Optimization and Simulation

Laboratory 6

Optimization

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Goals

- Understand the limitation of full enumeration
- Understand and apply optimization algorithms:
  - Greedy algorithm
  - Local search
  - Variable neighborhood search
  - Simulated annealing

Overview

Travelling salesman problem

Implementation functions

6.1 Full enumeration

6.2 Greedy algorithm

6.3 Local search

6.4 Variable neighborhood search

6.5 Simulated annealing

Travelling Salesman Problem

Travelling Salesman Problem

A salesman has to visit n cities

- She starts and ends the trip at her home city
- Assume cost of travel to be total trip length

What sequence of cities minimizes the travel cost?

Travelling Salesman Problem

Cities are consecutively numbered: $1, 2, \ldots, n$

We encode solutions as $x = (x_1\, x_2\, \ldots\, x_i \ldots\, x_n)$ where

- x_i is the index of ith city visited along the way
- x_n is the last city visited before returning home
- Every city must be visited exactly once

Implementation functions

Solution proposal function

Function to implement (.m)

```
GenerateNewCitySequence
```

Test the function (.m)

```
GenerateNewCitySequenceTest
```

Objective Function

Function to implement (.m)

```
EvaluateCitySequence
```

Test the function (.m)

```
EvaluateCitySequenceTest
```

6.1 Full enumeration

Exercise

Function to implement (.m)

```
GenerateNewCitySequence
FullEnumeration
```

Test the function (.m)

```
OptimizationTSPTest
```

Calculate the computational time limitations of the full enumeration.

What is the maximum problem size (number of cities) that you could solve with this approach?

6.2 Greedy algorithm

Exercise

Function to implement (.m)

```
GenerateNewCitySequence
GreedyAlgorithm
```

Test the function (.m)

```
OptimizationTSPTest
```

6.3 Local search

Exercise

Function to implement (.m)

```
GenerateNewCitySequence
LocalSearch
```

Test the function (.m)

```
OptimizationTSPTest
```

6.4 Variable neighborhood search

Exercise

Function to implement (.m)

```
GenerateNewCitySequence
GenerateNewCitySequence_...
GenerateNewCitySequence_k
VariableNeighborhoodSearch
```

Test the function (.m)

```
OptimizationTSPTest
```

6.5 Simulated annealing

Exercise

Function to implement (.m)

```
GenerateNewCitySequence
SimulatedAnnealing
```

Test the function (.m)

