Discrete Panel Data

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Outline

- Introduction
- Static model
- Static model with panel effect
- Dynamic model
- Dynamic model with panel effect
- Application

Introduction

- Type of data used so far: *cross-sectional*.
- Cross-sectional: observation of individuals at the same point in time.
- Time series: sequence of observations.
- **Panel data** is a combination of comparable time series.

Introduction

- Panel Data: data collected over multiple time periods for the same sample of individuals.
- Multidimensional:

Individual	Day	Price of stock 1	Price of stock 2	Purchase
n	t	x_{1nt}	x_{2nt}	i_{int}
1	1	12.3	15.6	1
1	2	12.1	18.6	2
1	3	11.0	25.3	2
1	4	9.2	25.1	0
2	1	12.3	15.6	2
2	2	12.1	18.6	0
2	3	11.0	25.3	0
2	4	9.2	25.1	1

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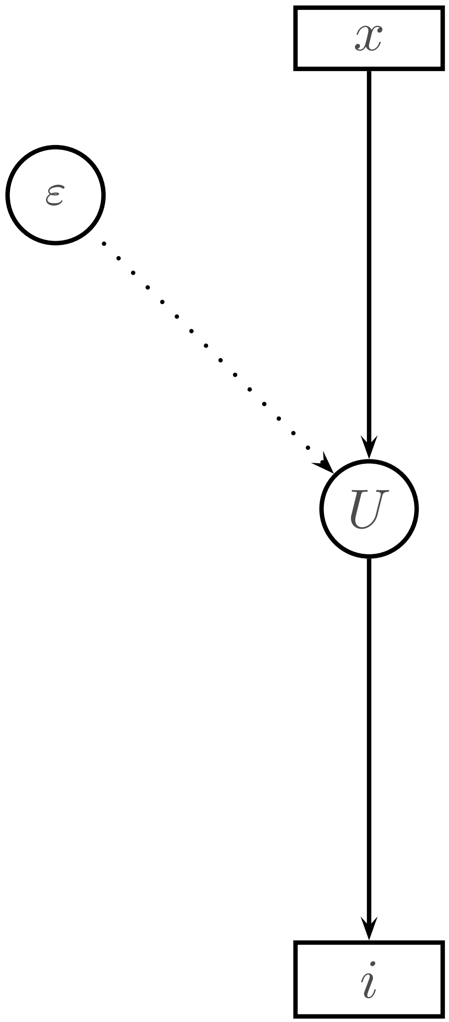
Introduction

