “Travel/no travel” model, y_n income

$$P(\text{travel}) = \frac{e^{-3+3y_n}}{1+e^{-3+3y_n}}$$

- Population: $N =$ 200’000 persons
- Sample: $S =$ 500 persons
- Sampling rate: $S/N =$ 1/400

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Illustration

s	y_s	S_s	N_s	P(travel)	PS_s	PN_s
1	0	150	20000	4.7%	7	949
2	0.5	200	30000	18.2%	36	5473
3	1	40	50000	50.0%	20	25000
4	1.5	10	50000	81.8%	8	40879
5	2	50	30000	95.3%	48	28577
6	2.5	50	20000	98.9%	49	19780
		500	200000		169	120657

$$120657 \neq 400 \times 169 = 67542$$

People with low probability of travel are oversampled

Forecasting

- Modify x_n in the sample to reflect anticipated modifications
- Apply the sample enumeration again
- Examples:
  - Socio-economic characteristics: scenarios of future demographics (level of education, modification of incomes, etc.)
  - Attributes of alternatives:
    - Policy variables: variables that we control (price, level of service, etc.)
    - Scenarios: scenarios about the competition

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Example: characteristics

s	y_s	S_s	P(travel)	W_s	Trips
1	0	150	4.74%	133.33	949
2	0.5	200	18.24%	150	5473
3	1	40	50.00%	1250	25000
4	1.5	10	81.76%	5000	40879
5	2	50	95.26%	600	28577
6	2.5	50	98.90%	400	19780
					120657

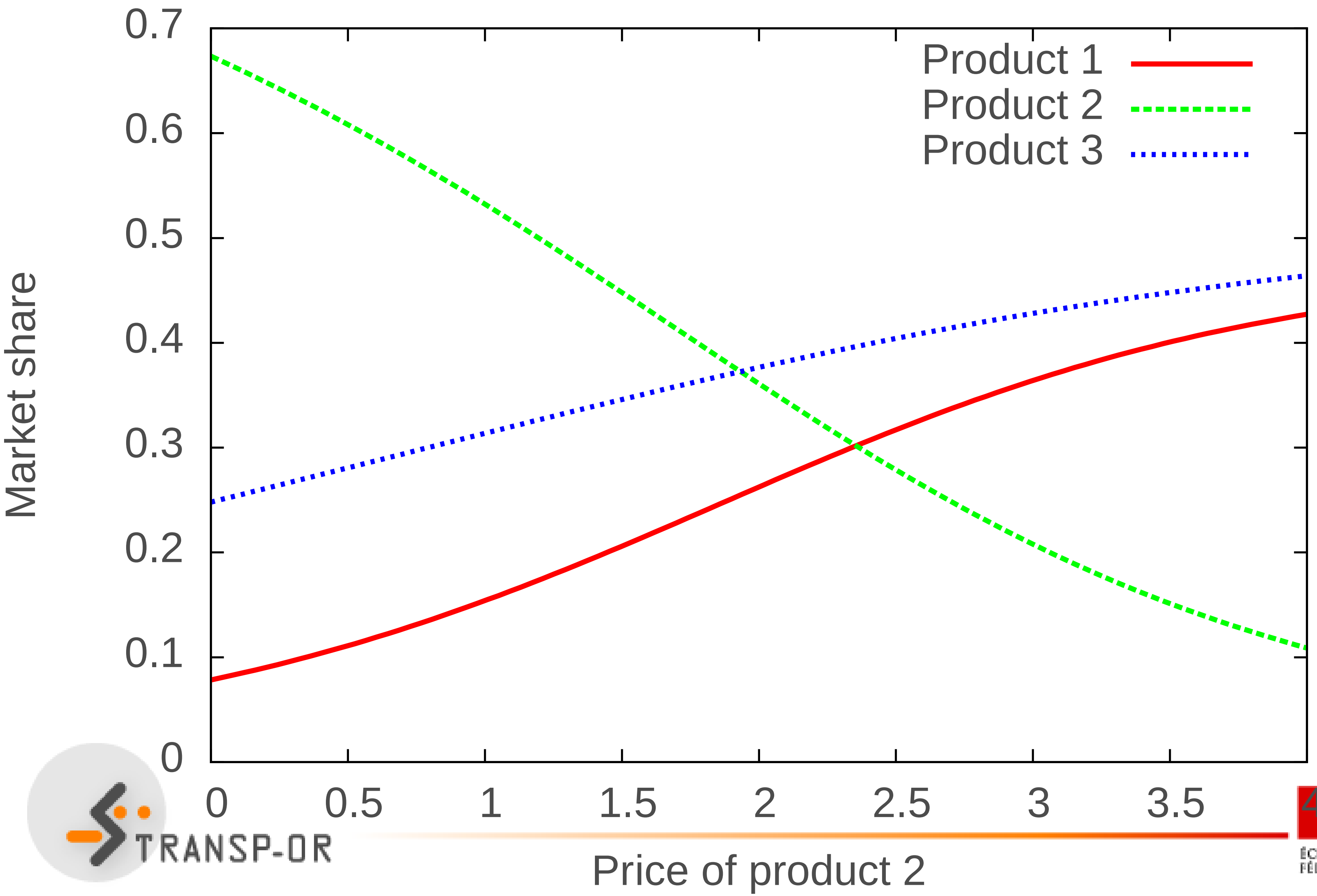
- Increase all salaries by 0.5
- What is the impact on the total number of trips?

