Choice theory

Michel Bierlaire

michel.bierlaire@epfl.ch

Transport and Mobility Laboratory

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Framework

Choice: outcome of a sequential decision-making process

- Definition of the choice problem: How do I get to EPFL?
- Generation of alternatives: car as driver, car as passenger, train
- Evaluation of the attributes of the alternatives: price, time, flexibility, comfort
- Choice: decision rule
- Implementation: travel

Framework

A choice theory defines

1. decision maker
2. alternatives
3. attributes of alternatives
4. decision rule

Framework

Decision-maker :

- Individual or a group of persons
- If group of persons, we ignore internal interactions
- Important to capture difference in tastes and decision-making process
- Socio-economic characteristics: age, gender, income, education, etc.

Framework

Alternatives :

- Environment: *universal choice set* ($\mathcal{U}$)
- Individual n: *choice set* ($\mathcal{C}_n$)

Choice set generation:

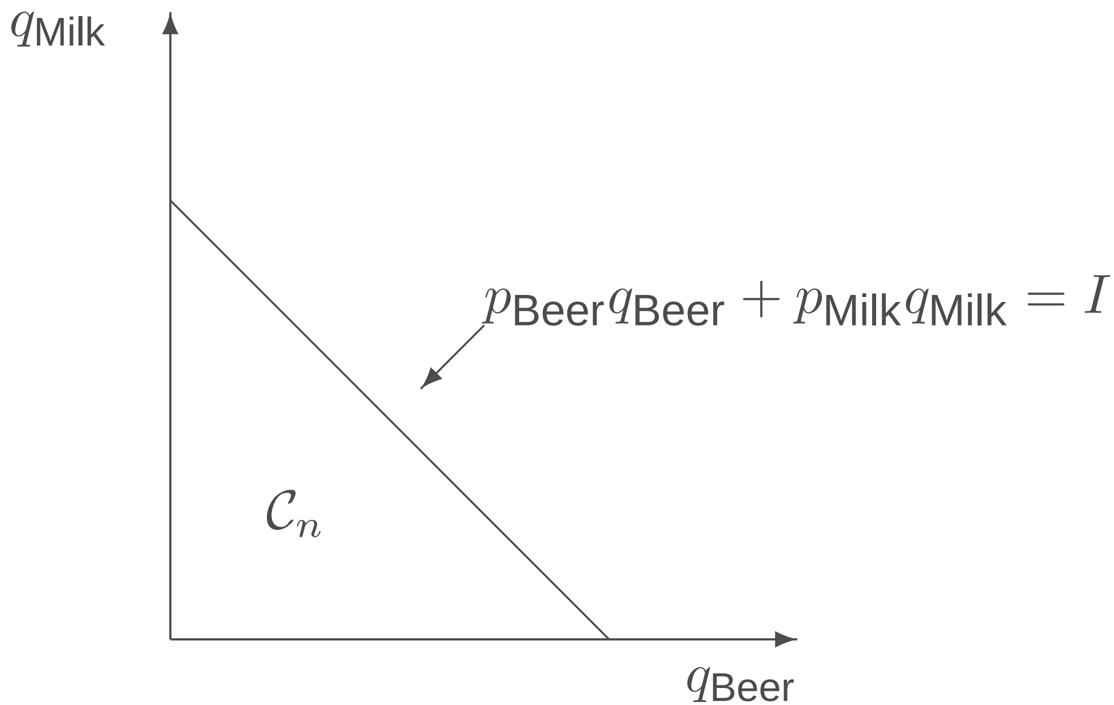
- Availability
- Awareness

Swait, J. (1984) *Probabilistic Choice Set Formation in Transportation Demand Models*
Ph.D. dissertation, Department of Civil Engineering, MIT, Cambridge, Ma.

Framework

Continuous vs. discrete

Continuous choice set:



Discrete choice set:

$$\mathcal{C}_n = \{\text{ Car, Bus, Bike}\}$$

Framework

Attributes

- ➜ cost
- ➜ travel time
- ➜ walking time
- ➜ comfort
- ➜ bus frequency
- ➜ etc.

