Results from midterm

Summary of results and common problems

Midterm contained 7 questions

- Q1: logit/probit
- Q2: IIA property
- Q3: constants/choice probability
- Q4: Log likelihood at zero
- Q5: Route choice
- Q6: Gender segmentation
- Q7: Non-linear transform

Summary of midterm results: critical questions

- Generally poor results (score was below 50% cutoff) for:
  - Q2: IIA property
  - Q4: Log likelihood at zero
  - Q6: Gender segmentation
  - Q7: Non-linear transform

Brief remarks on Q1, Q3, Q4

- Q1 (5 points)
  Logit – Error terms are i.i.d. across both individuals and alternatives;
  Probit – Error terms are i.i.d only across individuals (covariances among alternatives are not zero)!!!
- Q3 (10 points)
  Most students could write the logit formula for 3 alternatives and correctly compute the effect of shifting the normalized constant
  Choice probabilities are NOT affected by the normalization (almost all correct)
- Q4 (10 points)
  LL(0) is NOT the log likelihood in the beginning of the estimation (iteration 0). It is the likelihood of the null (naive) model where all the parameters are zero. It is equivalent to having NO model at all – all the alternatives have equal probability to be chosen.

Q2: IIA property (15 points)

- ***Question***

  1) Describe the Independence from Irrelevant Alternatives property (IIA).

  2) Prove that it holds in case of a logit model.

  3) Explain the red bus/blue bus paradox and why the IIA property doesn't hold in this case.

- ***Problems***

  - Most students did not fully understood the core of the property.

  - There was no proper answer to the red bus/ blue bus paradox question.

  - **NOTE:** The logit model and the *resulting* IIA do NOT imply that if we have 2 alternatives the probability of each one would be $\frac{1}{2}$!!! This is only the case if we **assume** that the utilities of the 2 alternatives are *equal*.

Q2: Interpretation of the IIA

1) The IIA implies that for a specific individual the ratio of the choice probabilities of any two alternatives is entirely unaffected by the presence or absence of any other alternatives in the choice set and by their systematic utilities.

- IIA is violated when i) alternatives share *unobserved* attributes (error terms are correlated); ii) error terms of the alternatives have *different variances* (violation of the identically distributed assumption).
- IIA causes highly restrictive *substitution patterns* as the ratio of the choice probabilities of two alternatives is independent of any other alternative in the choice set, resulting in proportional substitution across alternatives.