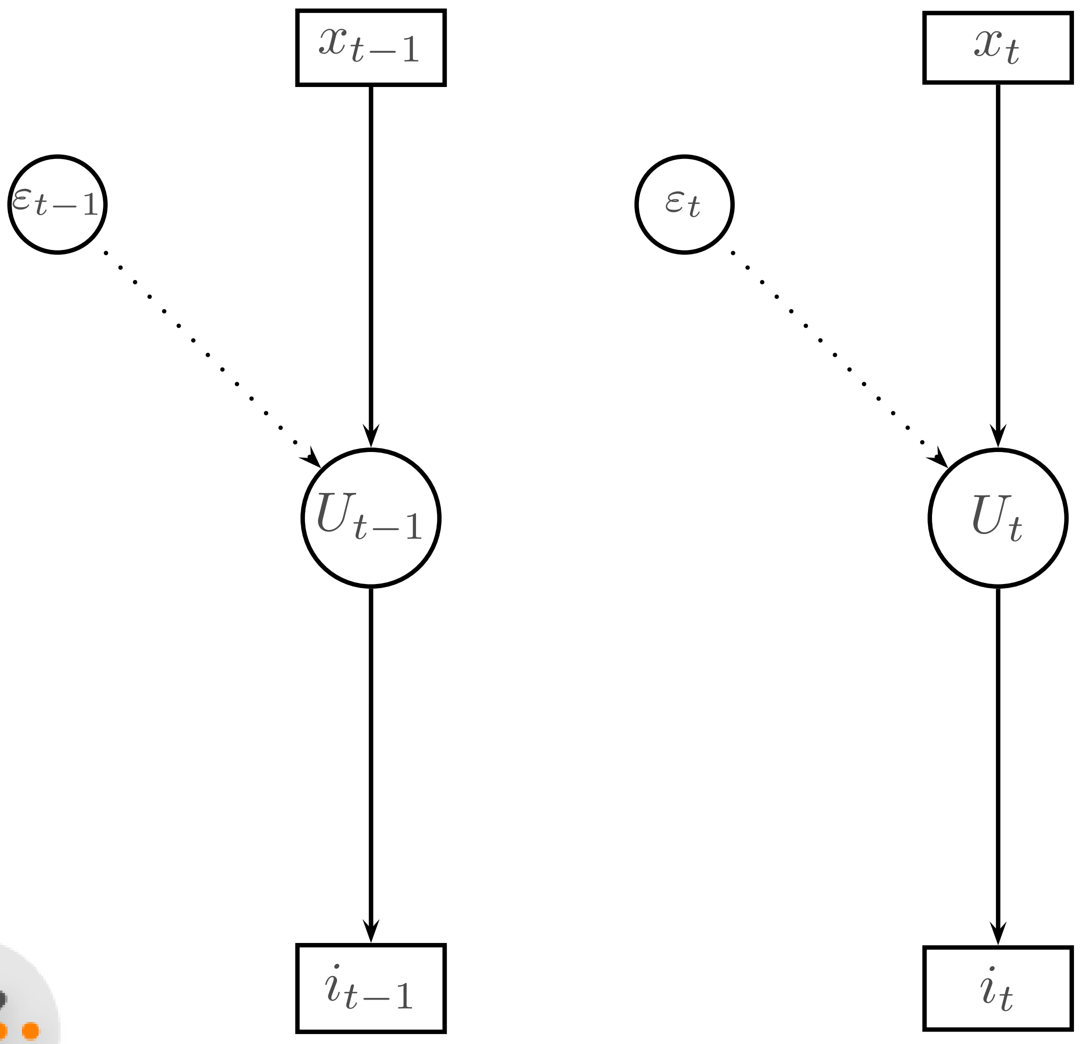
Examples of discrete panel data:

- People are interviewed monthly and asked if they are working or unemployed.
- Firms are tracked yearly to determine if they have been acquired or merged.
- Consumers are interviewed yearly and asked if they have acquired a new cell phone.
- Individual's health records are reviewed annually to determine onset of new health problems.

Model: single time period



Static model



Static model

The model:

- Utility:

$$U_{int} = V_{int} + \varepsilon_{int}, \; i \in \mathcal{C}_{nt}.$$

- Logit:

$$P(i_{nt}) = \frac{e^{V_{int}}}{\sum_{j \in \mathcal{C}_{nt}} e^{V_{jnt}}}$$

- Estimation: contribution of individual n to the log likelihood:

