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Weekly management of a hydraulic valley by robust optimization

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2. Robust optimization : key points
   a) Uncertainty set
   b) Robust counterpart of a constraint

3. Robust with Periodic Revisions (RPR)

   Short presentation and results

4. Robust with Linear Adjustments (RLA)

   Short presentation and results

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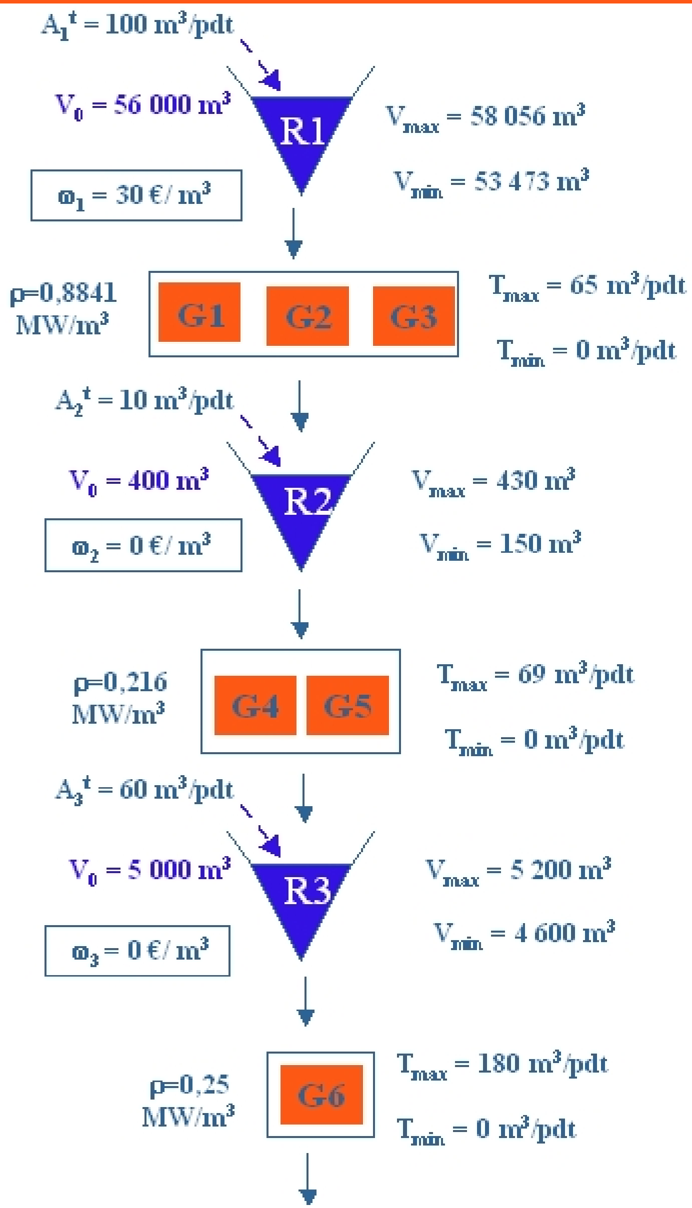
General introduction

1.1 – Presentation of the problem - Example



A hydraulic valley is composed of:

- **Reservoirs** (V_{min}, V_{max}, V_0, mean inflows , water value)
- **Groups (Release or pump water)** (T_{min}, T_{max}, Yields)

Goal : Find a strategy of hydroelectricity production for 7 days (84 time steps), respecting all technical constraints with a minimum cost.

Problem : uncertain inflows !!

1.2 – Deterministic formulation of the problem

$$\min_{T_{gt}} \quad \sum_g \rho_g \sum_{\tau \leq H} c_\tau T_{g\tau} + \sum_r \omega_r (V_r^0 - V_r^H)$$

$$V_r^t = V_r^0 + \sum_{\tau \leq t} a_r^\tau + \sum_{\tau \leq t} \sum_g \delta_{gr} T_{g\tau}, \ \forall r, \ \forall t \leq H$$

$$\underline{V}_r^t \leq V_r^t \leq \bar{V}_r^t, \ \forall r, \ \forall t \leq H$$

$$\underline{T}_g^t \leq T_{gt} \leq \bar{T}_g^t, \ \forall g, \ \forall t \leq H.$$

