

Optimization and Simulation

Laboratory 4

Variance Reduction

Riccardo Scarinci

Transport and Mobility Laboratory TRANSP-OR
École Polytechnique Fédérale de Lausanne EPFL

Goals

- Understand and apply variance reduction techniques

Overview

4.1: Antithetic Draws

4.2: Control Variates

4.1 Antithetic Draws

Antithetic Draws

Simulation example

Modelling vehicle queue on a single road

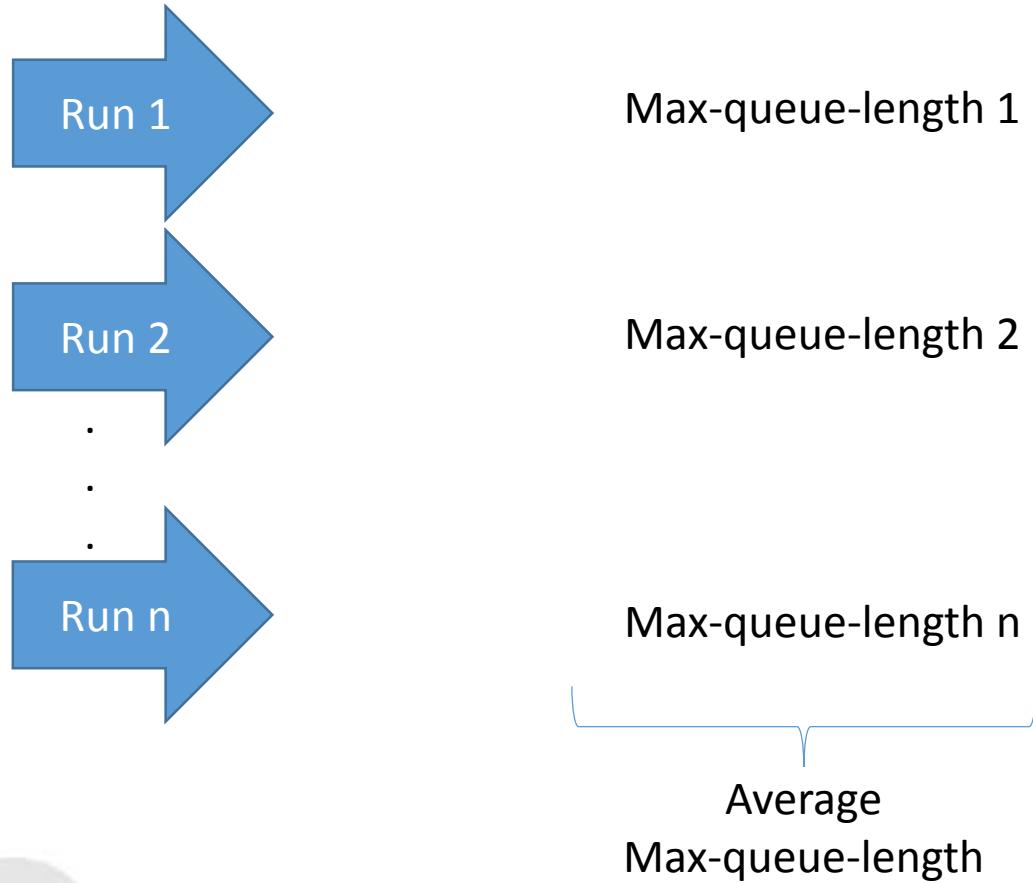
QueueingSimulation1

StatAnalysisAndBootstrapTest.m