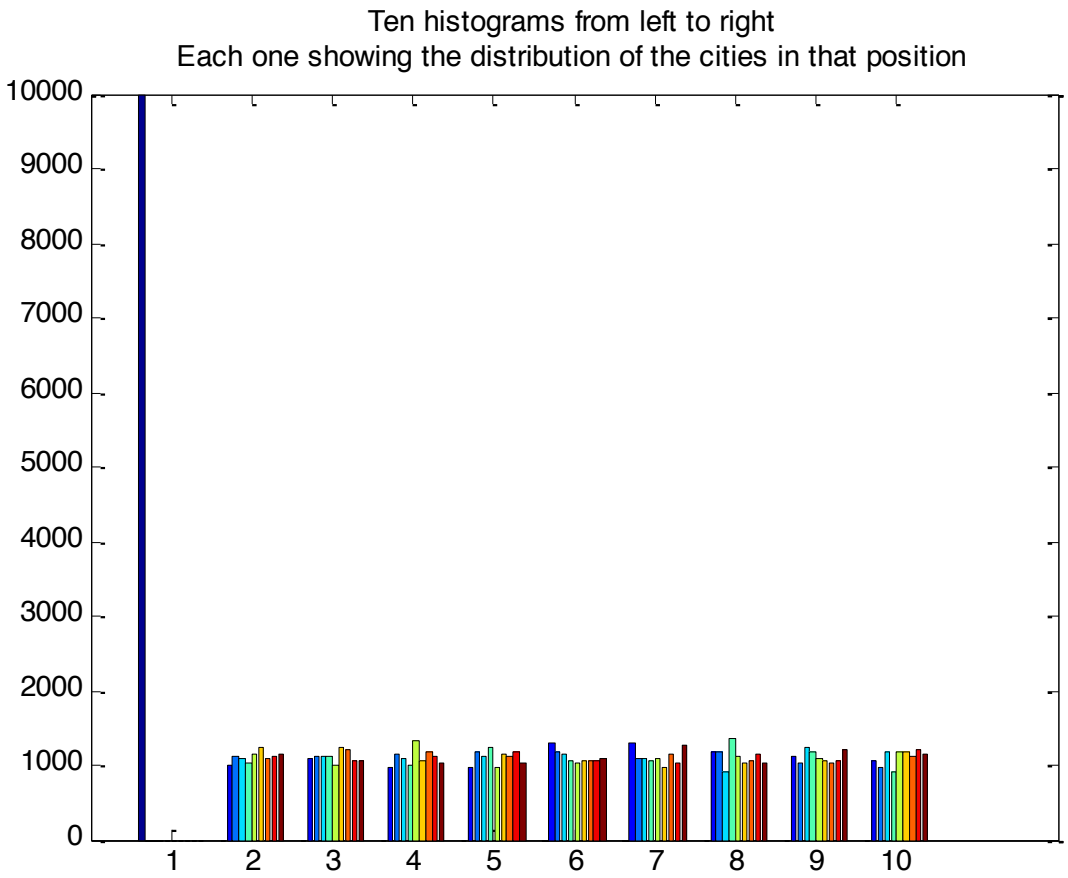
```
OptimizationTSPTest
```

- Investigate the effect of different parameters

My results

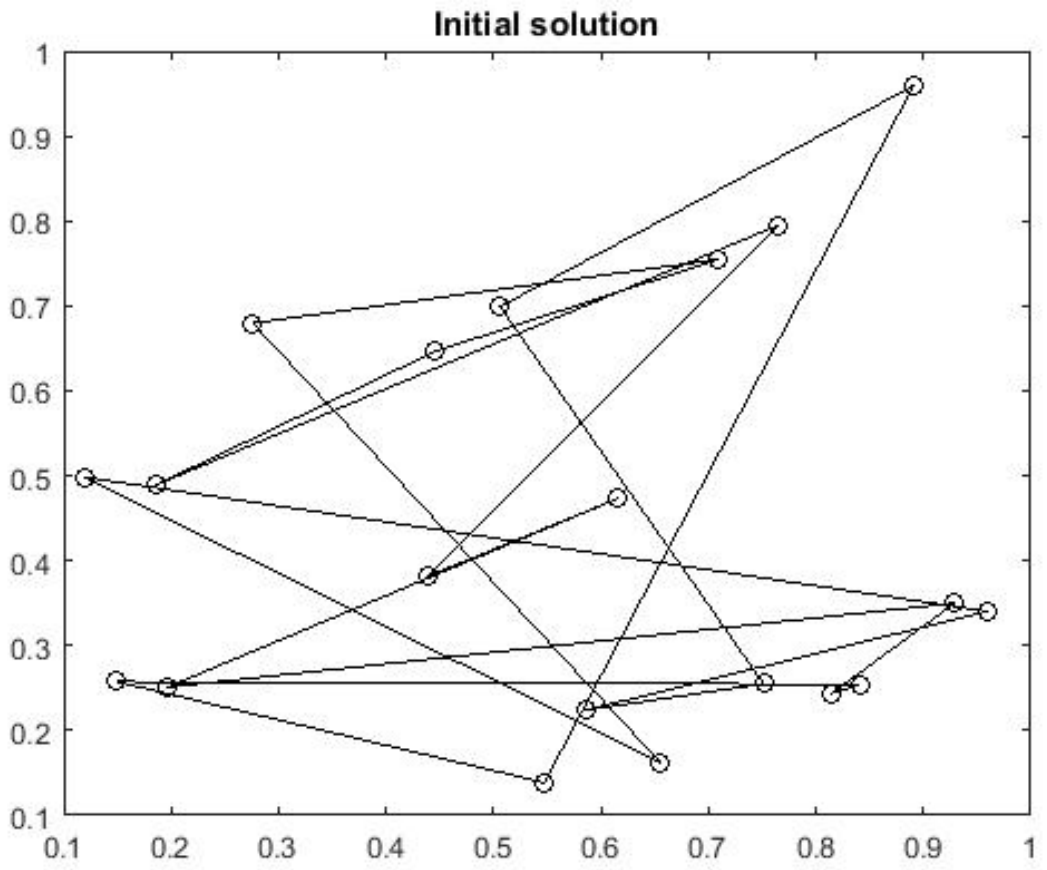
My results

RandomizeCitySequence



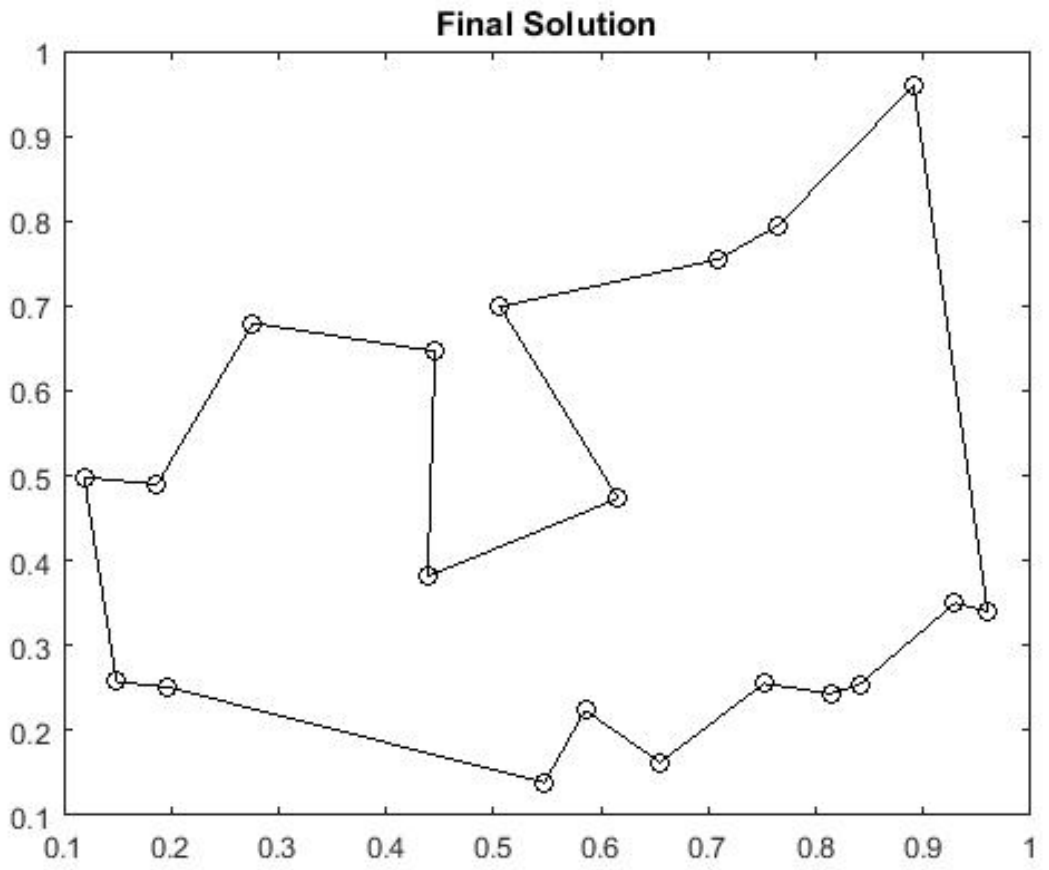
My results

An initial solution



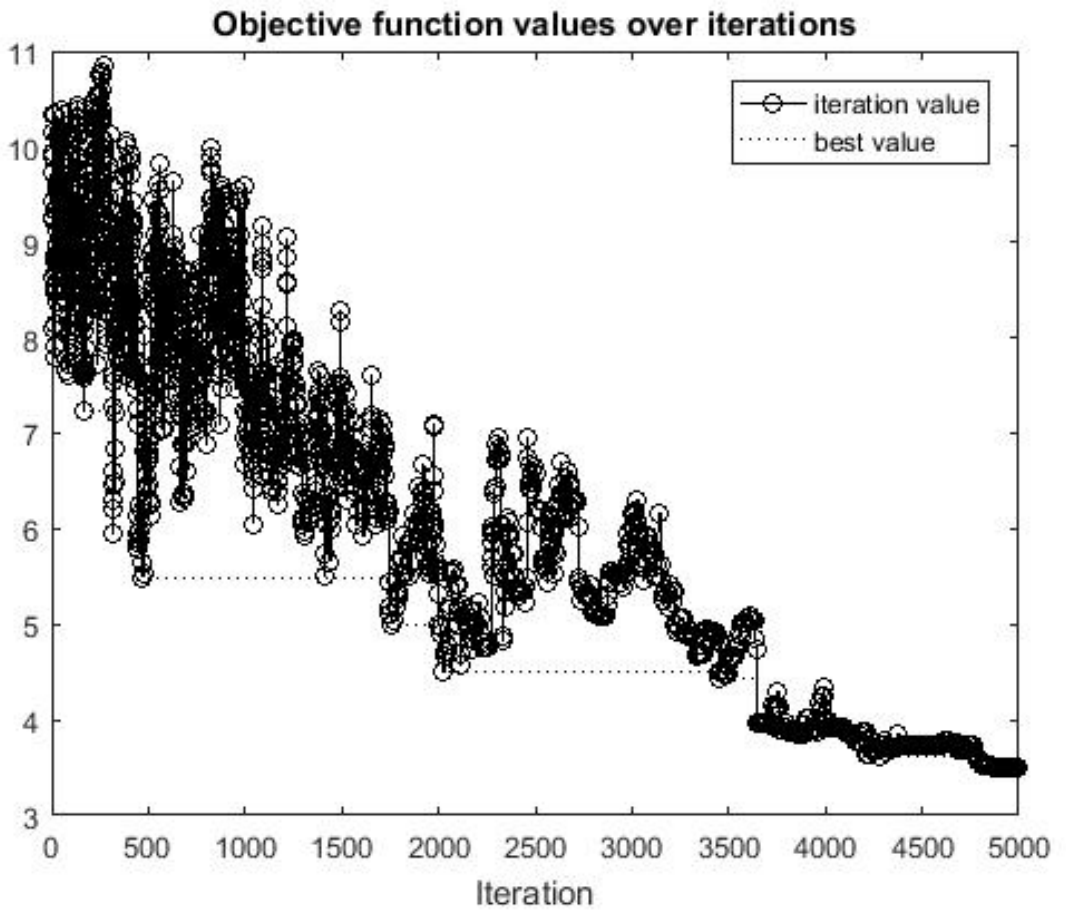
My results

A final solution



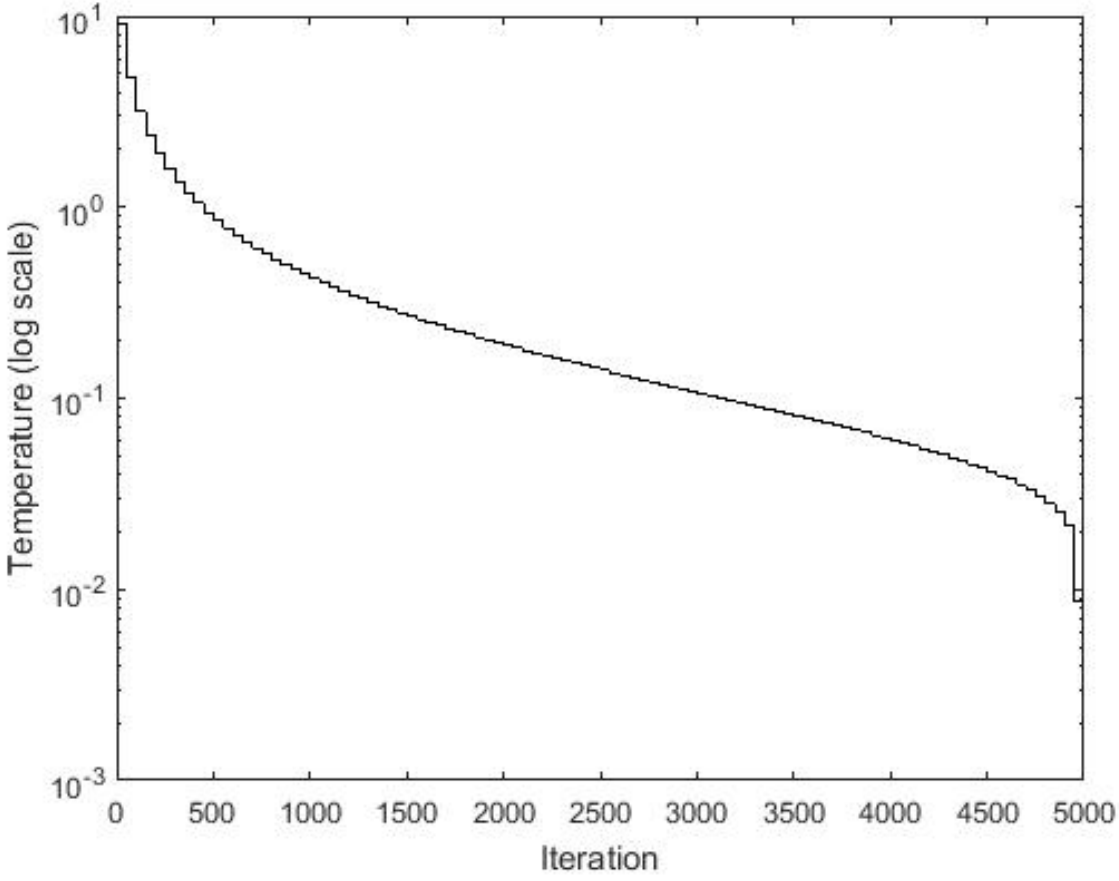
My results

Simulated Annealing



My results

Simulated Annealing



Knapsack Problem

- A salesman is planning his visit to the market and wants to select items he will try to sell.