H	:	Horizon (divided into H/T days)
r	:	Reservoir r
g	:	Group (release or pump water) g
T_g^t	:	Quantity of released water by g at t
$\underline{T}_g^t, \bar{T}_g^t$	:	Minimal and maximal quantities of water allowed to be release by the group g at t
V_r^0	:	Initial volume for the reservoir r
$\underline{V}_r^t, \bar{V}_r^t$	:	Minimal and maximal volumes allowed for r at t
a_r^t	:	Forecasted inflows in r at t
ρ_g	:	Yield of group g
c_t	:	Marginal costs of electricity at t
ω_r	:	Water value of reservoir r
δ_g^r	:	Topology of the valley (+/- 1 if g is connected to r, 0 otherwise)

1.3 – Simulation of the current process (DPR: Deterministic with Periodic Revisions)

10 000 simulations

While nb_opti $\neq 7$

mean inflows → **DETERMINISTIC OPTIMIZATION**

$$\begin{Bmatrix} T_{1,1} & T_{1,2} & \cdots & T_{1,12} & T_{1,13} & \cdots & \cdots & \cdots & \cdots & T_{1,84} \\ T_{2,1} & T_{2,2} & \cdots & T_{2,12} & T_{2,13} & \cdots & \cdots & \cdots & \cdots & T_{2,84} \\ \cdots & & \cdots & & \cdots & & \cdots & & & \cdots \\ T_{6,1} & T_{6,2} & \cdots & T_{6,12} & T_{6,13} & \cdots & \cdots & \cdots & \cdots & T_{6,84} \end{Bmatrix}$$

Application + Corrections

1 scenario of « realized » inflows

If $V_r^t > V_{max}$:

– *corrected commands if possible (bounds)*

– *If not enough, the overflow is moved in the next reservoir*

If $V_r^t < V_{min}$: *turbines are stopped until the violations are nil.*

$$\begin{matrix} T_{1,1}^* & T_{1,2}^* & \cdots & T_{1,12}^* \\ T_{2,1}^* & T_{2,2}^* & \cdots & T_{2,12}^* \\ \cdots & & \cdots & \\ T_{6,1}^* & T_{6,2}^* & \cdots & T_{6,12}^* \end{matrix}$$

1.3 – Results : DPR

Modes :

DPR : Deterministic with Periodic Revisions
FD : Fixed Deterministic policy (1st iteration of DPR)
PI : Perfect Information (10 000 optimizations)

Mode	PI	FD	DPR
Mean costs [euros]	-1 092 493	-1 092 579	-1 092 679
Mean viol. V_{max} [m^3/scén.]	0	76,2	57,2
Mean viol. V_{min} [m^3/scén.]	0	55,5	7,4

Conclusions :

- Constraints violations for FD are too much important !!
- DPR reduces constraints violations but not enough for an application
- Only the PI cost is representative

$\Rightarrow$ *Robust approach*

2

Robust optimization : key points

2.1 – Robust optimization : key points

1. We only consider the set of the most probable events Θ (called ***uncertainty set***).

   $\rightarrow$ *Extreme values are not considered. This set must be chosen ...*

2. We don't need a probabilistic distribution of events

3. The obtained strategy is always feasible for each realization of the events in the uncertainty set.

For a detailed presentation of the method, see the articles and the talks of R. Ben-Tal, A. Nemirovski, D. Bertsimas,…