Variance reduction of the index average max-queue-length

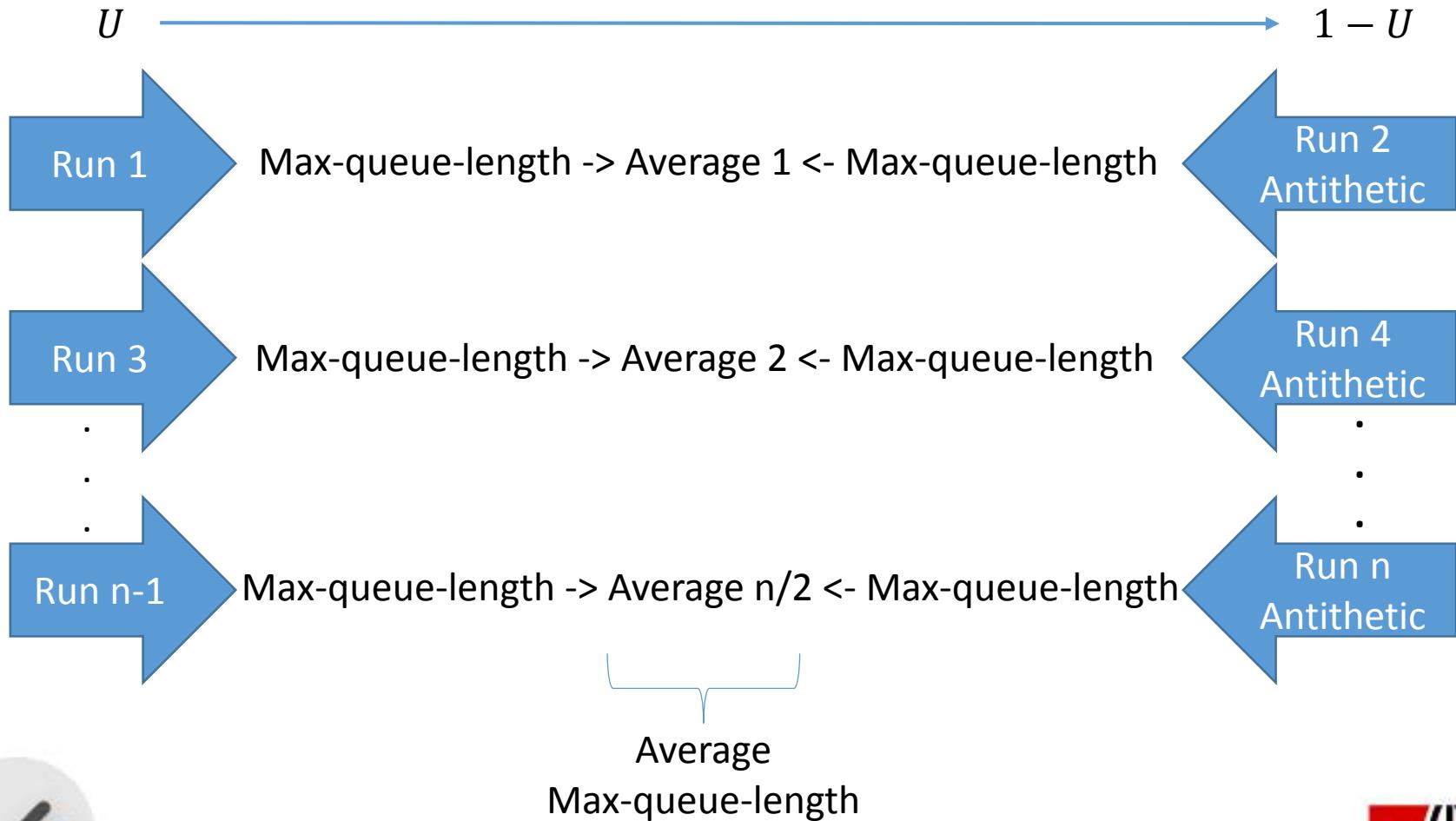
Antithetic Draws

Independent draws workflow



Antithetic Draws

Antithetic Draws workflow



Antithetic Draws

Normal run



So far, your single road queueing simulation was function

```
[times, queues] = QueueingSimulation1(scenario)
```

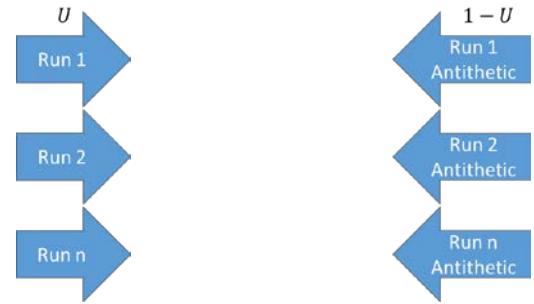
- Extend this into function

```
[times, queues, draws] =  
QueueingSimulation1_Independent(scenario)
```

