Mathematical modeling of behavior

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What kind of behavior can be mathematically modeled?
Psychohistory

Branch of mathematics which deals with the reactions of human conglomerates to fixed social and economic stimuli. The necessary size of such a conglomerate may be determined by Seldon’s First Theorem.

Encyclopedia Galactica, 116th Edition (1020 F.E.)
Encyclopedia Galactica Publishing Co., Terminus

Motivation: shorten the period of barbarism after the Fall of the Galactic Empire
Asimov, I. (1951) Foundation, Gnome Press
In this course...

- Individual behavior (vs. aggregate behavior)
- Theory of behavior which is
  - **descriptive**: how people behave and not how they should
  - **abstract**: not too specific
  - **operational**: can be used in practice for forecasting
- Type of behavior: **choice**
Motivations

"Well, each decision we make determines the range of choices we’ll face next."
Motivations

“It is our choices that show what we truly are, far more than our abilities” Albus Dumbledore

“Liberty, taking the word in its concrete sense, consists in the ability to choose.” Simone Weil (French philosopher, 1909-1943)

<table>
<thead>
<tr>
<th>Field</th>
<th>Type of behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marketing</td>
<td>Choice of a brand</td>
</tr>
<tr>
<td>Transportation</td>
<td>Choice of a transportation mode</td>
</tr>
<tr>
<td>Politics</td>
<td>Choice of a president</td>
</tr>
<tr>
<td>Management</td>
<td>Choice of a management policy</td>
</tr>
<tr>
<td>New technologies</td>
<td>Choice of investments</td>
</tr>
</tbody>
</table>
Applications

Case studies

- Choice-lab marketing
  - Context: B2B, data provider (financial, demographic, etc.)
  - Objective: understand why clients quit
- Quebec energy
  - Context: space and water heating in households
  - Objective: importance of the type of household and price
- Transportation mode choice in the Netherlands
  - Context: car vs rail in Nijmegen
  - Objective: sensitivity to travel time and cost, inertia.
Applications

- Swissmetro
  - Context: new transportation technology
  - Objective: demand pattern, pricing
- Residential telephone services
  - Context: flat rate vs. measured
  - Objective: offer the most appropriate service
Importance

Daniel L. McFadden
1937–

- Laureate of *The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel* 2000
- Owns a farm and vineyard in Napa Valley
- “Farm work clears the mind, and the vineyard is a great place to prove theorems”
Example

Voice over internet protocol (VoIP)

- What is the market penetration?
- How will the penetration change in the future?
- Assumption: level of education is an important explanatory factor

Data collection

- sample of 600 persons, randomly selected
- Two questions:
  1. Do you subscribe to voice over IP? (yes/no)
  2. How many years of education have you had? (low/medium/high)
Example

- Contingency table

<table>
<thead>
<tr>
<th>VoIP</th>
<th>Education</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low 10</td>
</tr>
<tr>
<td>Yes</td>
<td>230</td>
</tr>
<tr>
<td>No</td>
<td>140</td>
</tr>
<tr>
<td></td>
<td>370</td>
</tr>
<tr>
<td></td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>600</td>
</tr>
</tbody>
</table>

- Penetration in the sample: \( \frac{230}{600} = 38.3\% \)
- Forecasting: need for a model
Example: a model

Type of variables:
- dependent, or endogenous: what we explain
- independent, exogenous or explanatory: how we explain

Model:
- Causal relationship between the independent and independent variables
- Based on theory, assumptions.
- Probabilistic.
Example: a model

- Dependent variable:
  \[ y = \begin{cases} 
  1 & \text{if subscriber} \\
  2 & \text{if not subscriber} 
\end{cases} \]

  Discrete dependent variable

- Independent or explanatory variable
  \[ x = \begin{cases} 
  1 & \text{if level of education is low} \\
  2 & \text{if level of education is medium} \\
  3 & \text{if level of education is high} 
\end{cases} \]
Example: a model

- Market penetration in the sample: \( \hat{p}(y = 1) \)
- Market penetration in the population: \( p(y = 1) \) estimated by \( \hat{p}(y = 1) \)
- Joint probabilities: \( \hat{p}(y = 1, x = 2) = \frac{100}{600} = 0.1667 \)
- Marginal probabilities: \( \hat{p}(y = 1) = \sum_{k=1}^{3} \hat{p}(1, k) = \frac{10}{600} + \frac{100}{600} + \frac{120}{600} = 0.383 \)
- Conditional probabilities: \( \hat{p}(y = 1|x = 2) \)

\[
\hat{p}(y = 1, x = 2) = \hat{p}(y = 1|x = 2) \hat{p}(x = 2) \\
\hat{p}(y = 1|x = 2) = \frac{\hat{p}(y = 1, x = 2)}{\hat{p}(x = 2)} = \frac{0.1667}{0.5} = 0.333
\]
Example: a model

Similarly, we obtain

\[ \hat{p}(y = 1|x = 1) = 0.067 \]
\[ \hat{p}(y = 1|x = 2) = 0.333 \]
\[ \hat{p}(y = 1|x = 3) = 0.8 \]

We obtain a causal relationship.

- Behavioral model: \( \hat{p}(y = i|x = j) \)
- Forecasting assumption: stable over time
Example: forecasting

- Model:

\[
p(y = 1| x = 1) = \pi_1 = 0.067 \\
p(y = 1| x = 2) = \pi_2 = 0.333 \\
p(y = 1| x = 3) = \pi_3 = 0.8
\]

where \(\pi_1, \pi_2, \pi_3\) are estimated parameters

- Assumption: future level of education: 10%-60%-30%

\[
p(y = 1) = \sum_{i=1}^{3} p(y = 1|x = i)p(x = i) \\
= 0.1\pi_1 + 0.6\pi_2 + 0.3\pi_3 \\
= 44.67\%
\]
Example: forecasting

- If the level of education increases
- from 25%-50%-25% to 10%-60%-30%
- Market penetration of VoIP will increase
- from 38.33 % to 44.67%

In summary

- \( p(x = j) \) can be easily obtained and forecast
- \( p(y = i | x) \) is the behavioral model to be developed
Outline

- Introduction and examples
- Review of relevant concepts in probability and statistics
- Choice theory
- Binary choice
- Multiple alternatives
- Tests
- Nested Logit model
- Multivariate Extreme Value models
- Forecasting
- Sampling
- Mixtures of models
- Latent variables
Bibliography