Example: characteristics

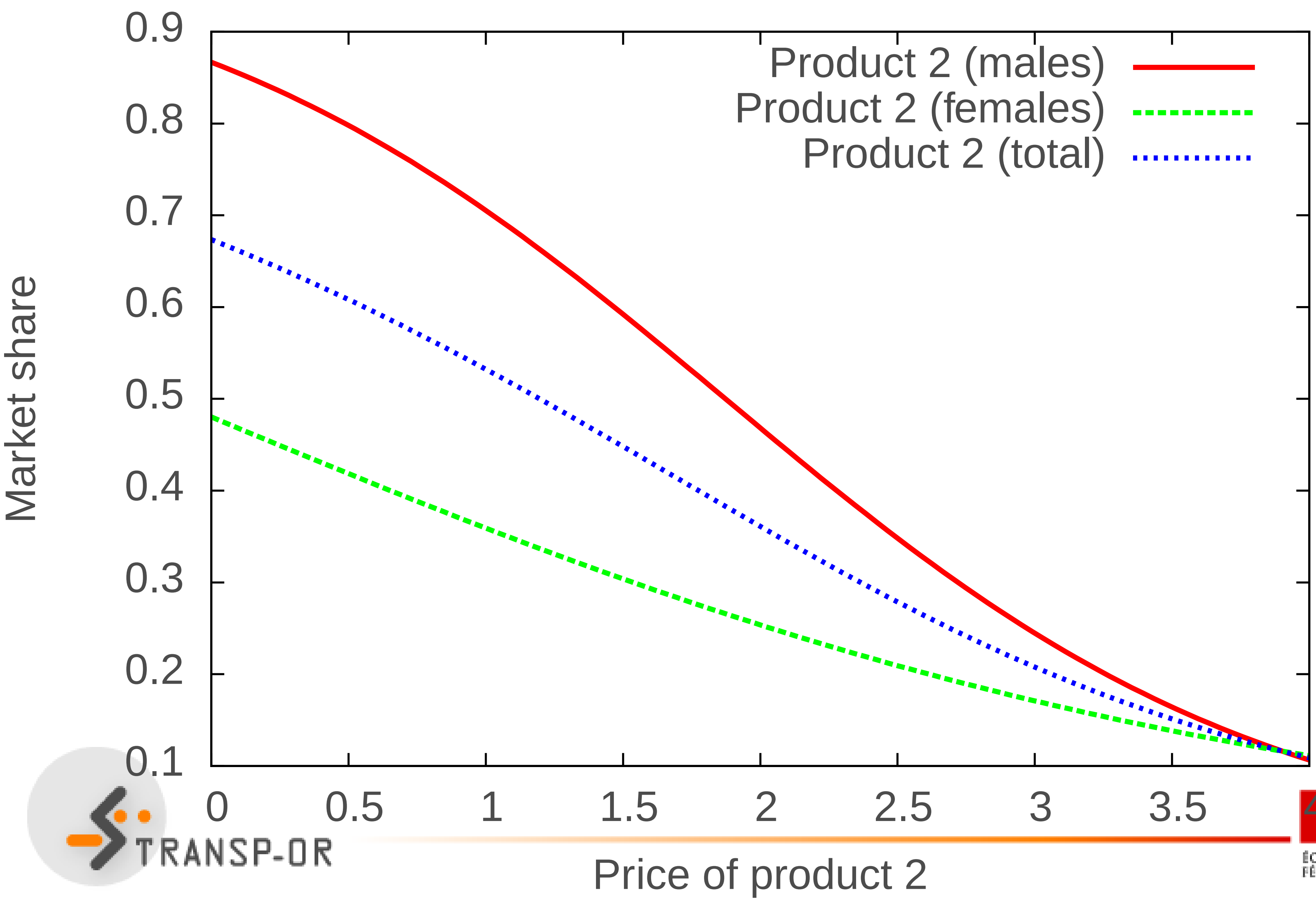
s	y_s	S_s	P(travel)	W_s	Trips
1	0.5	150	18.24%	133.33	3649
2	1	200	50.00%	150	15000
3	1.5	40	81.76%	1250	40879
4	2	10	95.26%	5000	47629
5	2.5	50	98.90%	600	29670
6	3	50	99.75%	400	19951
					156777