- Where *draws* is a structure that contains a row vector for each stream of generated random number

Antithetic Draws

Normal run -> Antithetic run



- The (negative) correlation between a run and its antithetic should be as big as possible
- The order of events could change between runs
- Different streams of random numbers for the different event types should be recorded

`draws.Generation`

`draws.Arrival`

`draws.Departure`

Antithetic Draws

Antithetic run



Extend QueueingSimulation1 into function

```
[times, queues] =  
QueueingSimulation1_Antithetic(scenario, draws)
```

- Where the random number streams in draws are used for the antithetic run ($1 - U$)

Test the function expanding

StatAnalysisAndBootstrapTest.m

Evaluate the improvement in variance reduction against the independent procedure

Antithetic Draws

Antithetic run

StatAnalysisAndBootstrapTest.m

main loop

```
[times, queues, draws] = QueueingSimulation1_Independent(scenario)
```

```
[times, queues] = QueueingSimulation1_Antithetic(scenario,draws)
```

```
max_queue = mean([max_queue_Indep max_queue_Antit])
```

end

Control Variates

Control Variates

Simulation example

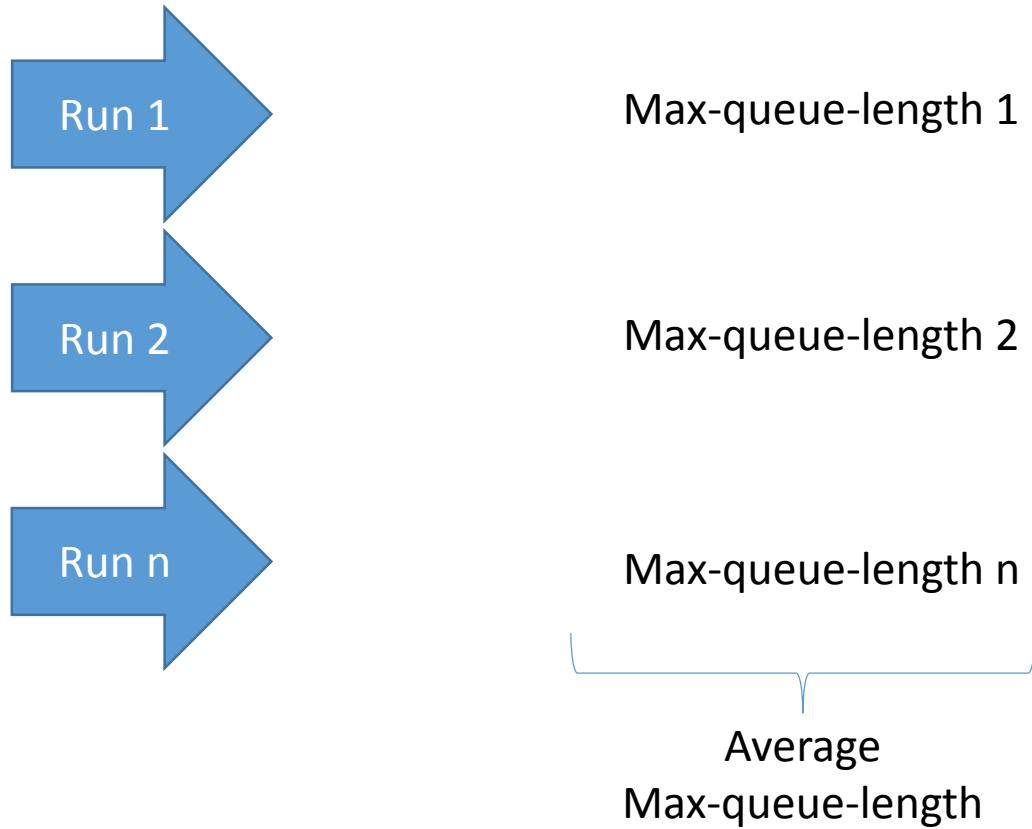
Modelling vehicle queue on a single road