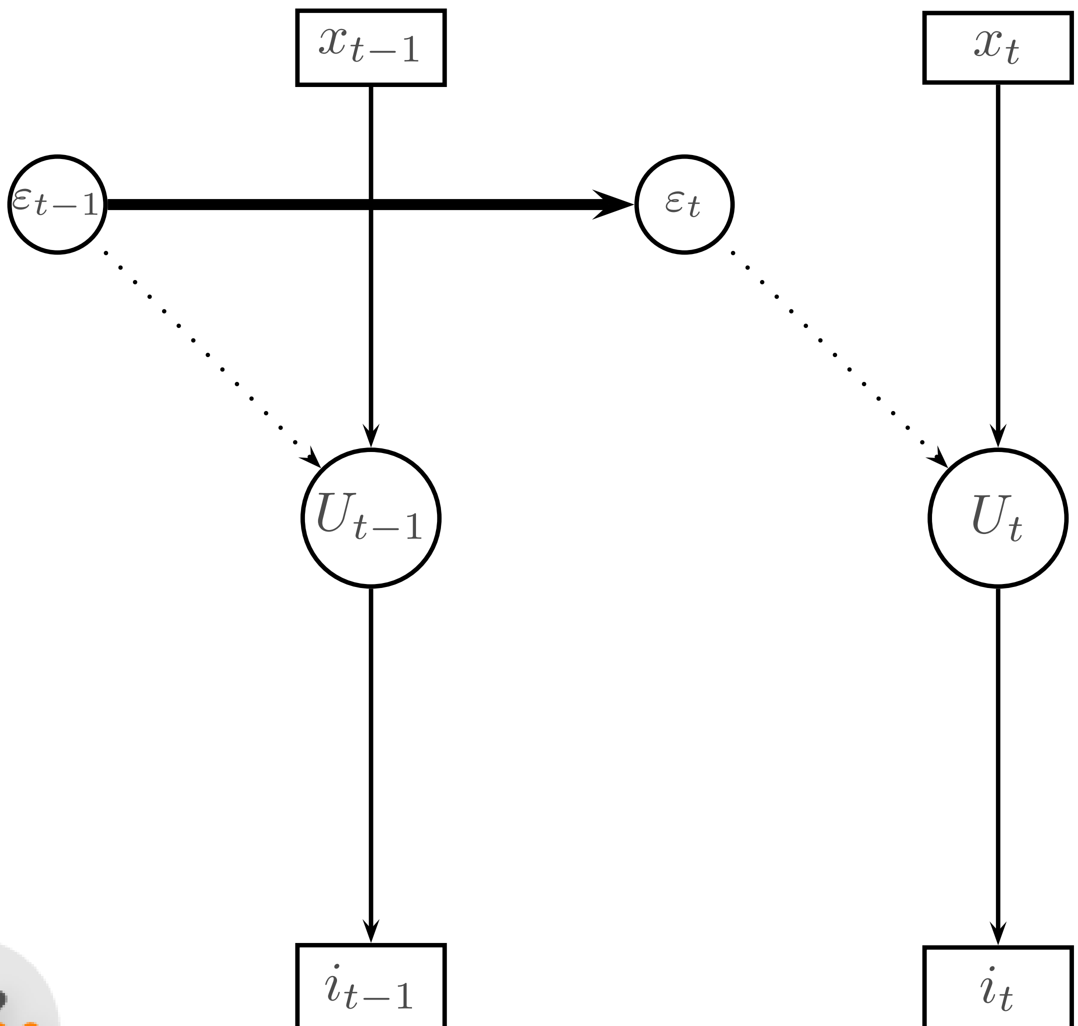
$$P(i_{n1}, i_{n2}, \ldots, i_{nT}) = P(i_{n1})P(i_{n2}) \cdots P(i_{nT}) = \prod_{t=1}^{T} P(i_{nt})$$

$$\ln P(i_{n1}, i_{n2}, \ldots, i_{nT}) = \ln P(i_{n1}) + \ln P(i_{n2}) + \cdots + \ln P(i_{nT}) = \sum_{t=1}^{T} \ln P(i_{nt})$$

Static model: comments

- Views observations collected through time as supplementary cross sectional observations.
- Standard software for cross section discrete choice modeling may be used directly.
- Simple, but there are two important limitations:
  1. Serial correlation:
     - unobserved factor persist over time,
     - in particular, all factors related to individual n,
     - $\varepsilon_{in(t-1)}$ cannot be assumed independent from ε_{int}.
  2. Dynamics:
     - Choice in one period may depend on choices made in the past.
     - e.g. learning effect, habits.

Dealing with serial correlation



Panel effect

- Relax the assumption that ε_{int} are independent across t.
- Assumption about the source of the correlation:
  - individual related unobserved factors,
  - persistent over time.
- The model:

$$\varepsilon_{int} = \alpha_{in} + \varepsilon'_{int}$$

- It is also known as
  - agent effect,
  - unobserved heterogeneity.

Panel effect

- Assuming that ε'_{int} are independent across t,
- we can apply the static model.
- Two versions of the model:
  - with fixed effect: α_{in} are unknown parameters to be estimated,
  - with random effect: α_{in} are distributed.