- Before: 120657
- After: 156777
- Increase: about 30%

Example: attributes



Example: attributes



Price optimization

- Choice model captures demand
- Demand is elastic to price
- Predicted demand varies with price, if it is a variable of the model
- In principle, the probability to use/purchase an alternative decreases if the price increases.
- The revenue per user increases if the price increases.
- Question: what is the optimal price to optimize revenue?

In short:

- Price$\uparrow \Rightarrow$ profit/customer$\uparrow$ and number of customers $\downarrow$
- Price$\downarrow \Rightarrow$ profit/customer$\downarrow$ and number of customers $\uparrow$
- What is the best trade-off?

Revenue calculation

Number of persons choosing alternative i in the population

$$\hat{N}(i) = \sum_{n \in \text{sample}} P(i|x_n, p_{in}) W_n$$

where

- p_{in} is the price of item i for individual n
- x_n gathers all other variables corresponding to individual n
- $P(i|x_n, p_{in})$ is the choice model
- W_n is the weight of individual n.

Revenue calculation

The total revenue from i is therefore:

$$R_i = \sum_{n \in \text{sample}} W_n P(i|x_n, p_{in}) p_{in}$$

If the price is constant across individuals, we have

$$R_i = p_i \sum_{n \in \text{sample}} W_n P(i|x_n, p_i)$$

Price optimization

Optimizing the price of product i is solving the problem

$$\max_{p_i} p_i \sum_{n \in \text{sample}} W_n P(i|x_n, p_i)$$

Notes:

- It assumes that everything else is equal
- In practice, it is likely that the competition will also adjust the prices

Illustrative example

A binary logit model with

$$
\begin{aligned}
V_1 &= \beta_p p_1 - 0.5 \\
V_2 &= \beta_p p_2
\end{aligned}
$$

