Simulation laboratory 4: Variance reduction

Rico Krueger

Transport and Mobility Laboratory
School of Architecture, Civil and Environmental Engineering
École Polytechnique Fédérale de Lausanne

10 March 2020

Goals

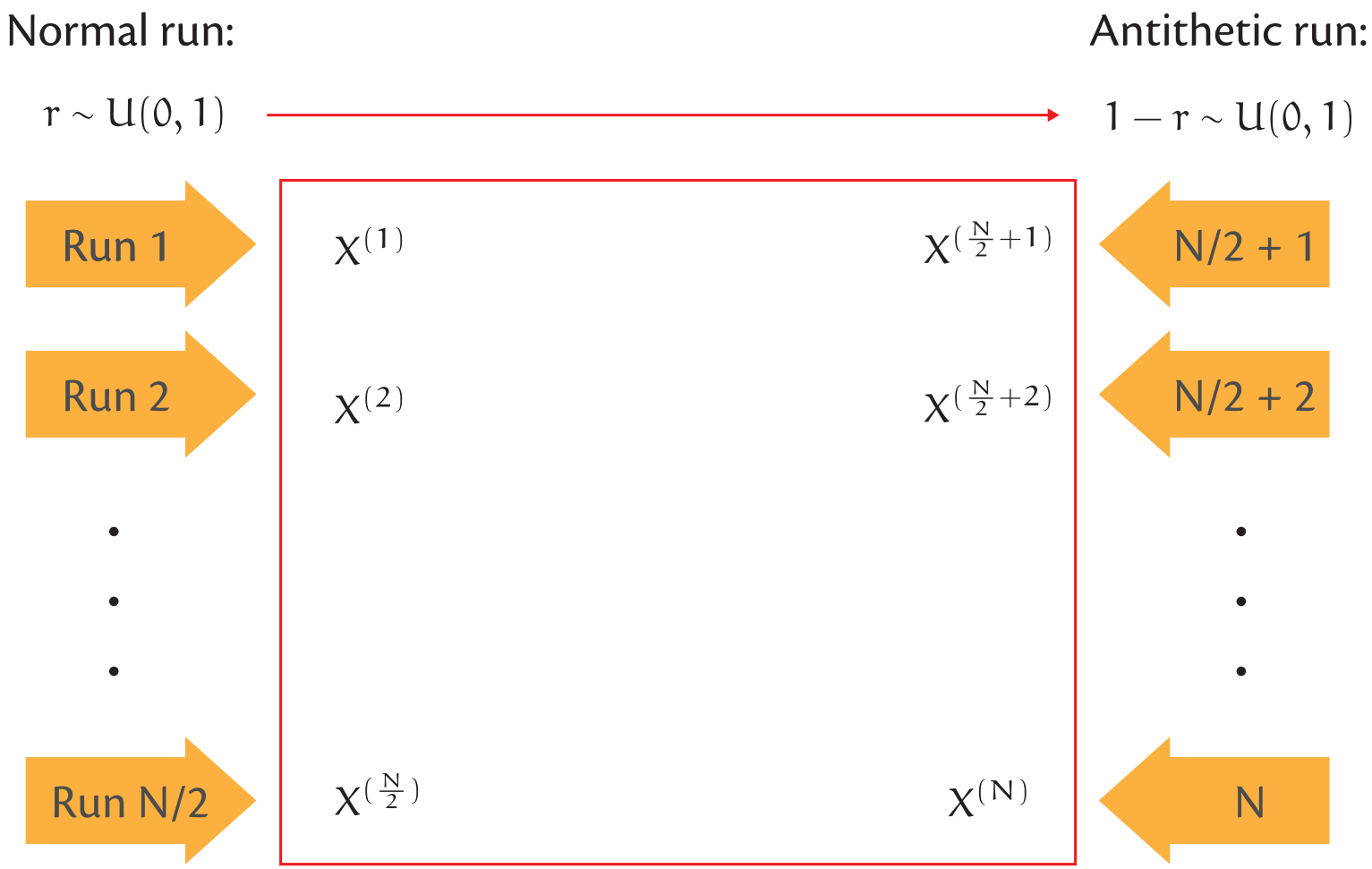
Variance reduction:

- Understand two different variance reduction techniques.
- Increase the precision of vehicle queue simulation result.

Implementation:

1. Antithetic draws
2. Control variates

Antithetic draws



Antithetic draws

Implementation:

- Modify the function `simulate`:
  - Keywords: `scenario`, `u`
  - Return: `times, queues`
- Conduct statistical analysis of maximum queue length using code developed in the previous lab.