QueueingSimulation1

StatAnalysisAndBootstrapTest.m

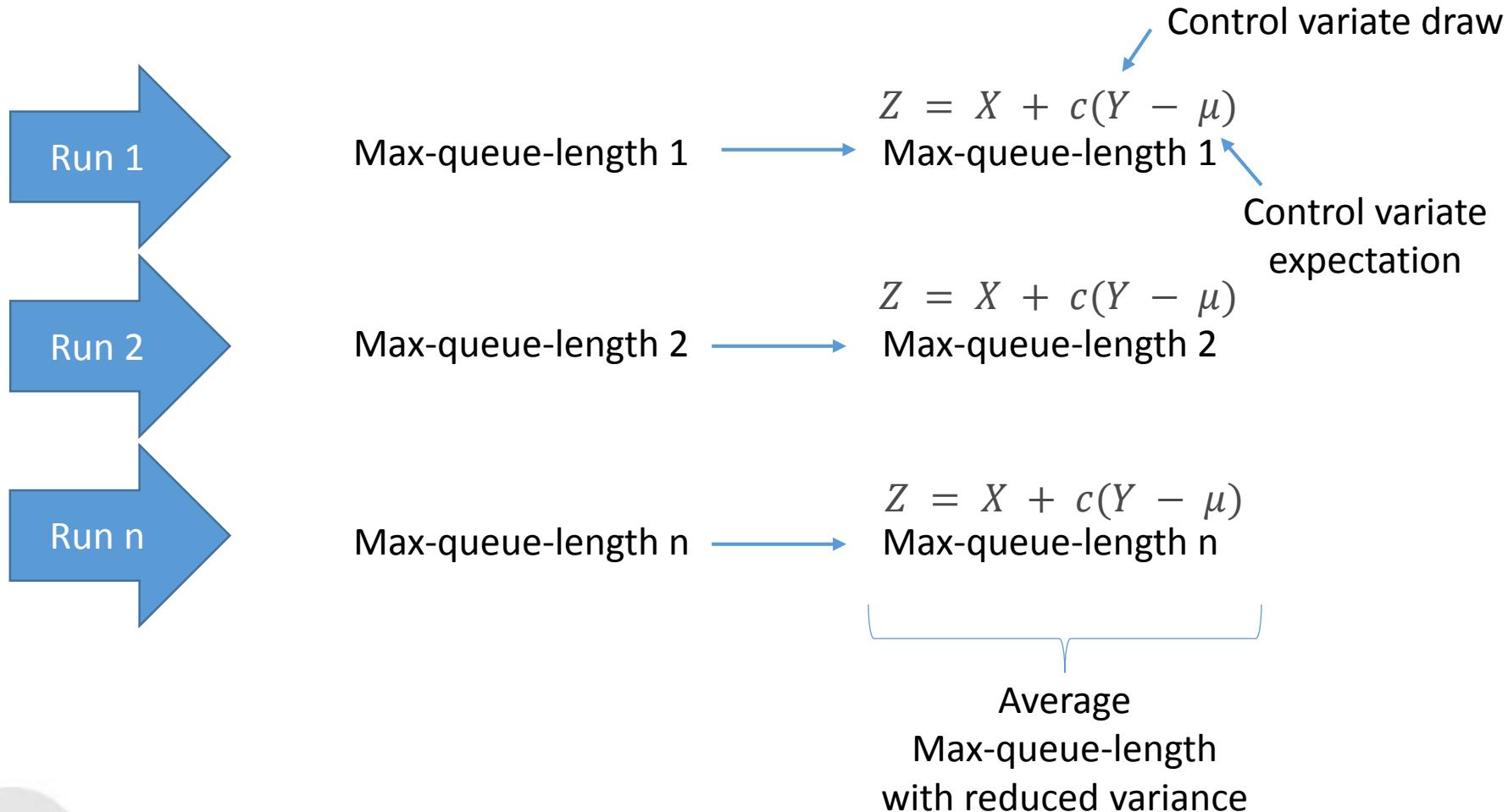
Variance reduction of the index average max-queue-length

Independent draws workflow



Control Variates

Control variates workflow



4.2 Control Variates

So far, your single road queueing simulation was function

```
[times, queues] = QueueingSimulation1(scenario)
```

- Extend this into function

```
[times, queues, control] =  
QueueingSimulation1_ControlVar(scenario)
```

- Where **control** is the realization of your control variate
- Choose the control variate you prefer. Higher the correlation bigger is the variance reduction.
- Control variate example: service time μ for each vehicle.