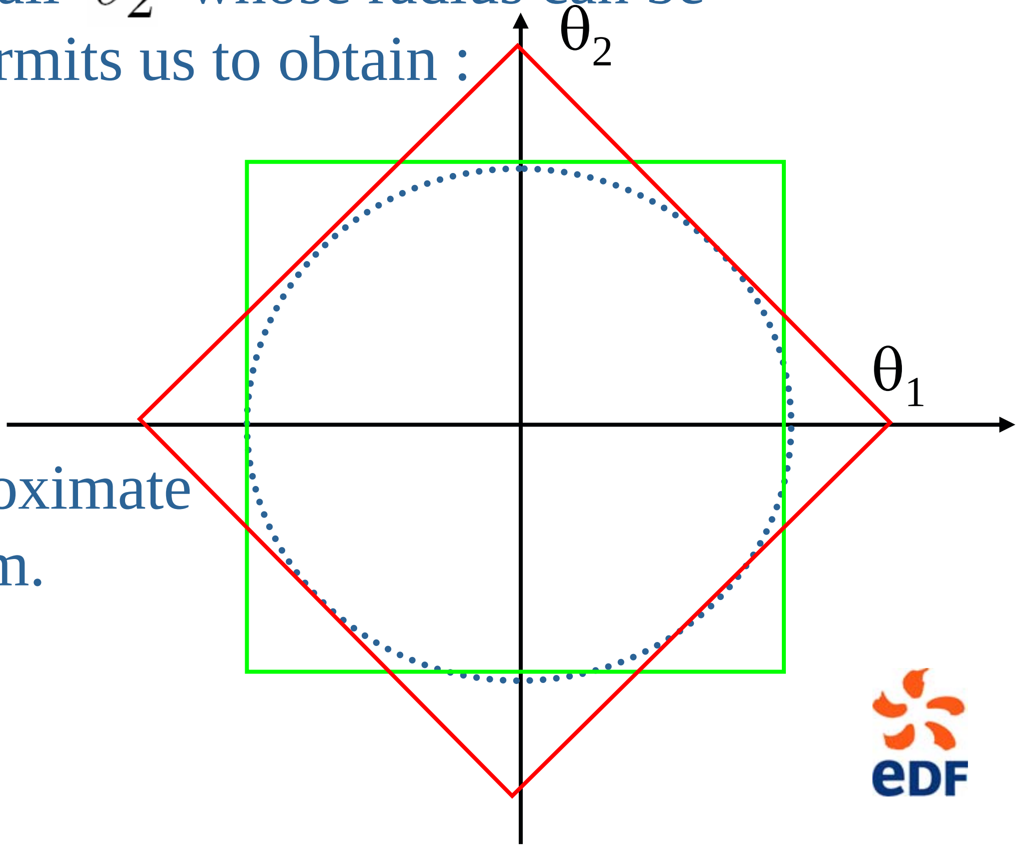
2.2 – Robust optimization : uncertainty set

In our study, uncertainties are:

- independent
- centered and normalized.

The chosen uncertainty set is a ball ℓ_2 whose radius can be adjusted with a Khi^2 law and permits us to obtain :

$$\mathrm{IP}(\ \theta \in \Theta\ )\ \leq\ \alpha$$



To stay in a linear case. we approximate the ball with a $\sqrt{n}\ell_1 \cap \ell_\infty$ norm.

2.3 – Robust optimization : application on a general constraint

We want to obtain the robust counterpart of the stochastic constraint :

$$a_0(x) + \sum_{i=1}^{n} \theta_i a_i(x) \leq 0, \quad \forall \theta \in \Theta = \{\theta : \sum_{i=1}^{n} |\theta_i| \leq k_1, \max |\theta_i| \leq k_2\}.$$

$$k_1 = \sqrt{n} k_2$$

PRIMAL

$$\max_{\theta^+,\theta^-} \quad \sum_{i=1}^{n} (\theta_i^+ a_i(x) - \theta_i^- a_i(x))$$

$$\sum_{i=1}^{n} (\theta_i^+ + \theta_i^-) \leq k_1 \quad \longleftarrow \quad v$$

$$\forall i = 1, \ldots, n : \quad \theta_i = \theta_i^+ - \theta_i^-$$

$$u_i^+ \longrightarrow \quad 0 \leq \theta_i^+ \leq k_2$$

$$u_i^- \longrightarrow \quad 0 \leq \theta_i^- \leq k_2.$$

DUAL

$$\min_{v,u^+,u^-} \quad k_1 v + k_2 (\sum_{i=1}^{n} u_i^+ + \sum_{i=1}^{n} u_i^-)$$

$$\forall i = 1, \ldots, n : \quad v + u_i^+ \geq a_i(x)$$

$$v + u_i^- \geq -a_i(x)$$

$$v \geq 0, \; u_i^+ \geq 0, \; u_i^- \geq 0.$$

2.3 – Robust optimization : application on a general constraint

The robust counterpart of the constraint

$$a_0(x) + \sum_{i=1}^{n} \theta_i a_i(x) \leq 0, \quad \forall \theta \in \Theta,$$

is :

$$\begin{aligned} & a_0(x) + k_1 v + k_2 (\sum_{i=1}^{n} u_i^+ + \sum_{i=1}^{n} u_i^-) \leq 0 \\ \forall i = 1, \ldots, n : \quad & v + u_i^+ \geq a_i(x) \\ & v + u_i^- \geq -a_i(x) \\ & v \geq 0, \; u_i^+ \geq 0, \; u_i^- \geq 0. \end{aligned}$$

3

Robust with Periodic Revisions approach (RPR)

3.1 – Robust Optimization – RPR Approach (Robust with Periodic Revisions)

