Augmetned Lagrangian Method

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General description of the algorithm

Objective

Find the local minimum of the non-linear optimization problem:

$$\min_{x \in \mathcal{R}^n} f(x)$$

such that $h(x) = 0$.

Input

- Function $f : \mathcal{R}^n \to \mathcal{R}$, two-times differentiable
- Gradient $\nabla f : \mathcal{R}^n \to \mathcal{R}^n$
- Hessian $\nabla^2 f : \mathcal{R}^n \to \mathcal{R}^{n \times n}$
- Function $h : \mathcal{R}^n \to \mathcal{R}^m$, two-times differentiable
- Gradient $\nabla h : \mathcal{R}^n \to \mathcal{R}^{n \times m}$
- Hessian $\nabla^2 h_i : \mathcal{R}^n \to \mathcal{R}^{n \times n}$ for every constraint $i = 1, \ldots, m$
- Initial solution (x_0, λ_0)
- Initial penalty parameter c_0
- Precision $\epsilon \in \mathcal{R}$, $\epsilon > 0$

Initialization

$k = 0$, $\tau = 10$, $\alpha = 0.1$, $\beta = 0.9$, $w_0 = 1/c_0$, $\eta_0 = 0.1$,$\hat{\eta}_0 = \eta_0 c_0^\alpha$

Iterations

1. Use the Newton method with linesearch for solving $x_{k+1} = \arg\min_{x\in\mathcal{R}^n} L_{c_k}(x, \lambda_k)$ where
$$L_{c_k}(x, \lambda_k) = f(x) + \lambda_k h(x) + \frac{c_k}{2}\|h(x)\|^2$$
using starting point x_k and precision w_k

2. If $\|h(x_k)\| \le \eta_k$, then update multipliers:
$$\lambda_{k+1} = \lambda_k + c_k h(x_k), c_{k+1} = c_k, w_{k+1} = w_k/c_k, \eta_{k+1} = \eta_k/c_k^{\beta}$$

3. If $\|h(x_k)\| > \eta_k$, then update penalty parameter:
$$\lambda_{k+1} = \lambda_k, c_{k+1} = \tau c_k, w_{k+1} = w_0/c_{k+1}, \eta_{k+1} = \hat{\eta}_0/c_{k+1}^{\alpha}$$

4. $k = k + 1$

Stopping criterion

$\|\nabla L(x_k, \lambda_k)\| \le \epsilon$ and $\|h(x_k)\| \le \epsilon$

Algorithm testing & analysis

The sutdents will implement and apply the above algorithm to the following problem:

$$\min_{x\in\mathcal{R}^2} \qquad \ln(1 + x_1^2) - x_2$$

$$(1 + x_1^2)^2 + x_2^2 = 4$$

$$-4 \le x_1 \le 4$$

$$-4 \le x_2 \le 4$$

Note that the students need to transform the above problem to the required format, i.e., $\min_{x\in\mathcal{R}^n} f(x)$ s.t., $h(x) = 0$. It is encouraged that the students to change the value of c_0 (e.g., $c_0 = 1, 10, 100$) and analyze its impact on the algorithm. The suggested starting point for the above question is $x_0 = [2, 2]$.