A Hierarchical Framework for Long-Term Power Planning Models

Tang & Ferris

René Glogg
Dasun Perera
Martin Repoux
Quanjiang Yu

Zinal Winter School 2017
Outline

• Context of the problem

• Energy-economic model

• Solving methods

• Numerical example
Context of the problem

• Where energy systems meet optimization
• Distributed generation is getting popular
  • Renewable energy integration
  • Flexible demand and generation
  • Real time pricing
  • Virtual power plants
• Number of parties involved
  • Power generation companies
  • Distributors + transmission
  • Regulators
  • Consumers
• Long term planning, system designing and operation is challenging
## Context of the problem

<table>
<thead>
<tr>
<th>Authors</th>
<th>Journal</th>
<th>Year</th>
<th>Method</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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</table>

1. Node dispatch
2. OPF
3. System sizing problem
4. Transmission planning A: Line sizing
5. Transmission Planning B: System sizing/locating
6. Transmission Planning C: Connectivity

![Image 1](image1.png)  
![Image 2](image2.png)
Context of the problem

Regional transmission organizations (RTO)

Independent system operators (ISO)

Firm

Nash Equilibrium

Model: Hierarchical framework

UPPER LEVEL: expansion

LOWER LEVEL: market = supply and demand
Expansion Model

- Minimization over expansion on specific transmission lines
- Uses information from the lower hierarchy, specifically what the LMP is and whether a generator is dispatched or not
Expansion Model (RTO)

\[
\min_{x \in X} \left( \psi(x) + \sum_{\omega \in \Omega} \pi_\omega \sum_{j \in N} \left( d_j^\omega p_j^\omega(x) + u_j^\omega(x) c_j^\omega(x) \right) \right)
\]

s.t. \( L(x) \leq 0. \) (1)


<table>
<thead>
<tr>
<th>Set</th>
<th>Description</th>
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<tbody>
<tr>
<td>( \omega \in \Omega )</td>
<td>Demand scenarios</td>
</tr>
<tr>
<td>( i, j \in N )</td>
<td>Network Vertices</td>
</tr>
<tr>
<td>( ij \in A )</td>
<td>Transmission lines. ( A \in {N, N} )</td>
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<tr>
<td>( f \in F )</td>
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</tr>
<tr>
<td>( G )</td>
<td>Generation nodes, ( G \subseteq N )</td>
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<td>( G_f \subseteq G )</td>
<td>Generation nodes belonging to firm ( f )</td>
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<tr>
<td>( E(j) \in N )</td>
<td>Edges connected to node ( j )</td>
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<table>
<thead>
<tr>
<th>Model</th>
<th>Variable</th>
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<tr>
<td>RTO</td>
<td>( x )</td>
<td>Investment in transmission line expansion</td>
</tr>
<tr>
<td>Firm</td>
<td>( y_j )</td>
<td>Investment in generator at ( j )</td>
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<tr>
<td>ISO</td>
<td>( z_{ij}^\omega )</td>
<td>Real power flowing along ( i - j ) in scenario ( \omega )</td>
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<tr>
<td></td>
<td>( \theta_j^\omega )</td>
<td>Voltage phase angle at ( j ) in scenario ( \omega )</td>
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<tr>
<td></td>
<td>( g_j^\omega )</td>
<td>Real power at ( j ) in scenario ( \omega )</td>
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<tr>
<td></td>
<td>( u_j^\omega )</td>
<td>Dispatch of generator at ( j ) in scenario ( \omega )</td>
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<table>
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<tr>
<th>Parameter</th>
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<th>Model</th>
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<tr>
<td>( d_j^\omega )</td>
<td>Demand at ( j ) in scenario ( \omega )</td>
<td>All</td>
</tr>
<tr>
<td>( \psi(x) )</td>
<td>Cost of line investment</td>
<td>RTO</td>
</tr>
<tr>
<td>( c_j^\omega(x) )</td>
<td>Generator uplift costs</td>
<td>RTO</td>
</tr>
<tr>
<td>( p_j^\omega(x) )</td>
<td>LMP value</td>
<td>RTO</td>
</tr>
<tr>
<td>( L(x) )</td>
<td>Budgetary/engineering limitations</td>
<td>RTO</td>
</tr>
<tr>
<td>( \pi_\omega )</td>
<td>Probability of scenario ( \omega )</td>
<td>RTO,Firm</td>
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<tr>
<td>( S_{ij} )</td>
<td>Susceptance of line ( i - j )</td>
<td>ISO</td>
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<tr>
<td>([g_j, \bar{g}_j])</td>
<td>Operating limits of generator at ( j )</td>
<td>ISO</td>
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<tr>
<td>( z_{ij} )</td>
<td>Capacity of transmission line ( i - j )</td>
<td>ISO</td>
</tr>
<tr>
<td>( C_j(g_j^\omega, y_j) )</td>
<td>Cost function of generator at ( j )</td>
<td>ISO,Firm</td>
</tr>
<tr>
<td>( \Phi(y) )</td>
<td>Cost of generator investment</td>
<td>Firm</td>
</tr>
<tr>
<td>( H_f(y) )</td>
<td>Budgetary constraints of firm ( f )</td>
<td>Firm</td>
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</table>
Optimal power flow model