10 000 simulations

While nb_opti ≠ 7

forecasted inflows → **DETERMINISTIC OPTIMIZATION**

$$\begin{Bmatrix} T_{1,1} & T_{1,2} & \cdots & T_{1,12} & T_{1,13} & \cdots & \cdots & \cdots & \cdots & T_{1,84} \\ T_{2,1} & T_{2,2} & \cdots & T_{2,12} & T_{2,13} & \cdots & \cdots & \cdots & \cdots & T_{2,84} \\ \cdots & & \cdots & & \cdots & & \cdots & & & \cdots \\ T_{6,1} & T_{6,2} & \cdots & T_{6,12} & T_{6,13} & \cdots & \cdots & \cdots & \cdots & T_{6,84} \end{Bmatrix}$$

Application + Corrections

1 scenario of « realized » inflows

If $V_r^t > V_{max}$:

– *corrected commands if possible (bounds)*

– *If not enough, the overflow is moved in the next reservoir*

$$\rightarrow \begin{matrix} T_{1,1}^* & T_{1,2}^* & \cdots & T_{1,12}^* \\ T_{2,1}^* & T_{2,2}^* & \cdots & T_{2,12}^* \\ \cdots & & \cdots & \\ T_{6,1}^* & T_{6,2}^* & \cdots & T_{6,12}^* \end{matrix}$$

If $V_r^t < V_{min}$: *turbines are stopped until violations are nil.*

3.1 – Robust Optimization – RPR Approach (Robust with Periodic Revisions)

10 000 simulations

While nb_opti ≠ 7

- Only the first day is robust
- Average view of the future

forecasted inflows → **ROBUST OPTIMIZATION**

$$\begin{Bmatrix} T_{1,1} & T_{1,2} & \cdots & T_{1,12} & T_{1,13} & \cdots & \cdots & \cdots & \cdots & T_{1,84} \\ T_{2,1} & T_{2,2} & \cdots & T_{2,12} & T_{2,13} & \cdots & \cdots & \cdots & \cdots & T_{2,84} \\ \cdots & & \cdots & & \cdots & & \cdots & & & \cdots \\ T_{6,1} & T_{6,2} & \cdots & T_{6,12} & T_{6,13} & \cdots & \cdots & \cdots & \cdots & T_{6,84} \end{Bmatrix}$$

Application + Corrections

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If $V_r^t > V_{max}$:

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$$\rightarrow \begin{matrix} T_{1,1}^* & T_{1,2}^* & \cdots & T_{1,12}^* \\ T_{2,1}^* & T_{2,2}^* & \cdots & T_{2,12}^* \\ \cdots & & \cdots & \\ T_{6,1}^* & T_{6,2}^* & \cdots & T_{6,12}^* \end{matrix}$$

3.2 – Results DPR vs. RPR : Constraints violations

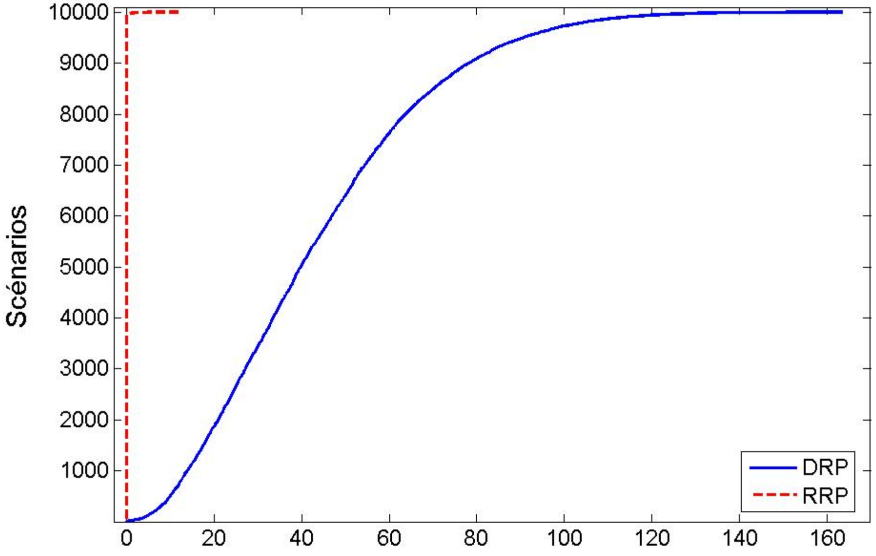
Technics	Mean overflow [m^3/scenario]	Worst scenario of cumulated overflow [m^3]	Max. Inst. overflow [m^3]	Mean number of time steps with overflow [time steps/scen.]	Number of scen. with overflow [scenarios]
DPR	57,2	163,5	37,82	8,86 (10,55%)	9994/10000
RPR	0,008	11,78	7,87	0,005 (0,01%)	41/10000
Δ(DPR,RPR)	-99,98%	-92,8%	-79,2%	-99,95%	-99,6%

