Mathematical modeling of behavior

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Introduction

- What kind of behavior can be mathematically modeled?

Introduction

Psychohistory

Branch of mathematics which deals with the reactions of human conglomerates to fixed social and economic stimuli. The necessary size of such a conglomerate may be determined by Seldon's First Theorem.

Encyclopedia Galactica, 116th Edition (1020 F.E.)
Encyclopedia Galactica Publishing Co., Terminus

Motivation: shorten the period of barbarism after the Fall of the Galactic Empire

Asimov, I. (1951) *Foundation*, Gnome Press

In this course...

- Individual behavior (vs. aggregate behavior)
- Theory of behavior which is
  - descriptive: how people behave and not how they should
  - abstract: not too specific
  - operational: can be used in practice for forecasting
- Type of behavior: choice

Motivations

Motivations

"It is our choices that show what we truly are, far more than our abilities" Albus Dumbledore

"Liberty, taking the word in its concrete sense, consists in the ability to choose." Simone Weil (French philosopher, 1909-1943)

Field :	Type of behavior:
▶Marketing	▶Choice of a brand
▶Transportation	▶Choice of a transportation mode
▶Politics	▶Choice of a president
▶Management	▶Choice of a management policy
▶New technologies	▶Choice of investments

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Applications

Case studies

- Choice-lab marketing
  - Context: B2B, data provider (financial, demographic, etc.)
  - Objective: understand why clients quit
- Quebec energy
  - Context: space and water heating in households
  - Objective: importance of the type of household and price
- Transportation mode choice in the Netherlands
  - Context: car vs rail in Nijmegen
  - Objective: sensitivity to travel time and cost, inertia.

Applications

- Swissmetro
  - Context: new transportation technology
  - Objective: demand pattern, pricing
- Residential telephone services
  - Context: flat rate vs. measured
  - Objective: offer the most appropriate service
- Airline itinerary choice
  - Context: questionnaire about itineraries across the US
  - Objective: help airlines and aircraft manfacturer to design a better offer

Importance

Daniel

L.

McFadden

1937–

- UC Berkeley 1963, MIT 1977, UC Berkeley 1991
- Laureate of *The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel 2000*
- Owns a farm and vineyard in Napa Valley
- “Farm work clears the mind, and the vineyard is a great place to prove theorems”

Example

Voice over internet protocol (VoIP)

- What is the market penetration?
- How will the penetration change in the future?
- Assumption: level of education is an important explanatory factor

Data collection

- sample of 600 persons, randomly selected
- Two questions:
  1. Do you subscribe to voice over IP? (yes/no)
  2. How many years of education have you had? (low/medium/high)

Example

- Contingency table

	Education			
VoIP	Low	Medium	High	
Yes	10	100	120	230
No	140	200	30	370
	150	300	150	600

- Penetration in the sample: $230/600 = 38.3\%$
- Forecasting: need for a model

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Example: a model

Type of variables:

- dependent, or endogenous: what we explain
- independent, exogenous or explanatory: how we explain

Model:

- Causal relationship between the independent and independent variables
- Based on theory, assumptions.
- Probabilistic.

Example: a model

- Dependent variable:

$$y = \begin{cases} 1 & \text{if subscriber} \\ 2 & \text{if not subscriber} \end{cases}$$

Discrete dependent variable

- Independent or explanatory variable

$$x = \begin{cases} 1 & \text{if level of education is low} \\ 2 & \text{if level of education is medium} \\ 3 & \text{if level of education is high} \end{cases}$$

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Example: inference

- Market penetration in the sample: $\hat{p}(y = 1)$
- Market penetration in the population: $p(y = 1)$ estimated by $\hat{p}(y = 1)$
- Joint probabilities: $\hat{p}(y = 1, x = 2) = 100/600 = 0.1667$
- Marginal probabilities:$\hat{p}(y = 1) = \sum_{k=1}^{3} \hat{p}(1, k) = 10/600 + 100/600 + 120/600 = 0.383$

Example: causal model

- Behavioral assumption: choice is explained by level of education
- Causal relationship assumed to be stable over time
- Conditional probabilities: $p(y = 1|x = 2)$
- Warning: not all conditional probabilities represent causal relationships
- Model parameters:

$$\begin{aligned} p(y = 1|x = 1) &= \pi_1 \\ p(y = 1|x = 2) &= \pi_2 \\ p(y = 1|x = 3) &= \pi_3 \end{aligned}$$

Example: estimation

$$\begin{aligned} \hat{p}(y=1, x=2) &= \hat{p}(y=1|x=2)\hat{p}(x=2) \\ \hat{p}(y=1|x=2) &= \hat{p}(y=1, x=2)/\hat{p}(x=2) \\ &= 0.1667/0.5 = 0.333 \end{aligned}$$

Estimates of the parameters:

$$\begin{aligned} p(y=1|x=1) &= \pi_1 = 0.067 \\ p(y=1|x=2) &= \pi_2 = 0.333 \\ p(y=1|x=3) &= \pi_3 = 0.8 \end{aligned}$$

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Example: testing

- Do these estimates make sense?
- Do they match our a priori expectations?
- As the number of years of education increases, the market penetration increases