- ✔ Generic vs. specific
- ✔ Quantitative vs. qualitative
- ✔ Perception

Framework

Decision rules

Neoclassical economic theory

Preference-indifference operator $\succsim$

(i) reflexivity

$$a \succsim a \quad \forall a \in \mathcal{C}_n$$

(ii) transitivity

$$a \succsim b \text{ and } b \succsim c \Rightarrow a \succsim c \quad \forall a, b, c \in \mathcal{C}_n$$

(iii) comparability

$$a \succsim b \text{ or } b \succsim a \quad \forall a, b \in \mathcal{C}_n$$

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Framework

Decision rules

Neoclassical economic theory (ctd)

- Numerical function

$$\exists\, U_n : \mathcal{C}_n \longrightarrow \mathbb{R} : a \rightsquigarrow U_n(a) \text{ such that}$$

$$a \gtrsim b \Leftrightarrow U_n(a) \geq U_n(b) \quad \forall a, b \in \mathcal{C}_n$$

Utility

Framework

Decision rules

- Utility is a latent concept
- It cannot be directly observed

Framework

Continuous choice set

- $Q = \{q_1, \ldots, q_L\}$ consumption bundle
- q_i is the quantity of product i consumed
- Utility of the bundle:

$$U(q_1, \ldots, q_L)$$

- $Q_a \gtrsim Q_b$ iff $U(q_1^a, \ldots, q_L^a) \geq U(q_1^b, \ldots, q_L^b)$
- Budget constraint:

$$\sum_{i=1}^{L} p_i q_i \leq I.$$

Framework

Decision-maker solves the optimization problem

$$\max_{q \in \mathbb{R}^L} U(q_1, \ldots, q_L)$$

subject to

$$\sum_{i=1}^{L} p_i q_i = I.$$

Example with two products...

Framework

$$\max_{q_1, q_2} U = \beta_0 q_1^{\beta_1} q_2^{\beta_2}$$

subject to

$$p_1 q_1 + p_2 q_2 = I.$$

Lagrangian of the problem:

$$L(q_1, q_2, \lambda) = \beta_0 q_1^{\beta_1} q_2^{\beta_2} + \lambda(I - p_1 q_1 - p_2 q_2).$$

Necessary optimality condition

$$\nabla L(q_1, q_2, \lambda) = 0$$

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Necessary optimality conditions

$$\begin{aligned}
\beta_0\beta_1 q_1^{\beta_1-1} q_2^{\beta_2} & - & \lambda p_1 & = & 0 \\
\beta_0\beta_2 q_1^{\beta_1} q_2^{\beta_2-1} & - & \lambda p_2 & = & 0 \\
p_1 q_1 + p_2 q_2 & - & I & = & 0.
\end{aligned}$$

We have

$$\begin{aligned}
\beta_0\beta_1 q_1^{\beta_1} q_2^{\beta_2} & - & \lambda p_1 q_1 & = & 0 \\
\beta_0\beta_2 q_1^{\beta_1} q_2^{\beta_2} & - & \lambda p_2 q_2 & = & 0
\end{aligned}$$

so that

$$\lambda I = \beta_0 q_1^{\beta_1} q_2^{\beta_2} (\beta_1 + \beta_2)$$

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Therefore

$$\beta_0 q_1^{\beta_1} q_2^{\beta_2} = \frac{\lambda I}{(\beta_1 + \beta_2)}$$

As $\beta_0 \beta_2 q_1^{\beta_1} q_2^{\beta_2} = \lambda p_2 q_2$, we obtain (assuming $\lambda \neq 0$)

$$q_2 = \frac{I\beta_2}{p_2(\beta_1 + \beta_2)}$$

Similarly, we obtain

$$q_1 = \frac{I\beta_1}{p_1(\beta_1 + \beta_2)}$$

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$$q_1 = \frac{I\beta_1}{p_1(\beta_1 + \beta_2)}$$