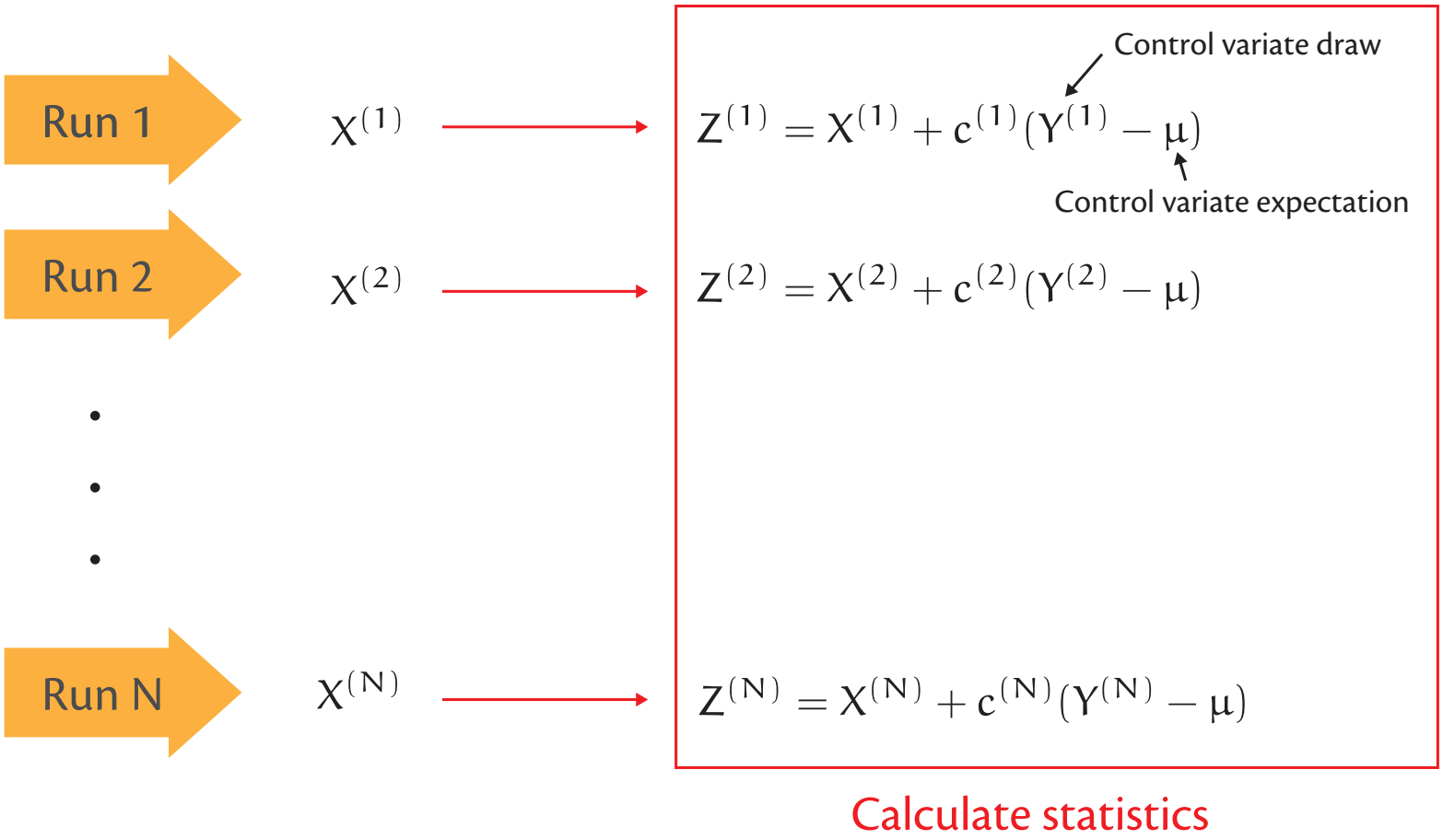
Antithetic draws workflow

1. For $r = 1, \ldots, N/2$:
   1. Independent simulation:
      1. Generate array of uniform random numbers u.
      2. Run `simulate(scenario, u)` and obtain max-queue-length $q_{\max}^{\text{ind}}$.
   2. Antithetic simulation:
      1. Set $u = 1 - u$.
      2. Run `simulate(scenario, u)` and obtain max-queue-length $q_{\max}^{\text{ant}}$.
      3. Compute $q_{\max}^{(r)} = \frac{q_{\max}^{\text{ind}} + q_{\max}^{\text{ant}}}{2}$.
2. Analyse the statistics of $[q_{\max}^{(1)}, \ldots, q_{\max}^{(\frac{N}{2})}]$

Keep draws

- The negative correlation between a run and its antithetic should be as big as possible.
- The order of events could change between the two runs.
- **Different streams of random numbers for the different event types should be used**.

Control variates



Control variates

Implementation:

- Modify the function `simulate`:
  - Keywords: `scenario`,
  - Return: `times, queues, service_time_mean`
- Conduct statistical analysis of maximum queue lengths.

