Preconditioned Projected Gradient Method

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General description of the algorithm

Objective

Find (an approximation of) a local minimum of the following problem:

$$\min_{x \in X \subseteq \mathcal{R}^n} f(x)$$

where X is closed, convex and not empty.

Input

- Function $f : \mathcal{R}^n \to \mathcal{R}$, differentiable
- Gradient $\nabla f : \mathcal{R}^n \to \mathcal{R}^n$
- Projection operator on X, $[\cdot]^P$
- First approximation of the solution $x_0 \in \mathcal{R}^n$
- Parameter $\gamma > 0$ (for example, $\gamma = 1$)
- Precision $\epsilon \in \mathcal{R}$, $\epsilon > 0$

Output

An approximation of the solution $x^* \in \mathcal{R}^n$

Iterations

1. $y_k = [x_k - \gamma \nabla f(x_k)]^P$
2. $d_k = y_k - x_k$
3. determine α_k by applying linesearch with $\alpha_0 = 1$
4. $x_{k+1} = x_k + \alpha_k d_k$
5. $k = k + 1$

Stopping criterion

If $\|d_k\| \leq \epsilon$, then $x^* = x_k$

Algorithm testing & analysis

The sutdents will implement and apply the above algorithm to the following non-linear problem:

$$\min_{x \in X} x_1^2 - 12x_1 + 10\cos(\frac{\pi}{2}x_1) + 8\sin(5\pi x_1) - \frac{\exp(-(x_2 - \frac{1}{2})^2/2)}{\sqrt{5}}$$

Please consider the following two cases:

1. $X = \{(x_1, x_2) \,|\, -30 \leq x_1 \leq 30, -10 \leq x_2 \leq 10\}$
2. $X = \{(x_1, x_2) \,|\, -x_1 + 2x_2 = -20\}$

It is encouraged that the students to change the value of the step γ (e.g., $\gamma = 0.1, 1, 10$) as well as to test different starting points x_0. Besides, please compare the performance of the algorithm with and without preconditioning.