- Each item i is characterized by its price p_i and weight w_i, which are given on the next slide
- There are 60 items in total.
- Logically, he would like to take as many items as possible but his van has a weight limitation of max. 1500 kilograms.

The set of which items will maximize the profit?

Knapsack Problem

- Price p_i and weight w_i for each item i:

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
p_i	80	31	48	17	27	84	34	39	46	58	23	67	62	79	38	44	31	50	72	71
w_i	84	27	47	22	21	96	42	46	54	53	32	78	64	82	33	49	28	56	77	69

i	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
p_i	73	32	49	21	25	91	36	19	8	74	56	41	62	89	48	54	11	41	52	54
w_i	79	29	51	19	23	94	39	16	8	69	61	38	59	92	43	59	18	44	58	67

i	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
p_i	9	21	58	26	57	65	54	48	36	49	34	75	59	86	24	51	41	63	72	45
w_i	14	26	57	33	51	72	62	52	44	55	43	83	70	93	23	56	40	66	75	49

Tasks

- Try to solve the problem with the full enumeration
- Test the following optimization algorithms:
  - Variable neighborhood search (VNS), by implementing at least 3 neighborhood structures, and
  - Simulated annealing (SA)
- Use the full enumeration solution as a benchmark, if obtained.

Problem Encoding

- Let us label each item with an integer: $1, 2, \ldots, n$
- We encode solutions as $x = (x_1\, x_2\, \ldots\, x_i \ldots\, x_n)$ where
  - x_i is the 0-1 variable denoting if the item i is selected into the knapsack

Results presentation

- Average number of appearances of each item in the knapsack
- Solutions achieved with all three algorithms
- Trend of the objective function value and temperatures in the case of SA