so that

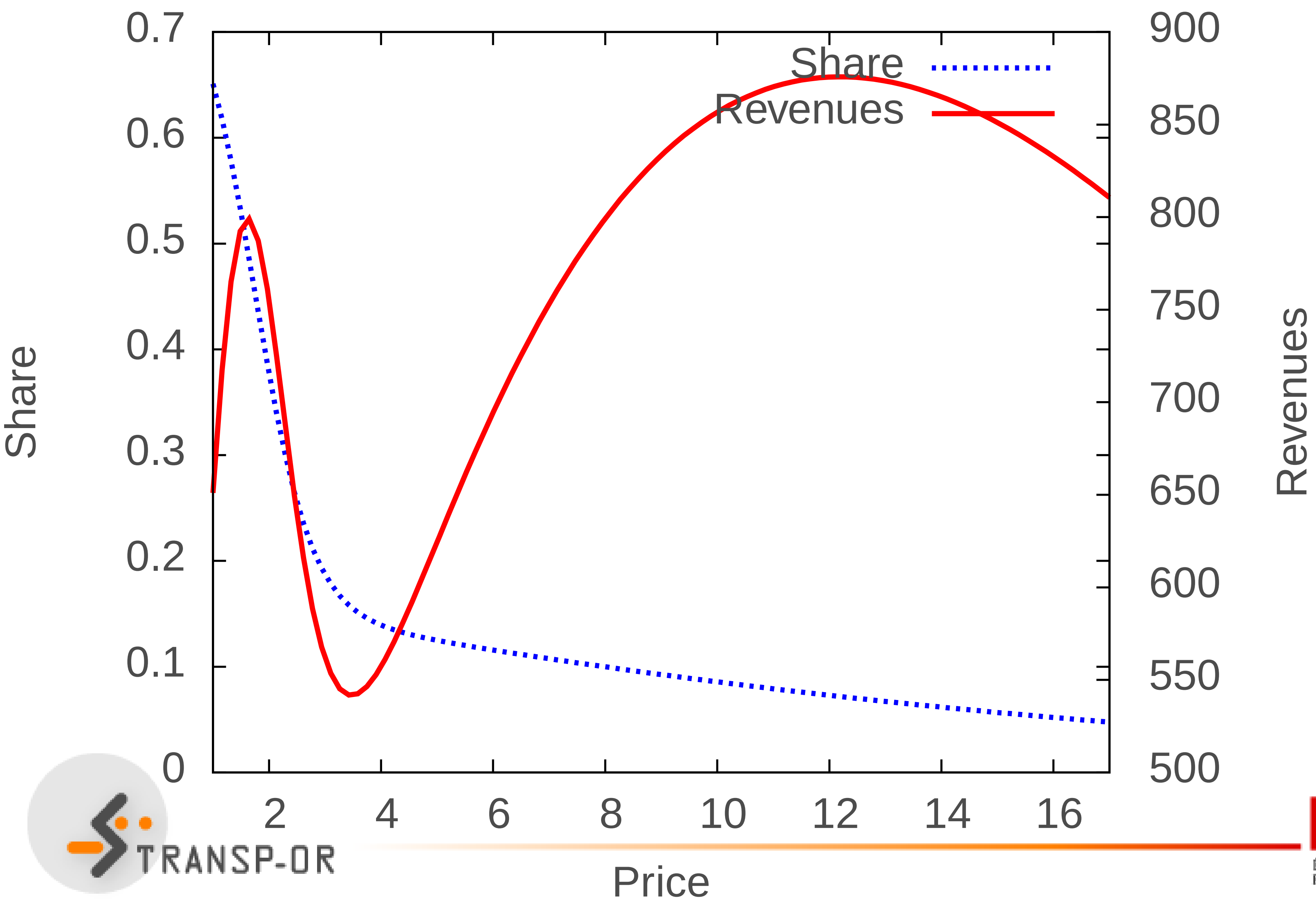
$$P(1|p) = \frac{e^{\beta_p p_1 - 0.5}}{e^{\beta_p p_1 - 0.5} + e^{\beta_p p_2}}$$

Two groups in the population:

- Group 1: $\beta_p = -2$, $N_s = 600$
- Group 2: $\beta_p = -0.1$, $N_s = 400$

Assume that $p_2 = 2$.

