Decomposition for network design: Exercises 2 **Bernard Gendron**

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1 (50 points)

Consider the following integer programming model:

$$\max \sum_{j \in N} c_j x_j, \tag{1}$$

$$\sum_{j \in N} a_{1j} x_j \leq b_1, \tag{2}$$

$$\sum_{j \in N} a_{2j} x_j \leq b_2, \tag{3}$$

$$x_j \in \{0,1\}, \quad j \in N. \tag{4}$$

1. Let Z_{LP} be the optimal value of the LP relaxation. Let Z_{LD} be the optimal value of the Lagrangian dual corresponding to the Lagrangian relaxation of constraint (2). Show that $Z_{LD} \leq Z_{LP}$.

2. Consider the following reformulation of the model:

$$\max \sum_{j \in N} c_j x_j,$$

$$\sum_{j \in N} a_{1j} x_j \leq b_1,$$

$$x_j \in \{0,1\}, \quad j \in N,$$

$$x_j = w_j, \quad j \in N, \tag{5}$$

$$\sum_{j \in N} a_{2j} w_j \leq b_2,$$

$$w_j \in \{0,1\}, \quad j \in N.$$

   Let Z_{LD}^C be the optimal value of the Lagrangian dual corresponding to the Lagrangian relaxation of constraints (5). Show that $Z_{LD}^C \leq Z_{LD}$.

3. Show that $Z_{LD}^C < Z_{LD} < Z_{LP}$ on the following example:

$$\max 2x_1 + 3x_2 + 6x_3,$$

$$3x_1 + 3x_2 + 4x_3 \leq 6,$$

$$x_1 + 4x_2 + 5x_3 \leq 8,$$

$$x_1,\, x_2,\, x_3, \in \{0,1\}.$$

2 **(50 points)**

Consider the following model for the capacitated facility location problem with single-assignment (CFLPS):

$$\min \sum_{j \in J} f_j y_j + \sum_{k \in K} \sum_{j \in J} c_{jk} x_{jk}, \tag{6}$$

$$\sum_{j \in J} x_{jk} = 1, \quad k \in K, \tag{7}$$

$$\sum_{k \in K} d_k x_{jk} \leq u_j y_j, \quad j \in J, \tag{8}$$

$$x_{jk} \leq y_j, \quad k \in K,\ j \in J, \tag{9}$$

$$x_{jk} \in \{0, 1\}, \quad k \in K,\ j \in J, \tag{10}$$

$$y_j \in \{0, 1\}, \quad j \in J. \tag{11}$$

1. Formulate the Lagrangian dual corresponding to the Lagrangian relaxation of constraints (8) and (9). How would you solve the Lagrangian subproblem?

2. Formulate the Lagrangian dual corresponding to the Lagrangian relaxation of constraints (7). How would you solve the Lagrangian subproblem?

3. Let Z^1_{LD} be the optimal value of the Lagrangian dual of number 1. Let Z^2_{LD} be the optimal value of the Lagrangian dual of number 2. Show that $Z^1_{LD} \leq Z^2_{LD}$. Give an example where $Z^1_{LD} < Z^2_{LD}$.