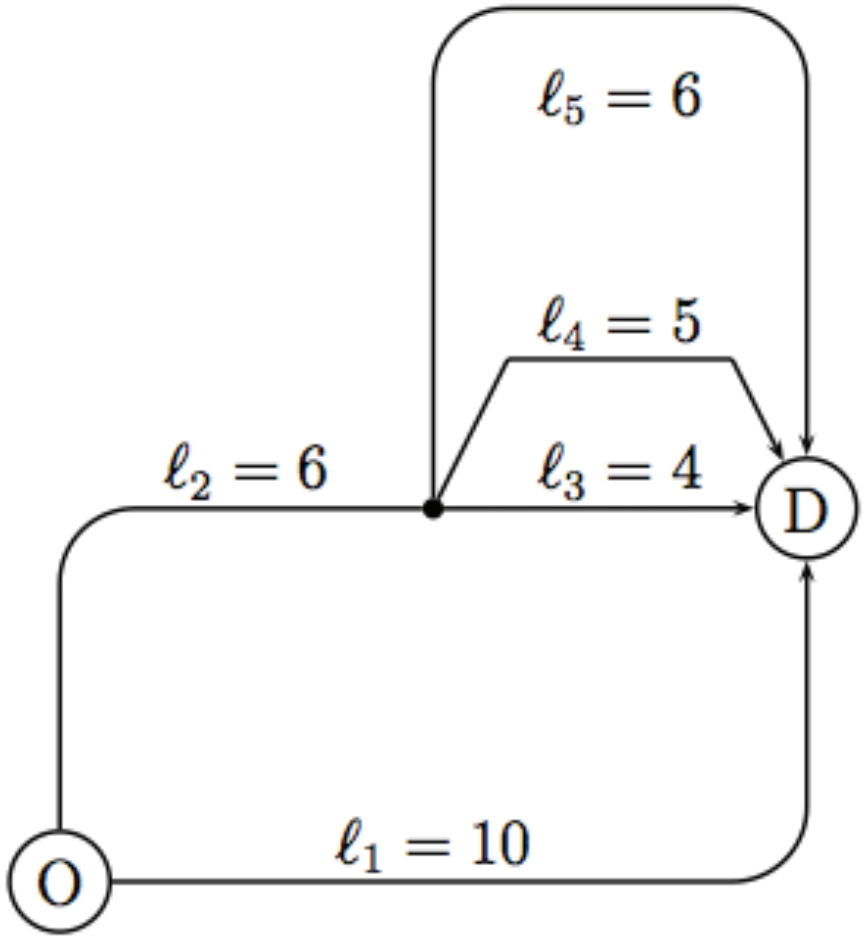
2) The logit model exhibits the IIA

$$\frac{P_n(i)}{P_n(l)} = \frac{\frac{e^{\mu V_{in}}}{\sum_{j \in C_n} e^{\mu V_{jn}}}}{\frac{e^{\mu V_{ln}}}{\sum_{j \in C_n} e^{\mu V_{jn}}}} = \frac{e^{\mu V_{in}}}{e^{\mu V_{ln}}} = e^{\mu (V_{in} - V_{ln})}$$

Q2: Interpretation of the IIA (cont.)

3) IIA is a *resulting* property of the logit formula deriving from the assumption about the distribution of the unobserved utility. The core of the problem is that the sources of errors contributing to the disturbances must do so in a way such that the total disturbances are independent. This does not hold in the case of red buses and blue buses, as both these alternatives share all the **unobserved** characteristics of buses (the disturbances are more reasonably assumed to be perfectly correlated).

Q5: Route choice (15 points)



Q5.1: Route choice

Q: What is the choice set?

A: $C = \{\{1\}, \{2,3\}, \{2,4\}, \{2,5\}\}$, where $\{i,j\}$ means that the decision maker chooses first link i and then link j to reach destination D.

Q5.2: Route choice

Q: Specify the utility function for each alternative.

A: As the decision maker are considering only the length of the route as an attribute, we can write:

$$\begin{aligned} U_1 &= ASC_1 + \beta_1 \cdot l_1 + \varepsilon_1 \\ U_{2,j} &= ASC_j + \beta_j (l_2 + l_j) + \varepsilon_j \end{aligned}$$

and we fix ASC_1 to 0.

Q5.3: Route choice

Q: Explain why a logit model is not appropriate in this context.

A: One of the properties of logit models is the Independence from Irrelevant Alternatives (IIA). All alternatives are supposed to be independent from each other. It is not the case in route choice when you have an overlapping segment. This is similar to the red bus / blue bus paradox. To overcome the IIA problem of logit, nested logit models were developed. It is useful when alternatives are correlated.

Q5.4: Route choice

Q: Propose a specification of another model to capture this situation. Describe the hypotheses associated with the specification. Explain the role and the interpretation of the parameters.

A: We use a nested logit model. The nest is composed of all routes going through link 2.

$$
\begin{aligned}
U_1 &= ASC_1 + \beta_1 \cdot l_1 + \varepsilon_1 \\
U_{2,3} &= ASC_3 + \beta_3 (l_2 + l_3) + \varepsilon_3 + \varepsilon_2 \\
U_{2,4} &= ASC_4 + \beta_4 (l_2 + l_4) + \varepsilon_4 + \varepsilon_2 \\
U_{2,5} &= ASC_5 + \beta_5 (l_2 + l_5) + \varepsilon_5 + \varepsilon_2
\end{aligned}
$$

where ε_1 and $+\varepsilon_2$ are nest-specific errors (shared by all alternatives in the nest) and ε_3, ε_4 and ε_5 are alternative-specific errors.

$$\varepsilon_1, \varepsilon_2 \sim EV(0, \mu_1)$$

Q6: Gender segmentation (20 points)

For this question we have segmented the population in 3 groups (female, male, NA) and we present 4 model outputs with an identical specification of utilities: one model for each sub-group and one for the full sample.

