A Recovery Algorithm for a Disrupted Airline Schedule

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In collaboration with APM Technologies
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Airline Scheduling Approach

- Route Choice
- Fleet Assignment
- Tail Assignment
- Crew Pairing
- Crew Roistering
- Passenger Routing (Catering)
Disrupted Schedule Recovery

$t_0 + T$

Schedule $S_0$

Recovery Decision

Disruption

$t_0$

$t_0$

$t_0$
Definitions

- **Disruption**
  event making a schedule unrealizable

- **Recovery**
  action to get back to initial schedule

- **Recovery Period (T)**
  time needed to recover initial schedule
Definitions

• *Recovery Plan*
  set of actions to recover disrupted schedule

• *Recovery Scheme (r)*
  set of actions for a resource (plane)
Hypothesis

- consider only fleet and tail assignment
- no repositioning flights
- no early departure for flights
- work with universal time (UMT)
- initial state of resources are known
- no irregularity until end of recovery period
- maintenance forced by resource consumption
Column Generation

- column = recovery scheme (schedule for a plane)

- recovery scheme $r = \text{way to link Initial State to Final State with succession of flights and maintenances}$

- suppose set of all possible schemes $R$ known

- find optimal combination of schemes
Master Problem (IMP)

\[
\begin{align*}
\min \quad & z_{\text{MP}} = \sum_{r \in R} c_r x_r + \sum_{f \in F} c_f y_f \\
\text{s.t.} \quad & \sum_{r \in R} b_r^f x_r + y_f = 1 \quad \forall f \in F \\
\quad & \sum_{r \in R} b_r^s x_r = 1 \quad \forall s \in S \\
\quad & \sum_{r \in R} b_r^p x_r \leq 1 \quad \forall p \in P \\
\quad & x_r \in \{0,1\} \quad \forall r \in R \\
\quad & y_f \in \{0,1\} \quad \forall f \in F
\end{align*}
\]
What is a column?

- Vector \( \mathbf{b}_r = (b_r^f, b_r^s, b_r^p)^T \)

Where

- \( b_r^f = 1 \) if flight \( f \) is covered by column \( r \)
- \( b_r^s = 1 \) if final state \( s \) is covered by \( r \)
- \( b_r^p = 1 \) if column \( r \) is affected to plane \( p \)
Example

\[ f_1 \text{ GVA to AMS} \]
\[ f_2 \text{ AMS to BCN} \]
\[ f_3 \text{ BCN to GVA} \]
\[ f_4 \text{ MIL to BUD} \]
\[ f_5 \text{ BUD to MIL} \]
\[ f_6 \text{ BCN to MIL} \]
Column Generation

Example

• flights: \[ F = \{f_1, f_2, f_3, f_4, f_5, f_6\} \]
• final states: \[ S = \{S^{GVA}, S^{MIL}\} \]
• planes: \[ P = \{p_1, p_2\} \]
• \( p_1 \) starts in GVA, \( p_2 \) starts in MIL
Column examples

\[ b_1 = (0,0,0,0,0,0,1,0,1,0)^T \]

\[ b_2 = (1,1,1,0,0,0,1,0,1,0)^T \]

\[ b_3 = (0,0,0,1,1,0,0,1,0,1)^T \]
Column Generation

Feasible Solution
Solving the Master Problem

I. Solve IMP with Branch and Bound

II. Solve linear relaxation LP at each node:

- Restrict LP to sub-set \( R' \subseteq R \)
- Solve RLP
- Find \( b_r \in R \setminus R' \) minimizing reduced cost
- Insert column if \( r.c. < 0 \) and resolve RLP
The Pricing Problem

Find column $b_r \in R \setminus R'$ minimizing reduced cost $\tilde{c}_r^p$

$$\min_{r \in R} \tilde{c}_r^p = c_p^r - \sum_{f \in F} b_r^f \lambda_f - \sum_{s \in S} b_r^s \eta_s - b_r^p \mu_p \quad \forall p \in P$$

Recovery Network Model

Solve Resource Constrained Elementary Shortest Path Problem (RCESPP)
The Recovery Network (Argüello et al. 97)

• Time-space network
• One network for every plane
• Source node corresponding to initial state
• Sinks corresponding to expected final states
• 3 arc types (NEVER horizontal):
  1. Flight arcs
  2. Maintenance arcs
  3. Termination arcs (vertical)
Source and Sink Nodes

Plane $p_1$, initial state = [GVA, 0800]
Expected States: [GVA, 1800] and [MIL, 1500]
Flight and Maintenance Arcs

flight F1: GVA to NY at 1200
Arc Costs

- Flight arcs: $c = c^f - \lambda_f$
- Maintenance arcs: $c = c^f + c^M - \lambda_f$
- Termination arcs: $c = -\eta_s$
Recovery Network Properties

• No horizontal arcs

• No vertical arcs except termination arcs

• Node only at earliest availability time

• Grounding time included in arc length (3 types)

• Maintenances are integrated before flight if possible
Preliminary Results

• implementation using COIN-OR BCP

• solve three problems of various sizes:
  1. 48 flights, 9 airports, 3 planes
  2. 84 flights, 15 airports, 11 planes
  3. 36 flights, 17 airports, 10 planes

• solved 1. to optimality (root node)

• promising results for instances 2. and 3.
Future Work

• Work on implementation

• Test more real instances

• Explore more widely RCESPP and CG algorithms

• Compare solutions to real recovery decisions

• Include Algorithm in APM Framework
Conclusions

- Colum Generation to solve DSRP
- Adapted model to solve pricing problem
- Get quick solutions for decision aid
- Still need real-instance validation
THANKS for your attention!

Any Questions?