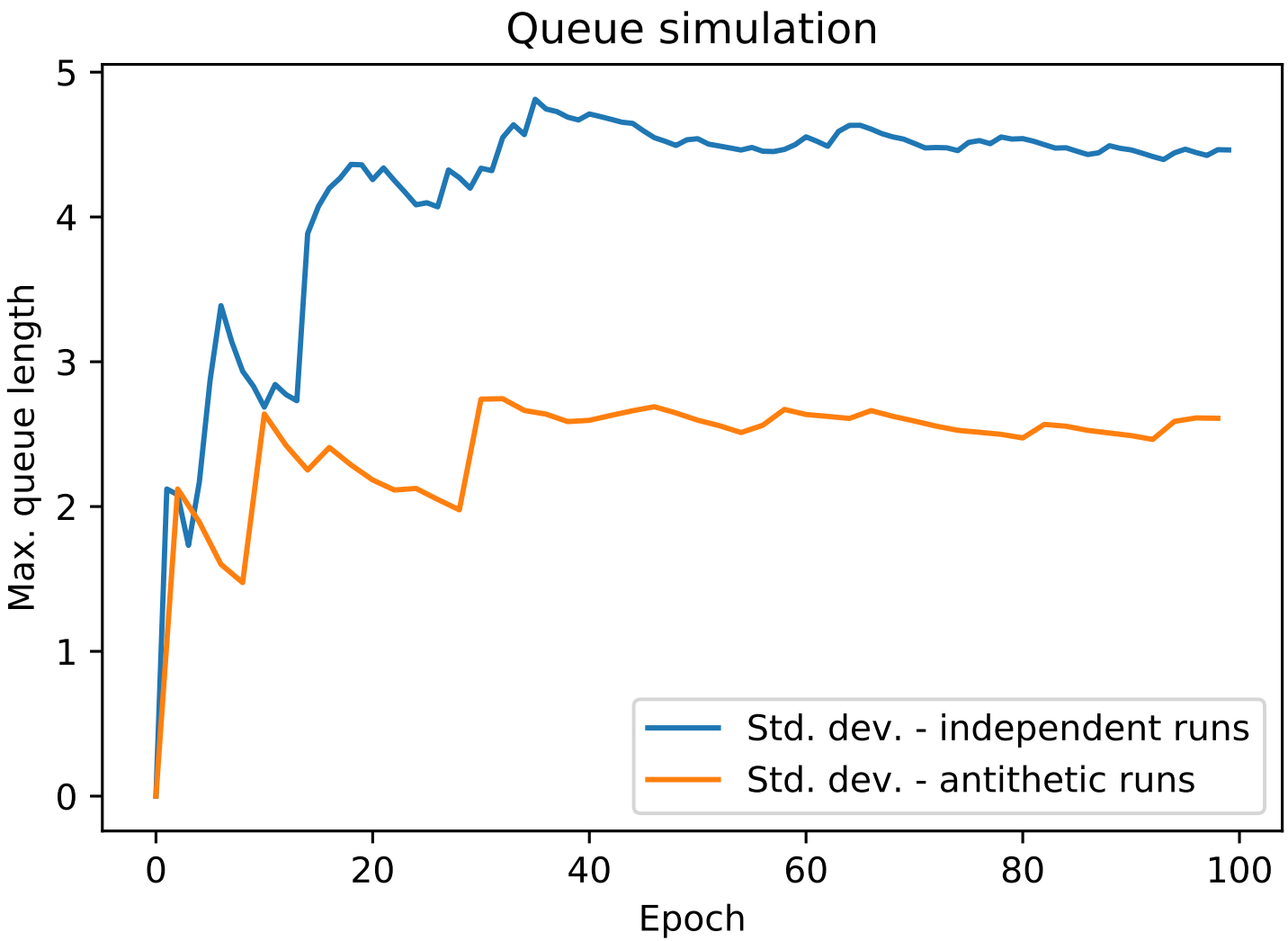
Control variates workflow

1. For $r = 1, \ldots, N$:
   1. Run `simulate` and output the control variate `control` $(= Y^{(r)})$.
   2. Obtain max-queue-length $q_{\max}$ $(= X^{(r)})$.
   3. Execute the function `controlled_mean`:
      1. Calculate variance $\mathrm{Var}(Y)$ and covariance $\mathrm{Cov}(X, Y)$
      2. Define the constant $c^* = -\mathrm{Cov}(X, Y)/\mathrm{Var}(Y)$
      3. Calculate $Z = X + c^*(Y - \mu)$
      4. Obtain the average and variance of $Z = [Z^{(1)}, \ldots, Z^{(r)}]$

Control variates

- Choose the control variate (Y) as you prefer. The higher the correlation is, the bigger the variance reduction is.
- Example: Y = mean of service time at bottleneck

Antithetic draws



Control variates