Project 4: Local Sequential Quadratic Programming Method

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General description of the algorithm

Objective

Find the local minimum of the non-linear optimization problem:

$$\min_{x \in \mathcal{R}^n} f(x)$$

such that $h(x) = 0$.

Input

- Function $f : \mathcal{R}^n \to \mathcal{R}$, two-times differentiable
- Gradient $\nabla f : \mathcal{R}^n \to \mathcal{R}^n$
- Hessian $\nabla^2 f : \mathcal{R}^n \to \mathcal{R}^{n \times n}$
- Function $h : \mathcal{R}^n \to \mathcal{R}^m$, two-times differentiable
- Gradient $\nabla h : \mathcal{R}^n \to \mathcal{R}^{n \times m}$
- Hessian $\nabla^2 h_i : \mathcal{R}^n \to \mathcal{R}^{n \times n}$ for every constraint $i = 1, \ldots, m$
- Initial solution (x_0, λ_0)
- Precision $\epsilon \in \mathcal{R}$, $\epsilon > 0$

Iterations

1. Compute $\nabla^2_{xx} L(x_k, \lambda_k) = \nabla^2 f(x_k) + \sum_{i=1}^{m} (\lambda_k)_i \nabla^2 h_i(x_k)$
2. Find d_x, d_λ by solving the following quadratic problem

$$\min_d \nabla f(x_k)^T d + \frac{1}{2} d^T \nabla^2_{xx} L(x_k, \lambda_k) d$$

$$s.t. \quad \nabla h(x_k)^T d + h(x_k) = 0$$

by using the analytic solution,

$$d_\lambda = H^{-1}(h(x_k) - \nabla h(x_k)^T \nabla^2_{xx} L(x_k, \lambda_k)^{-1} \nabla f(x_k))$$

with $H = \nabla h(x_k)^T \nabla^2_{xx} L(x_k, \lambda_k)^{-1} \nabla h(x_k)$

$$d_x = -\nabla^2_{xx} L(x_k, \lambda_k)^{-1} (\nabla h(x_k) d_\lambda + \nabla f(x_k))$$

3. $x_{k+1} = x_k + d_x$
4. $\lambda_{k+1} = d_\lambda$
5. $k = k + 1$

Stopping criterion

$\|\nabla L(x_k, \lambda_k)\| \leq \epsilon$

Algorithm testing & analysis

The sutdents will implement and apply the above algorithm to the following problem:

$$\min_{x \in \mathcal{R}^2} \qquad 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

$$x_1 - x_2^2 = \frac{1}{2}$$

The suggested starting point $(x_0, \lambda_0) = (-1, 0, 1)$. Meanwhile, analyze the computation complexity for the Step 2 in terms of flop (the number of floating point operations).