Static model with fixed effect

The model:

- Utility:

$$U_{int} = V_{int} + \alpha_{in} + \varepsilon'_{int},\ i \in \mathcal{C}_{nt}.$$

- Logit:

$$P(i_{nt}) = \frac{e^{V_{int}+\alpha_{in}}}{\sum_{j\in\mathcal{C}_{nt}} e^{V_{jnt}+\alpha_{jn}}}$$

- Estimation: contribution of individual n to the log likelihood:

$$P(i_{n1}, i_{n2}, \ldots, i_{nT}) = P(i_{n1})P(i_{n2})\cdots P(i_{nT}) = \prod_{t=1}^{T} P(i_{nt})$$

$$\ln P(i_{n1}, i_{n2}, \ldots, i_{nT}) = \ln P(i_{n1}) + \ln P(i_{n2}) + \cdots + \ln P(i_{nT}) = \sum_{t=1}^{T} \ln P(i_{nt})$$

Static model with fixed effect

Comments:

- α_{in} capture permanent taste heterogeneity.
- For each n, one α_{in} must be normalized to 0.
- The α's are estimated consistently only if $T \to \infty$.
- This has an effect on the other parameters that will be inconsistently estimated.
- In practice,
  - T is usually too short,
  - the number of α parameters is usually too high,

  for the model to be consistently estimated and practical.

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Static model with random effect

- Denote α_n the vector gathering all parameters α_{in}.
- Assumption: α_n is distributed with density $f(\alpha_n)$.
- For instance:

$$\alpha_n \sim N(0, \Sigma).$$

- We have a *mixture* of static models.
- Given α_n, the model is static, as ε'_{int} are assumed independent across t.

Static model with random effect

The model:

- Utility:

$$U_{int} = V_{int} + \alpha_{in} + \varepsilon'_{int},\ i \in \mathcal{C}_{nt}.$$

- Conditional choice probability:

$$P(i_{nt}|\alpha_n) = \frac{e^{V_{int}+\alpha_{in}}}{\sum_{j\in\mathcal{C}_{nt}} e^{V_{jnt}+\alpha_{jn}}}$$

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Static model with random effect

Estimation:

- Contribution of individual n to the log likelihood, given α_n

$$P(i_{n1}, i_{n2}, \ldots, i_{nT} | \alpha_n) = \prod_{t=1}^{T} P(i_{nt} | \alpha_n).$$

- Unconditional choice probability:

$$P(i_{n1}, i_{n2}, \ldots, i_{nT}) = \int_{\alpha} \prod_{t=1}^{T} P(i_{nt} | \alpha) f(\alpha) d\alpha.$$

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Static model with random effect

Estimation:

- Mixture model.
- Requires simulation for large choice sets.
- Generate draws $\alpha^1, \ldots, \alpha^R$ from $f(\alpha)$.
- Approximate

$$P(i_{n1}, i_{n2}, \ldots, i_{nT}) = \int_\alpha \prod_{t=1}^{T} P(i_{nt}|\alpha) f(\alpha) d\alpha \approx \frac{1}{R} \sum_{r=1}^{R} \prod_{t=1}^{T} P(i_{nt}|\alpha^r)$$

- The product of probabilities can generate very small numbers.

$$\sum_{r=1}^{R} \prod_{t=1}^{T} P(i_{nt}|\alpha^r) = \sum_{r=1}^{R} \exp\left(\sum_{t=1}^{T} \ln P(i_{nt}|\alpha^r)\right).$$

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Static model with random effect

