

# Errata for the book “Optimization: principles and algorithms

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## 1 Errors

**Page 16** Equation (1.31) is ambiguous. It should be

$$\operatorname{argmin}_{x \in X \subseteq \mathbb{R}^n} f(x) = \operatorname{argmin}_{x \in X \subseteq \mathbb{R}^n} h(x),$$

where  $h(x) = g(f(x))$ .

**Page 86** Theorem 3.40. The correct statement is “(Equivalence between vertices and feasible basic solutions) Let  $\mathcal{P} = \{x \in \mathbb{R}^n \mid Ax = b, x \geq 0\}$  be a polyhedron. The point  $x^* \in \mathcal{P}$  is a vertex of  $\mathcal{P}$  if and only if it is a feasible basic solution.”

**Page 86** Proof of Theorem 3.40. Replace “Consider  $m$  linearly independent columns of  $A$ , chosen arbitrarily, which form an invertible matrix  $B, \dots$ ” by “As  $x^*$  is not a feasible basic solution, there are strictly more than  $m$  non zero components in  $x^*$ . Consider  $m$  linearly independent columns of  $A$ , corresponding to non zero components of  $x^*$ , which form an invertible matrix  $B, \dots$ ”

**Page 86** Proof of Theorem 3.40. Replace “Since  $x_B > 0$  and  $x_k^* > 0$ ” by “Since  $x_k^* > 0$ ”. Indeed, the rest of the proof requires only the positivity of the  $k$ th component. Moreover, there is no guarantee that  $x_B > 0$  in the degenerate case discussed after.

**Page 99** After Eq. (4.15). “Therefore, we can eliminate  $\mu_1$  and  $\mu_2$  so that

$$X_q = \{\lambda \mid \lambda \leq 1\},$$

and the dual function becomes

$$q(\lambda) = \lambda.$$

The dual problem is written as

$$\max \lambda$$

subject to

$$\lambda \leq 1,$$

for which...”

**Page 124** Theorem 3.10, item 3. Instead of “there is an infinite number of local minima”, it should be “there is an infinite number of global minima”.

**Pages 315 and 316** There is a mistake in Eq. (13.13), that is reproduced in Algorithm 13.1. Instead of

$$H_k^{-1} = \left( I - \frac{d_{k-1}y_{k-1}^T}{d_{k-1}^T y_{k-1}} \right) H_{k-1}^{-1} \left( I - \frac{d_{k-1}y_{k-1}^T}{d_{k-1}^T y_{k-1}} \right) + \frac{d_{k-1}d_{k-1}^T}{d_{k-1}^T y_{k-1}},$$

it should be

$$H_k^{-1} = \left( I - \frac{d_{k-1}y_{k-1}^T}{d_{k-1}^T y_{k-1}} \right) H_{k-1}^{-1} \left( I - \frac{y_{k-1}d_{k-1}^T}{d_{k-1}^T y_{k-1}} \right) + \frac{d_{k-1}d_{k-1}^T}{d_{k-1}^T y_{k-1}}.$$

**Page 388** “The algorithm may detect an unbounded problem at step 15, or produce an optimal solution  $x^*, x_a^*$ . Consequently, one of the following three possibilities occurs:”. It should be “The algorithm cannot detect an unbounded problem at step 15, as the objective function of  $(\mathcal{A})$  is bounded below by 0. It always produces an optimal solution  $x^*, x_a^*$ . Consequently, one of the following two possibilities occurs:”

**Page 388** “In summary, if the original problem is bounded, solving the auxiliary problem....” should be “In summary, solving the auxiliary problem...”

**Page 546** Example 22.14: “The best deal is obtained by selling .... for a total of 32,000.01 kEuros” should be “The best deal is obtained by selling the Botticelli to Harry, the Bruegel to Giny, the Kandinsky to Ron and the last one to Hermione, for a total of 34,000.01 kEuros.”

**Page 570** Proof of Theorem 23.14. The result is correct, but the proof of property 1 is incorrect. Indeed, property 4 of Theorem 23.9 assumes that  $i \notin \mathcal{S}$ . If the algorithm treats  $i$ , it means that  $i \in \mathcal{S}$  and the property cannot be applied. This invalidates the proof for properties 1 and 4. A new proof by induction is provided below.

Property 4 holds for the first iteration, where node  $o$  is treated, with  $\lambda_o = 0$ . It is the only node in  $\mathcal{T}$  at the end of the iteration. All other labels are equal to  $\infty$ , except for nodes  $j$  such that  $(o, j) \in \mathcal{A}$ . As  $\lambda_j = c_{oj} \geq \lambda_o = 0$ , the property holds for these labels. As  $\mathcal{T} = \emptyset$  at the beginning of the iteration, property 1 trivially holds for the first iteration.