Technics	Mean violation [m^3/scenario]	Worst scenario of cumulated violations [m^3]	Max. Inst. violations [m^3]	Mean numbers of time steps with violations [time steps/scen.]	Number of scen. with violations [scenarios]
DPR	7,41	26,13	9,08	4,87 (5,8%)	9968/10000
RPR	0,05	5,75	2,98	0,11 (0,13%)	991/10000
Δ(DPR,RPR)	-99,32%	-78%	-67,2%	-97,75%	-90,06%

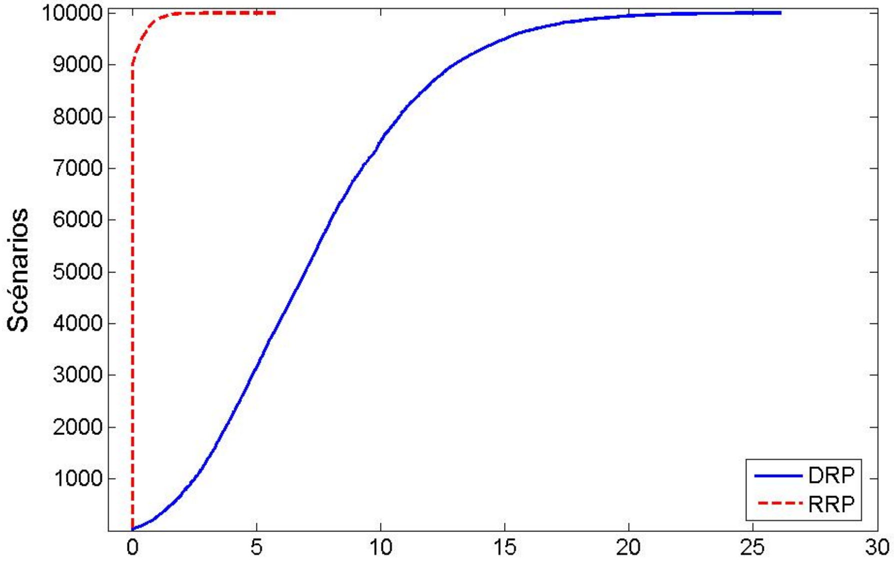
Conclusions :

- RPR removes mean overflow and divides maximal overflow by 5
- Frequency is drastically reduced: 0,4% instead of 99,94%
- Same conclusions for V_{min} and V_{max}

3.2 – Results DPR vs. RPR : Constraints violations



Overflows
[m³]



V_{min} Violations
[m³]

3.3 – Results DPR vs. RPR : Corrections and costs

Cumulated Corrections $[m^3]$	DPR	RPR
Mean	$0,13$	8.10^{-5}
Maximal	2	6.10^{-1}

Conclusion : we can apply RPR commands without corrections

Costs$[Euros]$	PI	RPR	Δ(PI,RPR)
Mean	-1 092 493	-1 086 250	+ 0,57%
Min	-1 104 743	-1 098 454	+0,57%
Max	-1 079 167	-1 072 146	+ 0,65%

Conclusion : the difference of costs between RPR and PI is only 0,57%.

eDF

3.4 – Results DPR vs. RPR : Sizes !!

Type	RPR	Deterministic model
Number of variables	1 513	505
Number of constraints	2 450	1 514

3 reservoirs, 6 groups, 84 time steps

Type	RPR	Deterministic model
Number of variables	13 681	8 641
Number of constraints	26 282	21 602

15 reservoirs, 60 groups, 144 time steps

3.5 – RPR : Conclusions !!

- Violations become almost nil (V_{max} and V_{min})
- The increase of mean cost is lower than 1%
- Commands don't need heuristic corrections
- RPR model is as tractable as a DPR one

4

Robust with Linear Adjustments approach (RLA)

4.1 – Robust Optimization – RLA approach (Robust with Linear Adjustments)

Now, we have to decide for a 7 days production planning with only one optimization.

The RPR model is adaptable by definition.