• Solved by the ISO to manage grid operations and provide LMP
• Modeled by short term competitive behavior of firms
• Minimize cost of unit commitment
• Solved to equilibrium with the firm models, where each firm has the option to improve efficiency of existing generators or of retiring them
• Provides LMPs and unit commitments to upper hierarchy
Equilibrium

\[ \text{OPF}^{\text{UC}} (\forall \omega \in \Omega) : \]
\[ \min_{u, g, z, \theta} \sum_{j \in G} u_j^\omega C_j (g_j^\omega, y_j) \]
\[ \text{s.t.} \quad g_j^\omega - d_j^\omega = \sum_{i \in E(j)} z_{ij}^\omega \perp p_j^\omega \quad \forall j \in N \]
\[ S_{ij} (\theta_i^\omega - \theta_j^\omega) = z_{ij} \quad \forall i, j \in A \]
\[ u_j^\omega g_j \leq g_j \leq u_j^\omega g_j \quad \forall j \in N \]
\[ z_{ij}^\omega \in [-z_{ij}(x), z_{ij}(x)] \quad \forall i, j \in A \]
\[ \text{free}, \ u \in \{0, 1\} \]

Firm \( (\forall f \in F) : \)

\[ \min_y \sum_{\omega \in \Omega} \sum_{j \in G_f} \left( u_j^\omega C_j (g_j^\omega, y_j) + \Phi(y_j) \right) \]
\[ \text{s.t.} \quad H_f(y) \leq 0. \]
Generator Upgrades

The cost of generating $g_j$ units of energy is given by:

$$C_j^0 \left( g_j^\omega \right) = \hat{c}_j \left( g_j^\omega \right)^2 + \hat{c}_j g_j^\omega + \hat{c}_j$$

The paper models the impact of an investment by a diminishing rewards function $\hat{c}_j^* (y_j)$, therefore:

$$C_j \left( g_j^\omega, y_j \right) = \hat{c}_j^* (y_j) g_j^2 + \hat{c}_j y_j + \hat{c}_j$$

New generator capacity or introduction may be accommodated by an additional hierarchy, but due to complexity it is preferable to use additional dispatch variables and constraints.
Solution method: principle

**UPPER LEVEL:** expansion

Derivative-free optimization

REQUIRES prices $p_j^\omega$

**LOWER LEVEL:** market

Multiple Optimization Problems

GIVES prices $p_j^\omega$
Lower level: MOPEC

KKT conditions of  
\[ \min_{u,g,z,\theta} \sum_{j \in G} u_j^\omega C_j(g_j^\omega, y_j) \forall \omega \in \Omega \]

+ 

KKT conditions of  
\[ \min_{y} \sum_{\omega \in \Omega} \pi_\omega \sum_{j \in F} (u_j^\omega C_j(g_j^\omega, y_j) + \Phi(y_j)) \forall f \in F \]

= 

CP

⚠️ KKT conditions cannot be written due to binary variables \( u \)
Approximation for generator costs

\[ u_j^\omega C_j(g_j^\omega, y_j) \rightarrow \tilde{C}_j(g_j^\omega, y_j) \]
Solution method: MCP*

- Smooth and continuous model – KKT conditions can be derived
- Non-convexity of objective functions

```
Procedure 1: MCP Solution Process

Initialization: \( g_j^\omega = 0, y_j = 0 \) \( \forall j \in N, \forall \omega \in \Omega \)

Loop
   // Obtain \( g_j^\omega \) starting points with \( y_j \) fixed, eqns.(11-15)
   \( \tilde{g}_j^\omega \leftarrow \text{OPF}^*(y_j) \) \( \forall \omega \in \Omega \)
   if \( \tilde{g}_j^\omega = g_j^\omega \) then // convergence
      break;
   else // Solve equilibrium with \( \tilde{g}_j^\omega \) starting points
      \( (g_j^\omega, y_j) \leftarrow \text{MCP with } \tilde{g}_j^\omega \) starting points
```