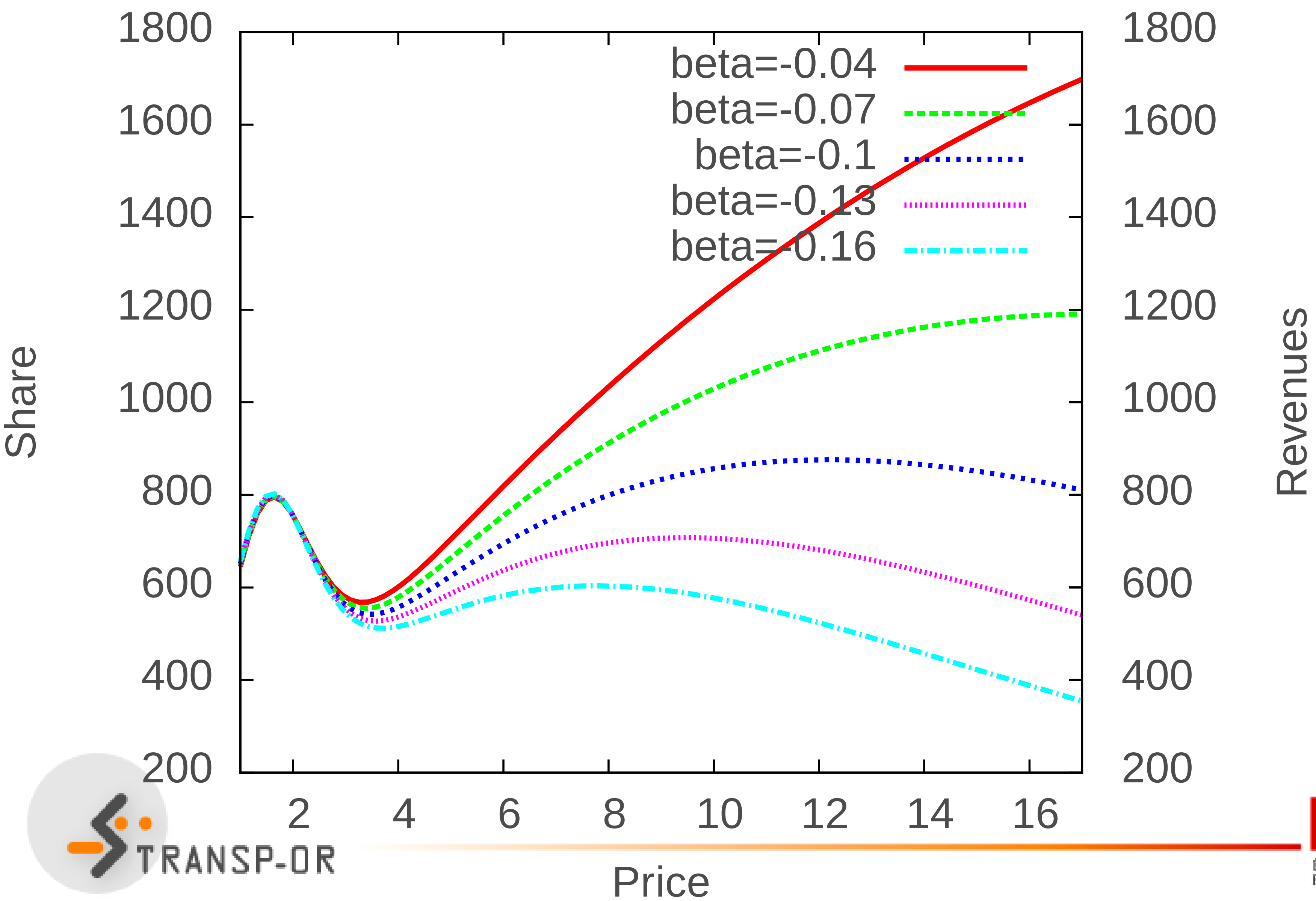
Illustrative example



Sensitivity analysis

- Parameters are estimated, we do not know the real value
- 95% confidence interval: $[\widehat{\beta}_p - 1.96\sigma, \widehat{\beta}_p + 1.96\sigma]$
- Perform a sensitivity analysis for β_p in group 2

Sensitivity analysis



Comments

- The estimation sample already contains a sample of the variables x.
- It is convenient to use the same sample for estimation and sample enumeration.
- It is valid only if revealed preference data (i.e. revealing real behavior) is used.
- Stated peference data (i.e. choice based on hypothetical situation) cannot be used for sample enumeration.