→ We introduce adaptations in the model with ***Linear Decision Rules (LDR)*** like :

$$\tilde{F}_g^t(\theta) = \sum_j \sum_r x_r^j \theta_r^j.$$

Bound constraints on commands have to be robust now…

4.1 – Robust Optimization – RLA approach (Robust with Linear Adjustments)

10 000 simulations

- Only the first day is robust
- Average view of the future

While nb_opti $\neq 7$

forecasted inflows → **ROBUST OPTIMIZATION**

$$\left\{\begin{array}{cccccccccc} T_{1,1} & T_{1,2} & \cdots & T_{1,12} & T_{1,13} & \cdots & \cdots & \cdots & \cdots & T_{1,84} \\ T_{2,1} & T_{2,2} & \cdots & T_{2,12} & T_{2,13} & \cdots & \cdots & \cdots & \cdots & T_{2,84} \\ \cdots & & \cdots & & \cdots & & \cdots & & & \cdots \\ T_{6,1} & T_{6,2} & \cdots & T_{6,12} & T_{6,13} & \cdots & \cdots & \cdots & \cdots & T_{6,84} \end{array}\right\}$$

Application + Corrections

1 scenario of « realized » inflows

If $V_r^t > V_{max}$:

– *corrected commands if possible (bounds)*

– *If not enough, the overflow is moved in the next reservoir* →

$$\begin{array}{cccc} T_{1,1}^* & T_{1,2}^* & \cdots & T_{1,12}^* \\ T_{2,1}^* & T_{2,2}^* & \cdots & T_{2,12}^* \\ \cdots & & \cdots & \\ T_{6,1}^* & T_{6,2}^* & \cdots & T_{6,12}^* \end{array}$$

If $V_r^t < V_{min}$: *turbines are stopped until violations are nil.*

4.1 – Robust Optimization – RLA approach (Robust with Linear Adjustments)

10 000 simulations

~~*While nb_opti ≠ 7*~~

- Only the first day is robust
- **Robust view of the future**

forecasted inflows → **ROBUST OPTIMIZATION**

$$\begin{Bmatrix} T_{1,1} & T_{1,2} & \dots & T_{1,12} & T_{1,13} & \dots & \dots & \dots & \dots & T_{1,84} \\ T_{2,1} & T_{2,2} & \dots & T_{2,12} & T_{2,13} & \dots & \dots & \dots & \dots & T_{2,84} \\ \dots & & \dots & & \dots & & \dots & & & \dots \\ T_{6,1} & T_{6,2} & \dots & T_{6,12} & T_{6,13} & \dots & \dots & \dots & \dots & T_{6,84} \end{Bmatrix}$$

Application + Corrections

+ Building LDR

1 scenario of « realized » inflows

If $V_r^t > V_{max}$:

– corrected commands if possible (bounds)

– If not enough, the overflow is moved in the next reservoir →

$$\begin{matrix} T_{1,1}^* & T_{1,2}^* & \dots & T_{1,84}^* \\ T_{2,1}^* & T_{2,2}^* & \dots & T_{2,84}^* \\ \dots & & \dots & \\ T_{6,1}^* & T_{6,2}^* & \dots & T_{6,84}^* \end{matrix}$$

If $V_r^t < V_{min}$: *turbines are stopped until violations are nil.*

4.2 – Results RPR vs. RLA : constraints violations

Technics	Mean overflow [m^3/scenario]	Worst scenario of cumulated overflow [m^3]	Max. Inst. overflow [m^3]	Mean numbers of time steps with overflow [time steps/scen.]	Number of scen. with overflow [scenarios]
DPR	*57,2*	*163,5*	*37,82*	*8,86 (10,55%)*	*9994/10000*
RPR	*0,008*	*11,78*	*7,87*	*0,005 (0,01%)*	*41/10000*
RLA	1,52	65,74	29,77	0,285 (0,34%)	1829/10000

Technics	Mean violations [m^3/scenario]	Worst scenario of cumulated violations [m^3]	Max. Inst. violations [m^3]	Mean numbers of time steps with violations [time steps/scen.]	Number of scen. with violations [scenarios]
DPR	*7,41*	*26,13*	*9,08*	*4,87 (5,8%)*	*9968/10000*
RPR	*0,05*	*5,75*	*2,98*	*0,11 (0,13%)*	*991/10000*
RLA	0,09	17,22	9,66	0,06 (0,07%)	566/10000

Conclusions :

- RLA divides mean overflow by 30 and the frequency by 5
- RPR < RLA << DPR

4.3 – Results RPR vs. RLA : Corrections and costs

Cumulated Corrections $[m^3]$	**RLA**
Mean	0, 9
Maximal	53, 66

Conclusion : we can apply RLA commands without corrections except for exceptional scenarios

Costs $[Euros]$	***PI***	***RPR***	**RLA**
Mean	*-1 092 493*	*-1 086 250*	-1 079 719
Min	*-1 104 743*	*-1 098 454*	-1 088 679
Max	*-1 079 167*	*-1 072 146*	-1 069 779

Conclusion : RLA cost is 0,6% greater than RPR cost and 1,2% than PI

4.4 – Results RPR vs. RLA : Sizes !!

