A multi-class framework for a pedestrian cell transmission model accounting for population heterogeneity

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Motivation

- PedFlux: a collaborative research project between the Swiss Federal Railways (SBB-CFF-FFS) and EPFL's transportation center.
- Objective: to analyze, model and optimize pedestrian flows in train stations.
- Dataset: a state-of-the-art pedestrian measurement system composed of visual, depth and infrared sensors that record pedestrian trajectories in the Lausanne train station.

Motivation

- Hänseler, F. S., Bierlaire M., Farooq, B. and Mühlematter T. (2013). *An aggregate dynamic model for multi-directional pedestrian flows.* Transportation Research Part B (in review.)
- Nikolic, M., Farooq, B., and Bierlaire, M. (2013). *Exploratory analysis of pedestrian flow characteristics in mobility hubs using trajectory data.* Proceedings of the Swiss Transportation Research Conference (STRC) 24-26 April, 2013.

Objectives

- Extend PedCTM to account for population heterogeneity through the development and implementation of a multi-class framework.

Multiclass Framework

- Let D be the set of pedestrian classes
- For $d \in D$ we have two defining characteristics:
- v_f^d is the free flow speed, and
- γ^d is a vector of all other class specific characteristics.

Space Mean Speed

- Let v^d be the space mean speed of pedestrian class d.
- Let k be the density of pedestrians in pedestrians per unit space and k^d be the density of only class d pedestrians.
- We have that: $v^d = v^d(k) = v^d\left(\sum_{d \in D} k^d\right)$

Specific Flow

- Let q be the specific flow of pedestrians and q^d the specific flow of class d pedestrians only.
- We calculate q^d as: $q^d = k^d v^d(k) = k^d v^d \left(\sum_{d \in D} k^d \right)$.

Discretization of time

- Time is discretized into a set of T intervals.
- The intervals $\tau \in \{1, 2, \ldots, |T|\}$ are of uniform length Δt.
- The value of Δt is left unspecified for the moment but is important.

Spatial Discretization

- Space is discretized into square cells ξ of uniform length ΔL.
- Each cell has an area A_ξ
- The cell network is represented by a directed graph $G = (\mathcal{X}, \mathcal{Y})$ where $\mathcal{X}$ is the set of cells ξ and $\mathcal{Y}$ is the set of links connecting cells one to another.
- The set of neighbors of a cell ξ is denoted N_ξ.

Route Structure

- Let R be the set of routes.
- A route $\rho \in R$ is defined as a sequence of areas without loops, $\rho = (\rho_0, \rho_1, \ldots, \rho_r)$
- An area ρ_i is a non-empty and connected subgraph of G.
- The first and last areas, ρ_0 and ρ_r, are boundary "areas".

Pedestrian aggregation

- A pedestrian group ℓ is defined by a route $\rho_\ell \in R$, a departure time interval $\tau_\ell \in T$ and a pedestrian class $d_\ell \in D$.
- The size of group ℓ is denoted $X_{\rho_\ell,\tau_\ell,d_\ell}$.
- The set of all pedestrian groups is denoted by $\mathcal{L} \subset R \times T \times D$

Pedestrian Quantities

- Let $M^{\ell}_{\xi,\tau}$ be the number of pedestrians in cell ξ at time interval τ belonging to group ℓ, given by $M^{\ell}_{\xi,\tau} = k^{\ell}_{\xi,\tau} A_{\xi}$.
- The total number of pedestrians in cell ξ at time interval τ is thus: $M_{\xi,\tau} = \sum_{\ell \in \mathcal{L}} M^{\ell}_{\xi,\tau}$
- The total number of class d pedestrians in a cell ξ at time interval τ is given by $M^{d}_{\xi,\tau}$ and is calculated as: $M^{d}_{\xi,\tau} = \sum_{\ell \in \mathcal{L}} M^{\ell}_{\xi,\tau} \mathbb{1}_{d=d_{\ell}}$

Choice of Δt and the update cycle

- Define $\Delta t^d = \Delta L / v_f^d$ as the update time interval associated to each pedestrian of class d.
- Now take $\Delta t = gcd(\{\Delta t^d\}_{d \in D})$.
- We thus have that for all $d \in D$, $\Delta t^d = \alpha^d \Delta t$ where $\alpha^d \in \mathbb{Z}^+$.
- For a group $\ell \in \mathcal{L}$, we have that the quantities of pedestrians belonging to ℓ throughout the network G are only updated every α^{d_ℓ} time steps after τ_ℓ.
- Further assumption: all groups with class $d \in D$ have departure time intervals that correspond to a constant multiple of $\alpha^d \Delta t$.

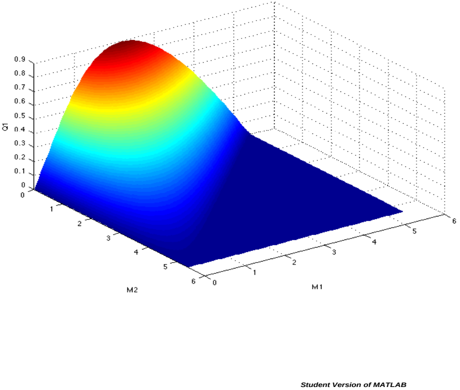
Cumulative Flow and Capacities

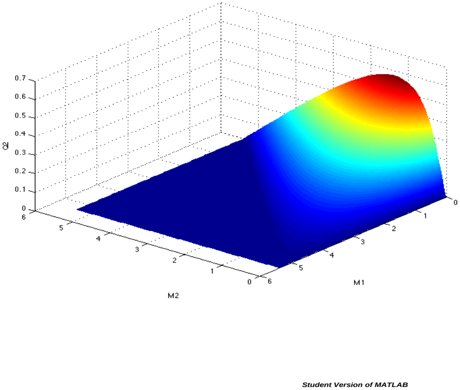
- Let $Q^d_{\xi,\tau}$ be the cumulative hydrodynamic flow of pedestrian class d through an arbitrary link of cell ξ at time interval τ.
- We calculate this quantity as:

$$Q^d_{\xi,\tau} = \int_0^{\Delta t^d} \int_0^{\Delta L} q^d dtdL = M^d_{\xi,\tau} \frac{v^d(M_{\xi,\tau})}{v^d_f}$$