Consider now another iteration, and assume that property 4 is true at the beginning of the iteration, that is  $\lambda_i \leq \lambda_j, \forall i \in \mathcal{T}, \forall j \notin \mathcal{T}$ . The iteration is treating node  $\ell$ . According to the rule of node selection,

$$\lambda_\ell \leq \lambda_j, \quad \forall j \notin \mathcal{T}. \quad (1)$$

As  $\ell \in \mathcal{S}, \ell \notin \mathcal{T}$  at the beginning of the iteration. When the iteration treats arc  $(\ell, m)$ , two cases must be considered:  $m \in \mathcal{T}$  and  $m \notin \mathcal{T}$ .

$m \in \mathcal{T}$  Using the assumption of the induction, we have  $\lambda_m \leq \lambda_\ell$ . As  $c_{\ell m} \geq 0$ , we have also  $\lambda_m \leq \lambda_\ell + c_{\ell m}$ . It means that no node in  $\mathcal{T}$  will see its label updated during the iteration. This proves property 1. As no label has been updated by the algorithm in this case, property 4 continues to hold.

$m \notin \mathcal{T}$  If the label of node  $m$  is not updated, nothing changes, and property 4 continues to hold. If the label is updated, we have, at the end of the iteration,  $\lambda_m = \lambda_\ell + c_{\ell m}$ . Take any node  $i \in \mathcal{T}$ . We have, at the end of the iteration,  $\lambda_i \leq \lambda_\ell$  by the assumption of the induction and the fact that the label of  $i$  has not been updated by the iteration. As  $c_{\ell m} \geq 0$ ,  $\lambda_i \leq \lambda_\ell + c_{\ell m} = \lambda_m$ , and the property holds after the iteration for all nodes that were in  $\mathcal{T}$  at the beginning of the iteration.

Finally, as  $\ell$  is in  $\mathcal{T}$  at the end of the iteration, we need to show that  $\lambda_\ell \leq \lambda_j, \forall j \notin \mathcal{T}$ . This is guaranteed by the rule of node selection (1), and the fact that  $\lambda_\ell$  has not been modified by the iteration.

## 2 Typos

**Page 20** Section 1.3. “Korte and Vygen, 2007” should be “Korte and Vygen (2007)”

**Page 51** Definition 3.1 “this concept is sometimes called *a feasible solution*” should be “this concept is sometimes called a *feasible solution*”

**Page 82** proof of Theorem 3.37 “for any  $i \in \mathcal{A}(x^*)$ ” should be “for any  $i \in \mathcal{A}(x_0)$ ”

**Page 83** Definition 3.38 “If, moreover,  $x = B^{-1}b \geq 0$ ” should be “If, moreover,  $x_B = B^{-1}b \geq 0$ ”

**Page 166** After Eq. (6.160) “ $b \in \mathbb{R}^n$ ” should be “ $b \in \mathbb{R}^m$ ”

**Page 159** Example 6.23 “ $x_1^2 + 4x_1 + 3 = 0$ ” should be “ $x_1^2 + 4x_1 - x_2 + 3 = 0$ ”

**Page 263** Introduction to Section 11.3 “a one-dimensional optimization problems” should be “a one-dimensional optimization problem”

**Page 265** Last paragraph “the improvement of objective function” should be “the improvement of the objective function”

**Page 371** Algorithm 16.2, step 22 “ $\alpha_q = \min_i \lambda_i$ ” should be “ $\alpha_q = \min_i \alpha_i$ ”

**Page 426** The Lagrangian before equation (18.16) should be written

$$L(x, \lambda, \mu) = c^T x - \varepsilon \sum_{i=1}^n \ln x_i + \lambda^T (Ax - b) - \mu^T x.$$

**Page 494** Definition 21.4 “where  $\mathcal{M} \subseteq \mathcal{N}$ ” should be “where  $\mathcal{M} \subset \mathcal{N}$ ”

**Page 501** biography of Leonhard Euler “..., and became friends with his two sons” should be “..., and became friend with his two sons”

**Page 506** Theorem 21.14 “a subset of nodes  $\mathcal{M} \subseteq \mathcal{N}$ ” should be “a subset of nodes  $\mathcal{M} \subset \mathcal{N}$ ”.

**Page 507** Line -17, line -9, line -2. “cost-additive” should be “link-additive”.

**Page 507** Line -11. “cost-additivite” should be “link-additive”.

**Page 532** Figure 22.2(c). “ $z_{31} = 2(0, +\infty)$ ” should be “ $y_{31} = 2(0, +\infty)$ ”.

**Page 535** Section 22.2 “The Karush-Kuhn-Tucker conditions (see Theorem 6.13) significantly simplify the transshipment problem” should be “The Karush-Kuhn-Tucker conditions (see Theorem 6.13) are significantly simpler for the transshipment problem”

**Page 616** Section 25.3 “about the time that as, elapsed since “homo erectus”” should be “about the time that has elapsed since “homo erectus””