4.2 Control Variates

Function to implement (.m)

ControlledMean.m

Use MATLAB functions: mean, var and cov

Test the function expanding

StatAnalysisAndBootstrapTest.m

Evaluate the improvement in variance reduction against the independent procedure

Control Variates

Control Variates run

StatAnalysisAndBootstrapTest.m

main loop

```
[times, queues, control] = QueueingSimulation1_ControlVar(scenario)
```

All max-queue-length
All control data

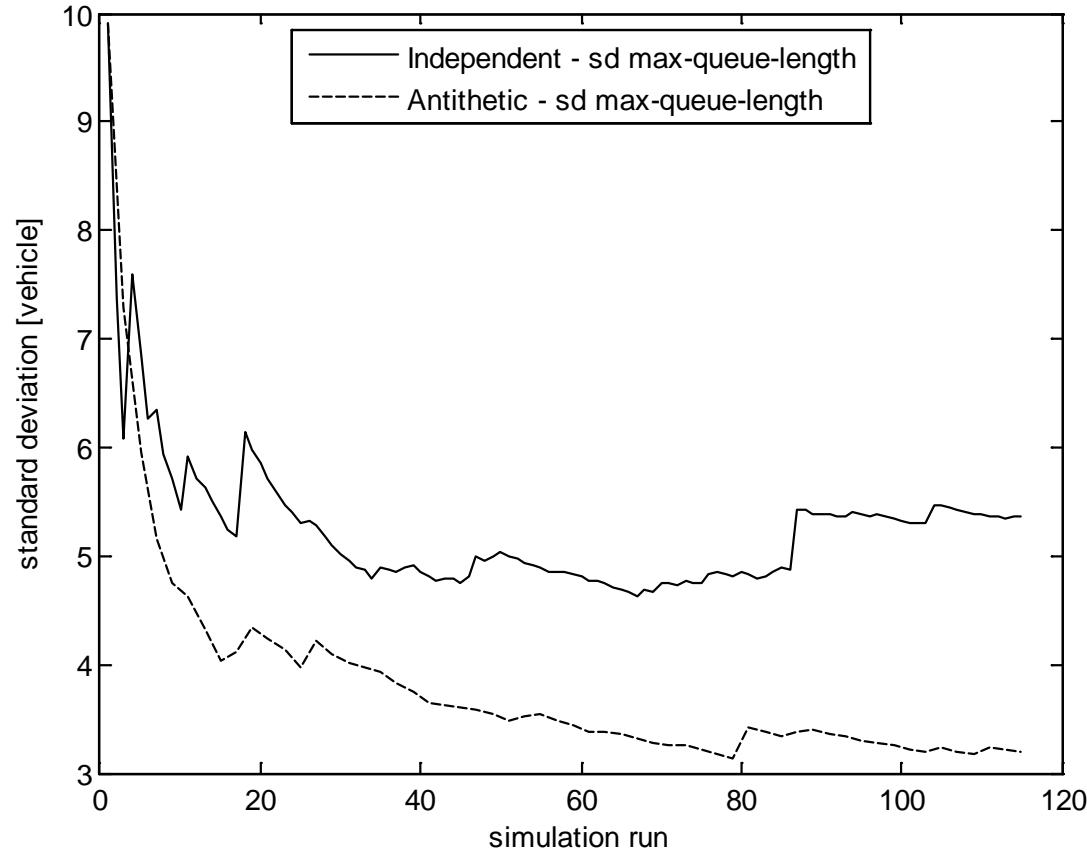
```
[avg, variance, newData] = ControlledMean(data, controlData, expectedControl)
```

end

My results

My results

Antithetic Draws



My results

Control Variates