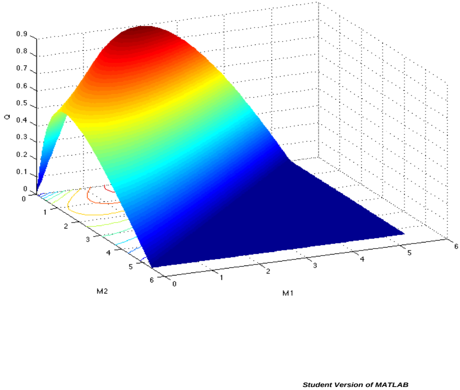
- Generalize the flow capacity and critical density of the homogenous model to the multi class framework. Consider the inflow capacity $\hat{Q}_{\xi,\tau}$ and outflow capacity $\tilde{Q}_{\xi,\tau}$ of the homogenous model.

Class Specific Cumulative Hydrodynamic Flow





Total Cumulative Hydrodynamic Flow



En-route Path Choice

- Define Θ_ξ^ρ to be the set of all neighboring cells to ξ which are part of route ρ.
- Define $\delta_{\xi \to \nu}^{\rho,d}$ to be the probability of a single agent of class d on route ρ in cell ξ to go to cell $\nu \in \Theta_\xi^\rho$ during time interval τ.
- Properties of $\delta_{\xi \to \nu}^{\rho,d}$: positive, $\sum_{\nu \in N_\xi} \delta_{\xi \to \nu}^{\rho,d} = 1$, values determined by relative strength of cell potentials, $\mathcal{P}_{\nu,\tau}^d$.

Flow Calculations

- Define the group-specific sending capacity as $S^{\ell}_{\xi\to\nu,\tau}$, of cell ξ to $\nu \in \Theta^{\rho_\ell}_{\xi}$ during time interval τ as the maximum number of associated pedestrians that flow through the link during τ and calculate it as,

$$S^{\ell}_{\xi\to\nu,\tau} = \delta^{\rho_\ell,d_\ell}_{\xi\to\nu,\tau}\min(M^{\ell}_{\xi,\tau}, M^{\ell}_{\xi,\tau}\tilde{Q}^{d}_{\xi,\tau}/M^{d}_{\xi,\tau}),$$

- We define the class-specific receiving capacity for an arbitrary cell ξ similarly and calculate it as

$$R^{d}_{\xi,\tau} = \min(N_\xi - M_{\xi,\tau}, \hat{Q}^{d}_{\xi,\tau}),$$

Flow Calculations

- The actual flow of group ℓ from ξ to $\nu \in \Theta_{\xi}^{\rho_\ell}$ during time interval τ is calculated as

$$\mathcal{Y}^{\ell}_{\xi\to\nu,\tau} = \begin{cases} S^{\ell}_{\xi\to\nu,\tau} & \textit{if } \sum_{\nu'\in N_\nu}\sum_{\ell'\in\mathcal{L}} S^{\ell'}_{\nu'\to\nu,\tau}\mathbb{1}_{d_{\ell'}=d_\ell} \le R^{d_\ell}_{\nu,\tau} \\ \zeta^{\ell}_{\xi\to\nu,\tau} R^{d_\ell}_{\nu,\tau} & \text{otherwise,} \end{cases}$$

- where $\zeta^{\ell}_{\xi\to\nu,\tau}$ controls the portion of pedestrians of group ℓ that "flow" if the receiving capacity of the target cell is reached.

$$\zeta^{\ell}_{\xi\to\nu,\tau} = \frac{S^{\ell}_{\xi\to\nu,\tau}}{\sum_{\nu'\in N_\nu}\sum_{\ell'\in\mathcal{L}} S^{\ell'}_{\nu'\to\nu,\tau}\mathbb{1}_{d_{\ell'}=d_\ell}}$$

Update Scheme

- The final update for the number of pedestrians of group ℓ in cell ξ at time $\tau + 1$ is

$$M^{\ell}_{\xi,\tau+1} = M^{\ell}_{\xi,\tau} + \sum_{\nu \in N_{\xi}} (\mathcal{Y}^{\ell}_{\nu\to\xi,\tau} - \mathcal{Y}^{\ell}_{\xi\to\nu,\tau}) + \mathcal{W}^{\ell}_{\xi,\tau},$$

  where $\mathcal{W}^{\ell}_{\xi,\tau}$ controls the inflow and outflow of pedestrians at boundary cells.
- The update cycle we employ is such that the equation above is used to update the quantity of a group throughout the network only when the current time step, τ, is divisible by $\alpha^{d_{\ell}}$.

Next Steps

- Extensive literature review of multi-class traffic models.
- Implementation of model