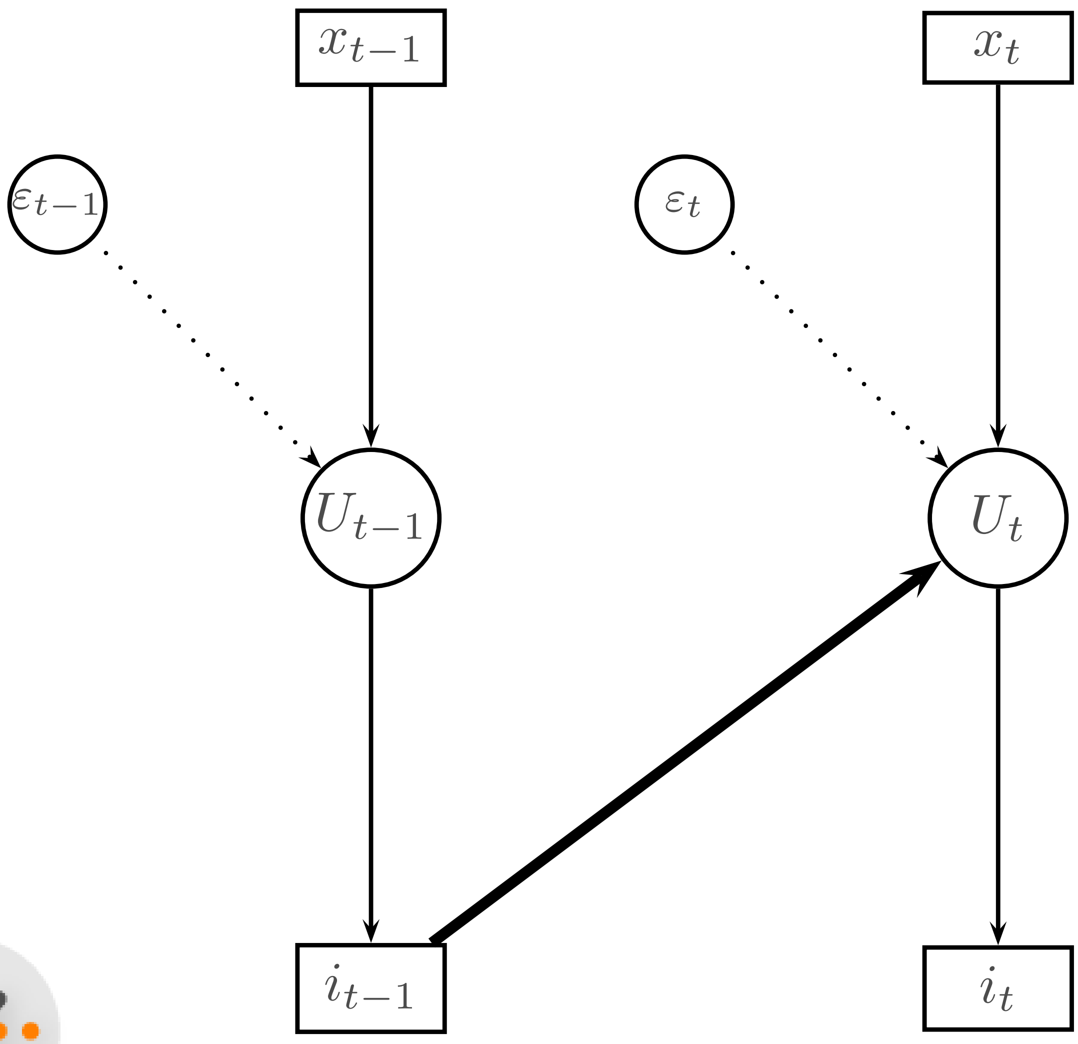
Comments:

- Parameters to be estimated: β's and σ's
- Maximum likelihood estimation leads to consistent and efficient estimators.
- Ignoring the correlation (i.e. assuming that α_n is not present) leads to consistent but not efficient estimators (not the true likelihood function).
- Accounting for serial correlation generates the true likelihood function and, therefore, the estimates are consistent and efficient.

Dynamics

- Choice in one period may depend on choices made in the past
- e.g. learning effect, habits.
- Simplifying assumption:
  - the utility of an alternative at time t
  - is influenced by the choice made at time $t-1$ only.
- It leads to a dynamic *Markov* model.