- Obtain good starting points for equilibrium
- Convergence test
- Solving the equilibrium
2\textsuperscript{nd} Solution method: RMCP

\textbf{OBSERVATION:} MCP does not change generators chosen in dispatch

\begin{itemize}
  \item Variables $u$ fixed inside each loop
  \item Convexity of the objectives functions
\end{itemize}

\begin{table}[h]
\centering
\begin{tabular}{|c|}
\hline
\textbf{Procedure 2: Restricted MCP Solution Process} \\
\hline
\textbf{Initialization:} $g_j^\omega = 0$, $y_j = 0$ \quad $\forall j \in N$, $\forall \omega \in \Omega$ \\
\hline
\textbf{Loop} \\
\hline
// Obtain $(\tilde{g}_j^\omega, u_j^\omega)$ with $y_j$ fixed, eqns.(3-8) \\
$(\tilde{g}_j^\omega, u_j^\omega) \leftarrow \text{OPF}^{\text{UC}}(y_j) \quad \forall \omega \in \Omega$
\hline
\textbf{if} $\tilde{g}_j^\omega = g_j^\omega$ \textbf{then} // convergence \\
\hspace{1cm} break; \\
\textbf{else} // Solve RMCP with $u_j^\omega$ fixed \\
\hspace{1cm} $(g_j^\omega, y_j) \leftarrow \text{MCP} u_j^\omega$
\hline
\end{tabular}
\end{table}

- Convergence test
- Solving the equilibrium
Results and observations

• Solved using PATH

• Same solutions with the two procedures

• RMCP converges faster

• Uniqueness issues Need to break symmetry
Numerical Example

Graphical depiction of 14-bus example.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Coefficient for $g_j$ as a fraction of $\bar{g}_j$</td>
<td></td>
<td>0.3</td>
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<tr>
<td>Coefficient for $\hat{c}_j$ as a fraction of $C_j (g_j = g_j, y_j = 0)$</td>
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<tr>
<td>Firm investment budget ($\text{$ units}$)</td>
<td>Firm $f_1$ ($G_1$, $G_3$, $G_8$)</td>
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</tr>
<tr>
<td></td>
<td>Firm $f_2$ ($G_2$, $G_6$)</td>
<td>200</td>
</tr>
<tr>
<td>Generator upgrade parameter, $\gamma$ ($\text{$ units}$)</td>
<td>Large generator ($G_1$)</td>
<td>200</td>
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<td></td>
<td>Medium generator ($G_2$)</td>
<td>150</td>
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<td></td>
<td>Small generator ($G_3$, $G_6$, $G_8$)</td>
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<table>
<thead>
<tr>
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<th>$\omega_1$</th>
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<th>$\omega_3$</th>
<th>$\omega_4$</th>
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<td>0.1</td>
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<td>1.8</td>
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<td>2.2</td>
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Function Evaluation for Fixed $x$

Set $y=0$

**OPF\textsuperscript{UC}, iteration 1:**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$g_1$</th>
<th>$g_2$</th>
<th>$g_3$</th>
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<td>0.42</td>
<td></td>
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<td>1.00</td>
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<tr>
<td>$\omega_2$</td>
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<td>0.42</td>
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<td></td>
<td>0.84</td>
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<tr>
<td>$\omega_3$</td>
<td>2.39</td>
<td>0.44</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$\omega_4$</td>
<td>2.43</td>
<td>0.75</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$\omega_5$</td>
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<td>0.52</td>
<td>0.74</td>
<td>1.00</td>
<td>1.00</td>
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**RMCP, iteration 1:**

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<th>$g_3$</th>
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</thead>
<tbody>
<tr>
<td>$\omega_1$</td>
<td>2.17</td>
<td>0.42</td>
<td></td>
<td></td>
<td>0.67</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>2.50</td>
<td>0.46</td>
<td></td>
<td></td>
<td>1.00</td>
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<tr>
<td>$\omega_3$</td>
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<td>0.61</td>
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<td>0.75</td>
<td>1.00</td>
<td>1.00</td>
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<tr>
<td>$\omega_5$</td>
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<td>0.54</td>
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<td>1.00</td>
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**OPF\textsuperscript{UC}, iteration 2 and 3 (convergence):**

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<tr>
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<th>$g_2$</th>
<th>$g_3$</th>
<th>$g_6$</th>
<th>$g_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1$</td>
<td>2.17</td>
<td>0.42</td>
<td></td>
<td></td>
<td>0.67</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>2.50</td>
<td>0.50</td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
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**RMCP, iteration 2 and 3 (convergence):**