$$q_2 = \frac{I\beta_2}{p_2(\beta_1 + \beta_2)}$$

Demand functions

Framework

Discrete choice set

- Similarities with Knapsack problem
- Calculus cannot be used anymore

$$U = U(q_1, \ldots, q_L)$$

with

$$q_i = \begin{cases} 1 & \text{if product } i \text{ is chosen} \\ 0 & \text{otherwise} \end{cases}$$

and

$$\sum_i q_i = 1.$$

Framework

- Do not work with demand functions anymore
- Work with utility functions
- U is the “global” utility
- Define U_i the utility associated with product i.
- It is a function of the attributes of the product (price, quality, etc.)
- We say that product i is chosen if

$$U_i \geq U_j \quad \forall j.$$

Framework

Example: two transportation modes

$$
\begin{aligned}
U_1 &= -\beta t_1 - \gamma c_1 \\
U_2 &= -\beta t_2 - \gamma c_2
\end{aligned}
$$

with β, $\gamma > 0$

$$U_1 \geq U_2 \text{ iff } -\beta t_1 - \gamma c_1 \geq -\beta t_2 - \gamma c_2$$

that is

$$-\frac{\beta}{\gamma} t_1 - c_1 \geq -\frac{\beta}{\gamma} t_2 - c_2$$

or

$$c_1 - c_2 \leq -\frac{\beta}{\gamma}(t_1 - t_2)$$

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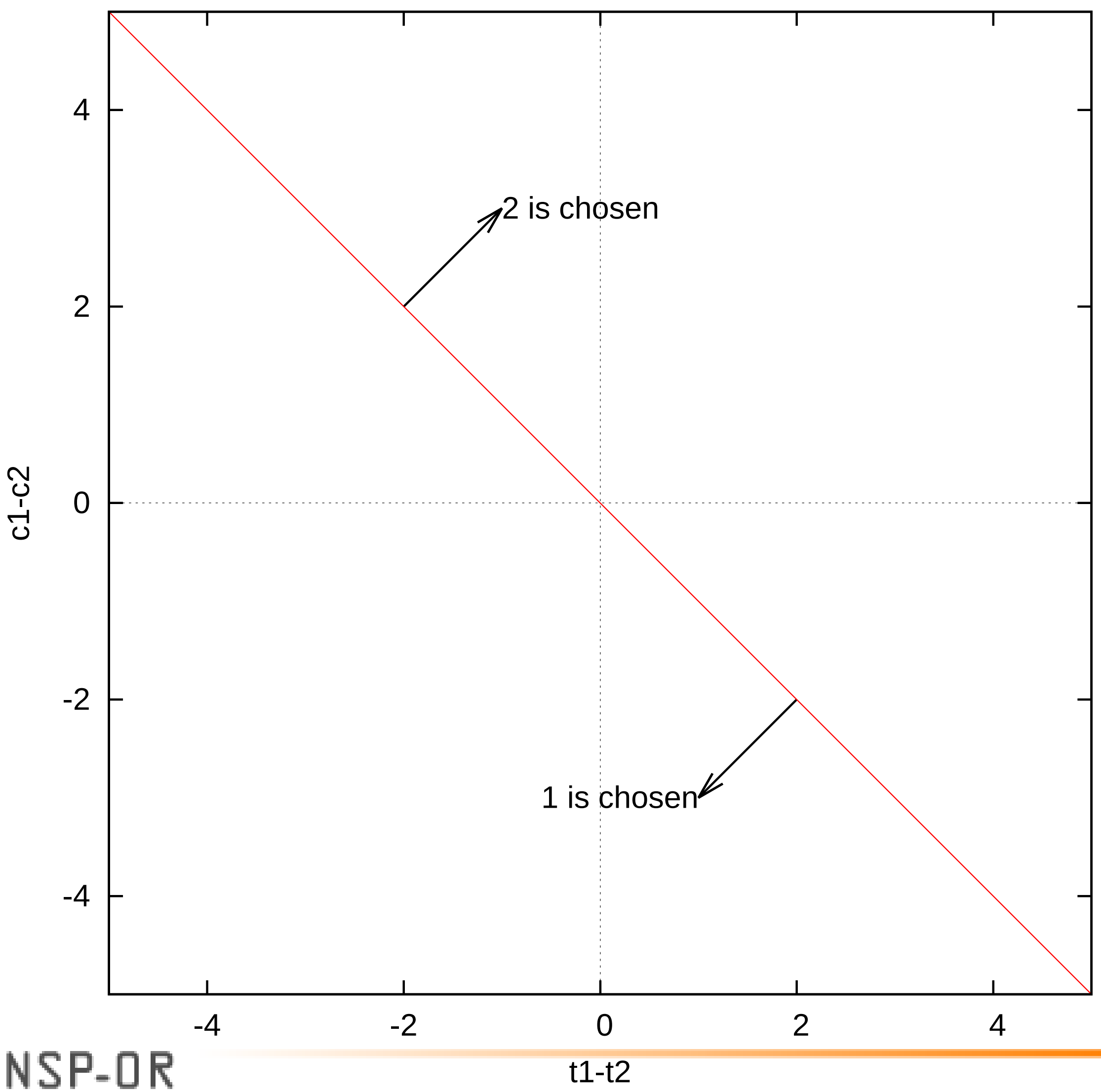
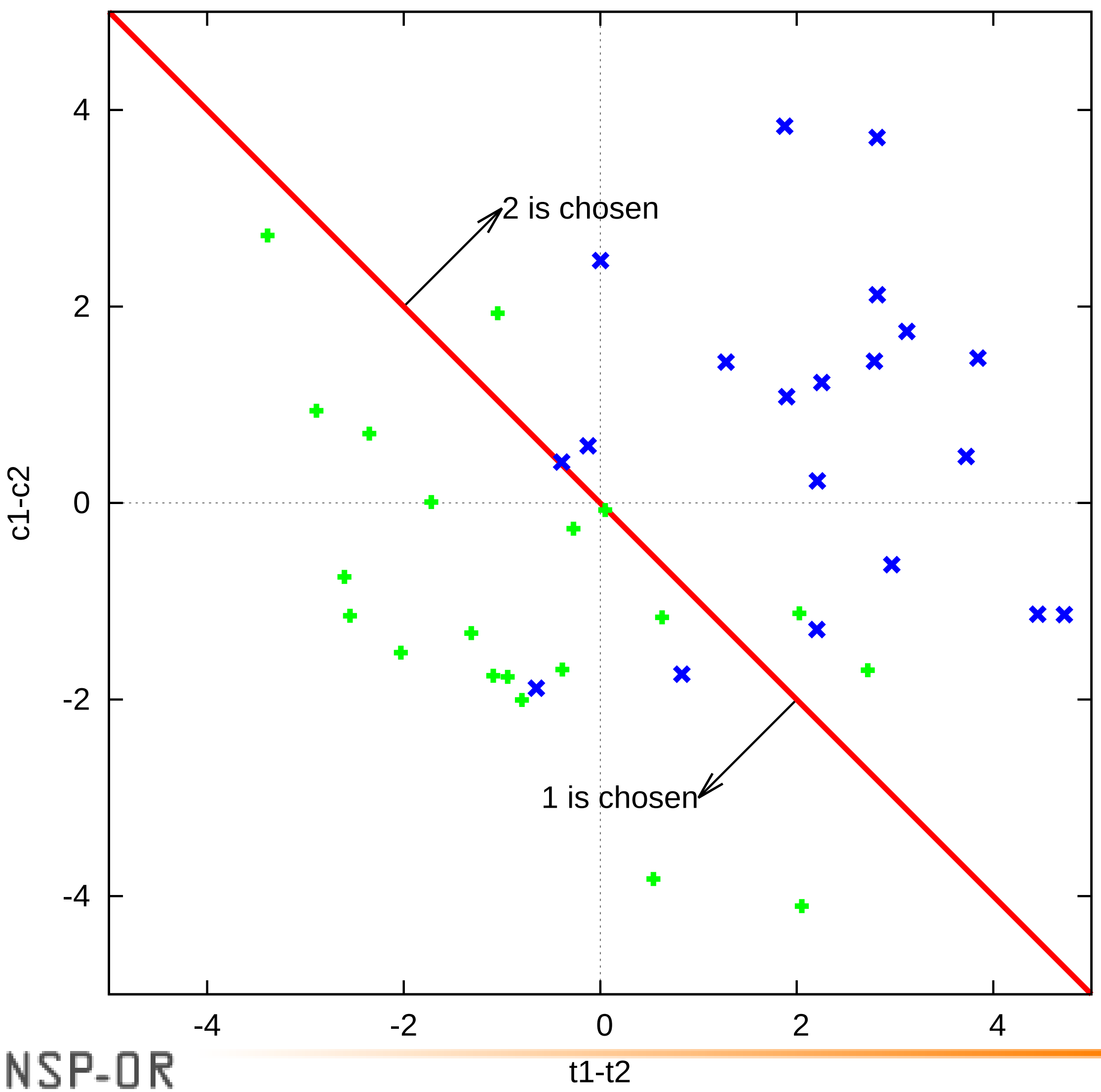
Obvious cases:

- $c_1 \geq c_2$ and $t_1 \geq t_2$: 2 dominates 1.
- $c_2 \geq c_1$ and $t_2 \geq t_1$: 1 dominates 2.
- Trade-offs in over quadrants

Framework



Framework



Assumptions

Decision rules

Neoclassical economic theory (ctd)

Decision-maker

- ✔ perfect discriminating capability
- ✔ full rationality
- ✔ permanent consistency

Analyst

- ✔ knowledge of all attributes
- ✔ perfect knowledge of $\gtrsim$ (or $U_n(\cdot)$)
- ✔ no measurement error

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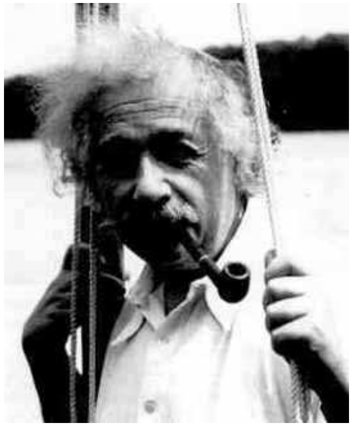
Assumptions

Uncertainty

Source of uncertainty?

- Decision-maker: stochastic decision rules
- Analyst: lack of information

- Bohr: *"Nature is stochastic"*
- Einstein: *"God does not play dice"*



Assumptions

Lack of information: random utility models

Manski 1973 The structure of Random Utility Models *Theory and Decision* 8:229–254

Sources of uncertainty:

- ☞ Unobserved attributes
- ☞ Unobserved taste variations
- ☞ Measurement errors
- ☞ Instrumental variables

For each individual n,

$$U_{in} = V_{in} + \varepsilon_{in}$$

and

$$P(i|\mathcal{C}_n) = P[U_{in} = \max_{j \in \mathcal{C}_n} U_{jn}] = P(U_{in} \geq U_{jn} \ \forall j \in \mathcal{C}_n)$$

Random utility models

$$U_{in} = V_{in} + \varepsilon_{in}$$

- Dependent variable is latent
- Only differences matter

$$\begin{aligned} P(i|\mathcal{C}_n) &= P(U_{in} \geq U_{jn} \ \forall j \in \mathcal{C}_n) \\ &= P(U_{in} + K \geq U_{jn} + K \ \forall j \in \mathcal{C}_n) \quad \forall K \in \mathbb{R} \end{aligned}$$

$$\begin{aligned} P(i|\mathcal{C}_n) &= P(U_{in} \geq U_{jn} \ \forall j \in \mathcal{C}_n) \\ &= P(\lambda U_{in} \geq \lambda U_{jn} \ \forall j \in \mathcal{C}_n) \quad \forall \lambda > 0 \end{aligned}$$