Dynamic Markov model



Dynamic Markov model

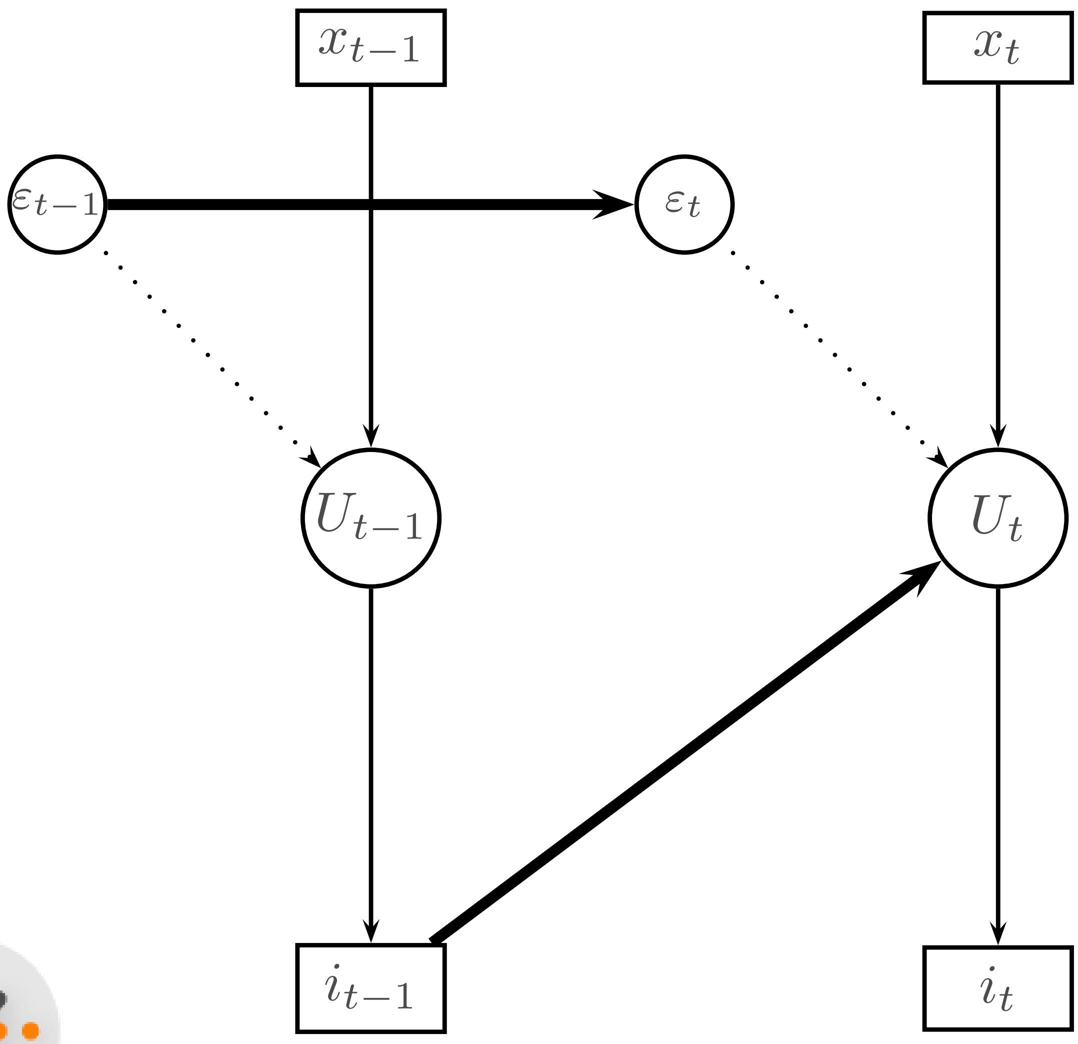
The model:

$$U_{int} = V_{int} + \gamma y_{in(t-1)} + \varepsilon_{int}, \; i \in \mathcal{C}_{nt}.$$

$$y_{in(t-1)} = \begin{cases} 1 & \text{if alternative } i \text{ was chosen by } n \text{ at time } t-1 \\ 0 & \text{otherwise.} \end{cases}$$

- Captures serial dependence on past realized state
  - Example - utility of bus today depends on whether consumer took bus yesterday (habit).
  - Fails if utility of bus today depends on permanent individual taste for bus (tastes) and whether consumer took bus yesterday. No serial correlation.
- Estimation: same as for the static model, except that observation $t = 0$ is lost.

Dynamic Markov model with serial correlation



Dynamic Markov model

- Extension: combine Markov with panel effect.

$$U_{int} = V_{int} + \alpha_{in} + \gamma y_{in(t-1)} + \varepsilon'_{int},\ i \in \mathcal{C}_{nt}.$$

- Dynamic Markov model with fixed effect.
  - Similar to the static model with FE.
  - Similar limitations.
- Dynamic Markov model with random effect.
  - Difficulties depending on how the Markov chain starts.
  - If the first choice i_0 is truly exogenous $\rightarrow$ similar to the static model with RE.