- An identical segmentation was shown in the lab.
- Many leave question empty or do not even propose a formal method to test for segmentation
- Of those who propose the correct LR test a full score is common (need to explain test procedure, χ^2 test, critical value with correct degrees of freedom, and give a comment on the test statistic
- For 6.2 very limited comments (many lost points)

Q6.1: Perfect answer by student, 20/20

Q: Test whether a market segmentation for men and women is relevant in this case

A: "To test if a market segmentation for men and women is relevant in this case we can do a **likelihood ratio test**.

log likelihood woman -596.017 (5 par)
log likelihood male -637.707 (5 par)
log likelihood NA -57.691 (5 par)
log likelihood full -1310.070 (5 par)

we consider the **unrestricted** model with gender segmentation which has a likelihood of $(-596.02\ -637.71\ -57.69) = -1291.42$ with 15 par. and the **restricted** model which has a likelihood of -1310.07 and 5 par., we compute:

$$-2 * (-1310 + 1291) = 37$$

and compare it with $\chi^2_{10,0.05}$ if $37 > \chi^2_{10,0.05}$ a market segmentation is relevant in this case, if $37 < \chi^2$ it is not relevant."

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Q6.2: Perfect answer by student, 20/20

Q: Interpret the results

A: "The first thing we have to check before starting to interpret the results is that convergence of the log likelihood function has been reached so that the parameters are valid. If in (6.1.) we had found that market segmentation is convenient then we would have to analyze the model with the β reported on the three first tables and if not we would have to interpret the last one (full).
(...)
with market segmentation: for the results of the **woman** table we can see that the β's associated with cost, distance and time are negative. It means that these attributes have a negative impact on the utility. We also notice the parameters are significantly different from zero
(...)

Q6.2: Perfect answer by student, 20/20 (cont.)

A contd. (...)

The larger value for β-time in the women segment than in the men means that **women are more sensitive to time** than men. (...)

We can also interpret the value of the **ASC**. For example for men there is an intrinsic preference towards the alternative 'car' compared to the reference (train) that is not captured by the other explanatory variables"

Q7: Box-Cox transform

- Very classical question, seen in labs in details.
- Most often, points are lost because answers are not precise nor complete enough.
- Question: “Describe assumptions associated with this specification and comment on estimation results”
- Full answer must mention:
  - Global properties of Box-Cox transform
  - Specific results in this case study (exactly similar to what was done in the lab)

Q7: Box-Cox transform

- Global properties of Box-Cox transform
  - Name the transformation: Box-Cox
  - Give the interpretation: not linear, not same marginal value of time e.g.
  - Mention extreme cases: $\lambda = 0, \lambda = 1$
  - Mention that it is not linear-in-parameter any more, λ is estimated
- Specific results in this case study
  - Test $\lambda = 0, \lambda = 1$
  - Interpret the sign of the corresponding β

Q7: Perfect answer by student, 20/20

"We assume that time is **not affecting our utility in a linear way**. It is a **Box-Cox** transformation. To test if this model is significantly better than the linear one we do a t-test to see if λ is **significantly different from 1**.
If $|\frac{0.375-1}{0.0374}| > 1.96$ then $H - 0 : \lambda = 1$ **would be rejected**.
(...)
Also, as expected the βs for cost, time and distance are **negative**.
With the t-test we reject $\lambda = 0$ so this mean that the Box-Cox transformation we did is **better than a logarithm transformation**.
The fact that λ is smaller than one means that utility decreases (because the sign of the β is negative) **at a slower rate than linear** when time is increased.
(...)"