<table>
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<th>$g_3$</th>
<th>$g_6$</th>
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<tbody>
<tr>
<td>$\omega_1$</td>
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<td>0.67</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>2.50</td>
<td>0.50</td>
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<td></td>
<td>1.00</td>
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<tr>
<td>$\omega_3$</td>
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<td>1.00</td>
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<td>$\omega_4$</td>
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<td>0.75</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$\omega_5$</td>
<td>2.52</td>
<td>1.18</td>
<td>1.00</td>
<td>1.00</td>
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**Firm**

<table>
<thead>
<tr>
<th>$f_1$</th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_3$</th>
<th>$y_6$</th>
<th>$y_8$</th>
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<tr>
<td>245.34</td>
<td>157.52</td>
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<td>54.67</td>
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</table>

<table>
<thead>
<tr>
<th>$f_2$</th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_3$</th>
<th>$y_6$</th>
<th>$y_8$</th>
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<tr>
<td>245.82</td>
<td>169.54</td>
<td>30.46</td>
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<td>54.18</td>
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</tbody>
</table>
Transmission Expansion

Nash equilibrium gives us the dual variables $p_j^\omega$ at the upper hierarchy.

Transmission Expansion Model (RTO)

\[
\min_{x \in X} \alpha \|x\|_1 + \sum_{\omega} \pi_\omega \sum_{j \in N} (d_j^\omega p_j^\omega (x) + u_j^\omega (x) \tilde{c}_j^\omega (x)) \\
\text{s.t. } \|x\|_1 - \xi \leq 0.
\]

Penalty function

\[
\min_{x \in X} \left\{ \alpha \|x\|_1 + \sum_{\omega} \pi_\omega \sum_{j \in N} d_j^\omega p_j^\omega (x) + u_j^\omega (x) \tilde{c}_j^\omega (x) \\
+ \eta \max (\|x\|_1 - \xi, 0) \right\}.
\]
Budgeted Transmission Expansion

<table>
<thead>
<tr>
<th>Description/Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set of decision variables</td>
<td>{x_{12}, x_{15}, x_{23}, x_{49}, x_{79}, x_{9,10}}</td>
</tr>
<tr>
<td>Physical bound, (\bar{x})</td>
<td>1.0</td>
</tr>
<tr>
<td>Investment budget, (\xi)</td>
<td>2.0</td>
</tr>
<tr>
<td>Line limits, (\bar{z}_{ij}(0))</td>
<td>1.0</td>
</tr>
<tr>
<td>Penalty value, (\eta)</td>
<td>1500</td>
</tr>
</tbody>
</table>

\[ \xi = 2 \]
\[
\hat{x} = \begin{pmatrix}
x_{12} \\
x_{15} \\
x_{23} \\
x_{49} \\
x_{79} \\
x_{9\,10}
\end{pmatrix} = \begin{pmatrix}
1.089 \\
0.060 \\
0 \\
0 \\
0.30 \\
0
\end{pmatrix}
\]

\[ f(\hat{x}) = 17558.715 \]
MCS iterations = 19
Procedure 2 calls = 1227.

\[ \xi = 1 \]
\[
\tilde{x} = \begin{pmatrix}
\tilde{x}_{12} \\
\tilde{x}_{15} \\
\tilde{x}_{23} \\
\tilde{x}_{49} \\
\tilde{x}_{79} \\
\tilde{x}_{9\,10}
\end{pmatrix} = \begin{pmatrix}
0.636 \\
0 \\
0.060 \\
0 \\
0 \\
0
\end{pmatrix}
\]

\[ f(\tilde{x}) = 18079.64 \]
MCS iterations = 19
Procedure 2 calls = 798.
Budgeted Transmission Expansion

• When increasing line limits, $x$, allow for a generator to be dropped from the active dispatch set in order to reduce operational cost.

• The remaining generators in the active set are forced to pick up the slack by increasing their production, which leads to an overall increase in the marginal cost.

• This has a direct impact on consumer cost because the LMP values, $p_j^\omega$, correspond of the marginal cost.

• When the line expansion variable is sufficiently increased, it is possible for the LMP prices $p_j^\omega$ to either increase or decrease.
Conclusion

• Framework which takes into account RTO, ISO and firms interactions

• Energy modelling issues
  • Generator expansion
  • RTO objective function
  • AC vs. DC power flow equations
  • Uncertainties

• Only 2 horizons: scaling of the problem and interactions?