Example: testing

- Are the estimates (based on a sample) far from the true values?
- Let's draw many samples of 600 observations in the population
- For each of them, estimate the parameters of the model
- We obtain a distribution
- π_k is the sample mean of Bernoulli random variables
- Unbiased estimator
- Standard deviation:

$$s_k = \sqrt{\frac{\sigma^2}{N_k}} = \sqrt{\frac{\pi_k(1-\pi_k)}{N_k}}$$

- Estimated standard deviation:

$$\widehat{s}_k = \sqrt{\frac{\widehat{\pi}_k(1-\widehat{\pi}_k)}{N_k}}$$

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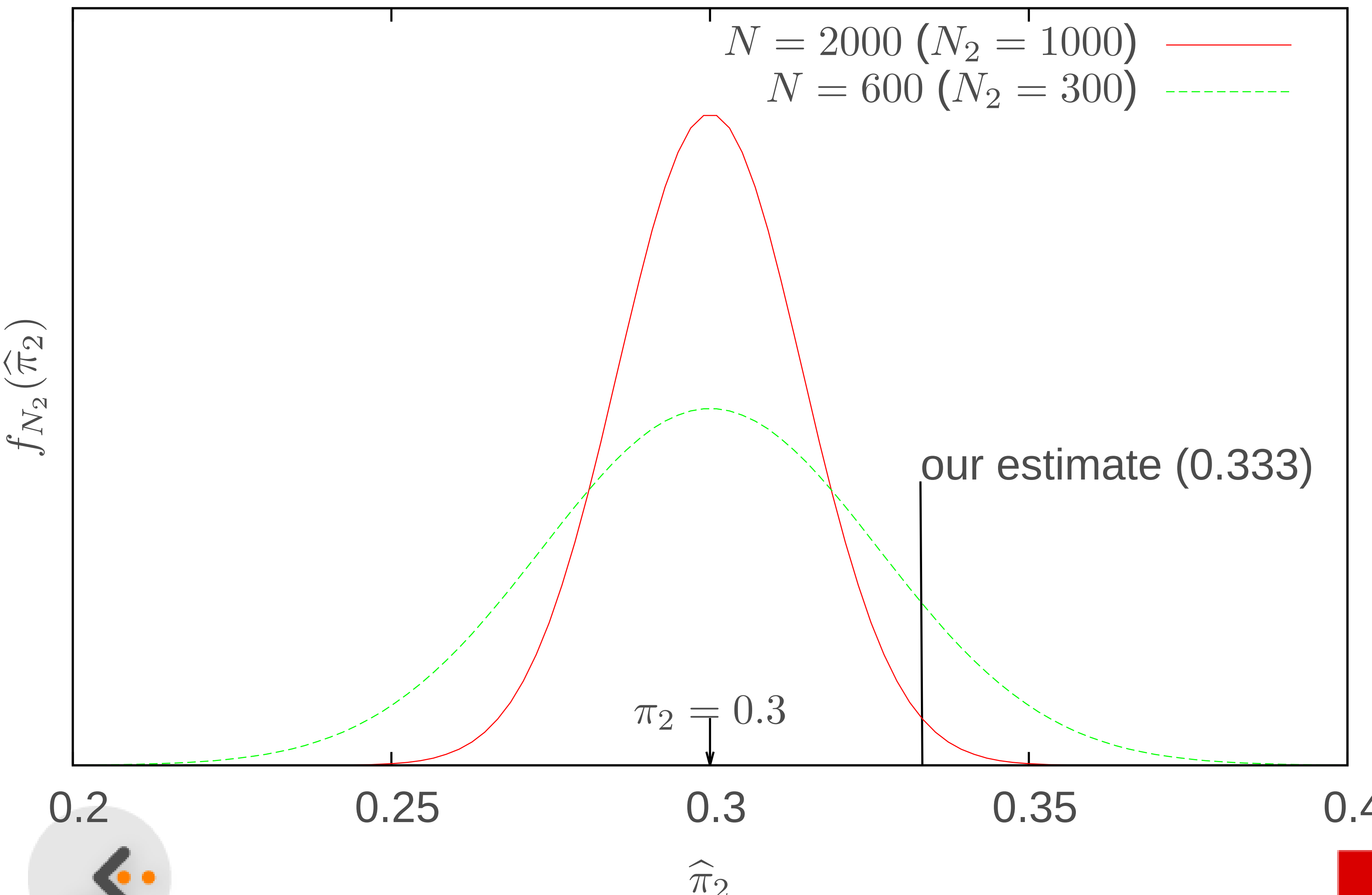
Example: testing



Example: forecasting

- Model:

$$\begin{aligned} p(y=1|x=1) &= \pi_1 = 0.067 \\ p(y=1|x=2) &= \pi_2 = 0.333 \\ p(y=1|x=3) &= \pi_3 = 0.8 \end{aligned}$$

  where π_1, π_2, π_3 are estimated parameters

- Assumption: future level of education: 10%-60%-30%

$$\begin{aligned} p(y=1) &= \sum_{i=1}^{3} p(y=1|x=i)p(x=i) \\ &= 0.1\pi_1 + 0.6\pi_2 + 0.3\pi_3 \\ &= 44.67\% \end{aligned}$$

Example: forecasting

- If the level of education increases
- from 25%-50%-25% to 10%-60%-30%
- Market penetration of VoIP will increase
- from 38.33 % to 44.67%

In summary

- $p(x = j)$ can be easily obtained and forecast
- $p(y = i|x)$ is the behavioral model to be developed

Outline

- Introduction and examples
- Review of relevant concepts in probability and statistics
- Choice theory
- Binary choice
- Multiple alternatives
- Tests
- Nested Logit model
- Multivariate Extreme Value models
- Forecasting
- Sampling
- Mixtures of models
- Latent variables