Type	RLA	RPR	Deterministic Model
Number of variables	18 812	1 513	505
Number of constraints	18 092	2 450	1 514

3 reservoirs, 6 groups, 84 time steps

Type	RLA	RPR	Deterministic model
Number of variables	566 732	13 681	8 641
Number of constraints	564 572	26 282	21 602

15 reservoirs, 60 groups, 144 time steps

Remark : problem for the RLA with Periodic Revisions…

4.5 – RLA : Conclusions !!

- Better respect of constraints than DPR but lower than RPR
- The increase of mean cost is just greater than 1% of Perfect Info
- Commands don’t need heuristic corrections in general case
- The size of the model explodes…

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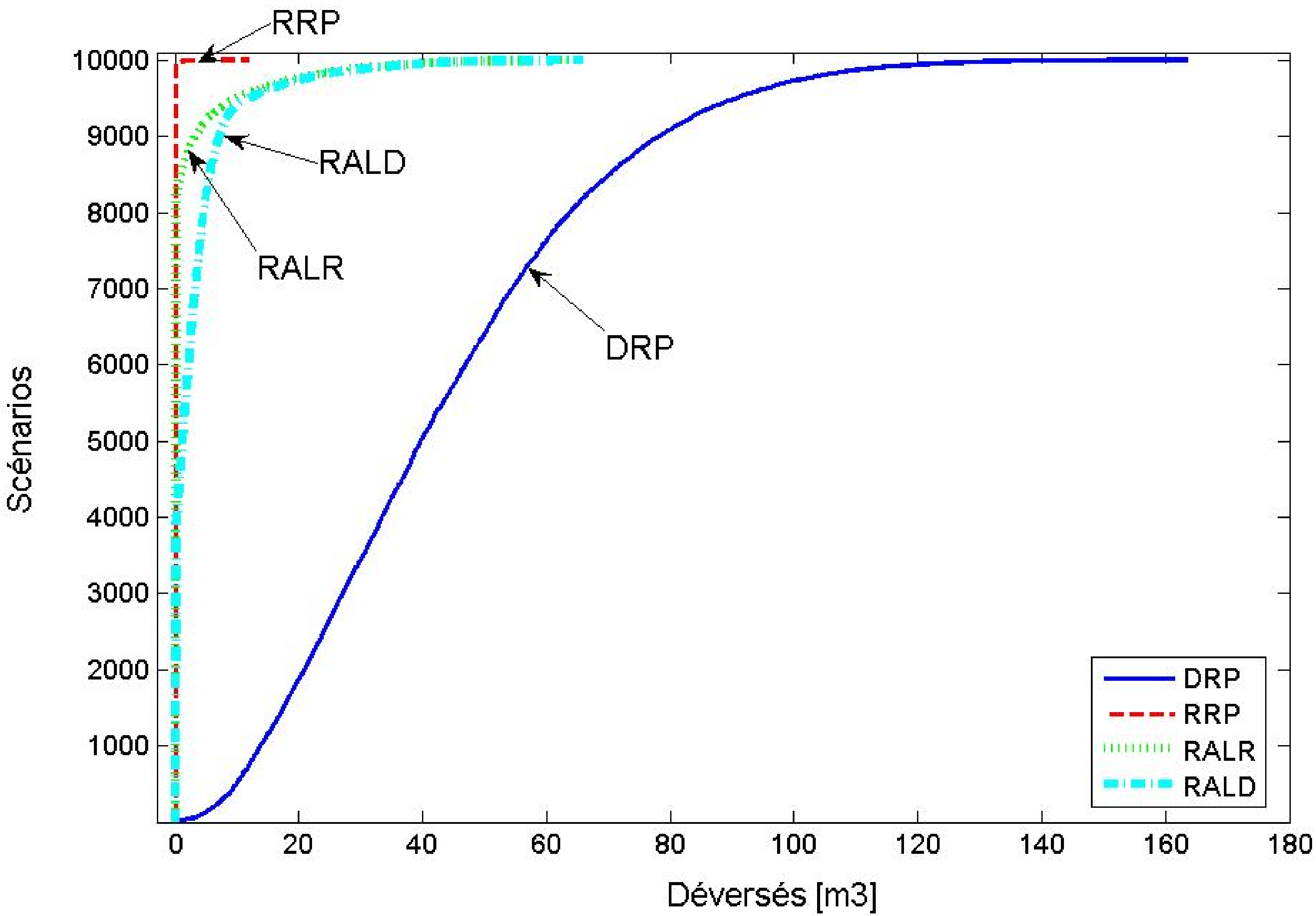
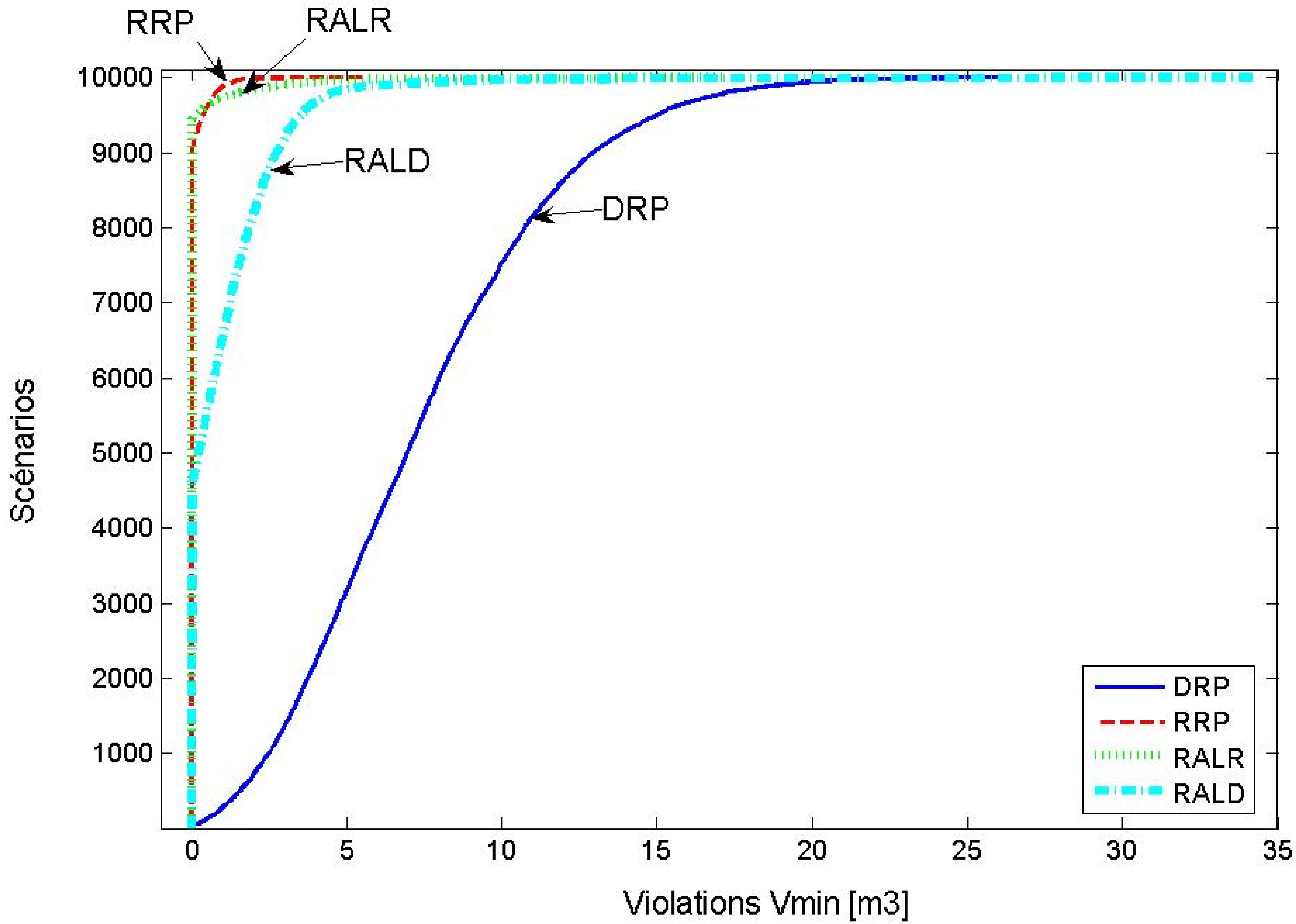
Thanks!

Results DPR vs. RPR vs. RLA : Constraints violations



Results DPR vs. RPR vs. RLA : Constraints violations



Results DPR vs. RPR vs. RLA : costs