Dynamic Markov model

What if i_{n0} is not exogenous (i.e. stochastic)?

$$U_{in1} = V_{in1} + \alpha_{in} + \gamma y_{in0} + \varepsilon'_{in1}, \; i \in \mathcal{C}_{n1}.$$

- The first choice i_{n0} is dependent on the agent's effect α_{in}.
- So, the explanatory variable y_{in0} is correlated with α_{in}.
- This is called *endogeneity*.
- Solution: use the Wooldridge approach.

Dynamic Markov model with RE - Wooldridge

- Conditional on y_{in0}, we have a dynamic Markov model with RE as before.

$$U_{int} = V_{int} + \alpha_{in} + \gamma y_{in(t-1)} + \varepsilon'_{int}, \; i \in \mathcal{C}_{nt}.$$

- Contribution of individual n to the log likelihood, given i_{n0} and α_n

$$P(i_{n1}, i_{n2}, \ldots, i_{nT} | i_{n0}, \alpha_n) = \prod_{t=1}^{T} P(i_{nt} | i_{n0}, \alpha_n).$$

- We integrate out α_n:

$$P(i_{n1}, i_{n2}, \ldots, i_{nT} | i_{n0}) = \int_{\alpha} \prod_{t=1}^{T} P(i_{nt} | i_{n0}, \alpha) f(\alpha | i_{n0}) d\alpha.$$

Dynamic Markov model with RE - Wooldridge

- The main difference between static model with RE and dynamic model with RE is the term

$$f(\alpha | i_{n0})$$

- It captures the distribution of the panel effects, knowing the first choice.
- This can be approximated by, for instance,

$$\alpha_n = a + by_{n0} + cx_n + \xi_n, \ \ \xi_n \sim N(0, \Sigma_\alpha).$$

  - a, b and c are vectors and Σ_α a matrix of parameters to be estimated.
  - x_n capture the entire history ($t = 1, \ldots, T$) for agent n.
  - This addresses the endogeneity issue.

Application

Cherchi and Ortuzar (2002) *Mixed RP/SP models incorporating interaction effects*, Transportation 29(4), pp. 371-395.

Application

Context

- Study done in 1998, Sardinia Island, Italy
- Cagliari-Assimini corridor (20km)
- Modal shares: car (75%), bus (20%), train (3%), other (2%)
- RP/SP data.
- Not time series, but panel structure of SP data.
- t is the index of the choice experiment instead of time.
- $t = 0$ corresponds to the RP observation.
- Panel effect is captured.

Application

Estimation results

	Logit		with panel effect	
Variable	Estimate	t-test	Estimate	t-test
Cte. train	-0.727	-3.130	-0.745	-3.047
Cte. car	-2.683	-6.378	-2.770	-5.775
Travel time (min)	-0.061	-4.120	-0.067	-3.722
Travel cost/wage rate (euros)	-1.895	-3.198	-2.364	-4.454
Waiting time (min)	-0.252	-6.247	-0.270	-6.705
Comfort low	-1.990	-7.328	-2.075	-6.219
Comfort avg.	-1.107	-6.330	-1.187	-5.546
Transfers	-0.286	-1.378	-0.316	-1.000
Panel effect std. dev.			0.840	6.348
Log likelihood	-511.039		-502.959	
ρ^2	0.116		0.130	

Application

Average value of time by purpose (euros/min)

		Logit	with panel effect
Work	321 obs.	0.20	0.17
Study	285 obs.	0.05	0.04
Personal business	164 obs.	0.13	0.11
Leisure	64 obs.	0.16	0.14

Application

Comments

- Panel effect is significant.
- Significant improvement of the fit.
- With small samples, the gain in efficiency obtained from the panel effect may significantly improve the estimates.

Summary

- Static model
  - Straightforward extension of cross-sectional specification.
  - Two main limitations: serial correlation and dynamics.
- Panel effect
  - Deals with serial correlation.
  - Fixed effect:
    - Static model with additional parameters.
    - Not operational in most practical cases.
  - Random effect:
    - Modifies the log likelihood function.
    - Must integrate the product of the choice probabilities over time.

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Summary

- Dynamic model, with a Markov assumption.
  - Static model with an additional variable: the previous choice.
- Dynamic model with panel effect
  - Both can be combined.
  - Must capture the relation between the first choice and the panel effect.
- Application
  - Illustrates the importance of the panel effect.