Modelling Growth Principles of Metropolitan Public Transport Networks

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Abstract

The development of metropolitan public transport networks often involve choosing between investing in extending radial lines and constructing ring connections. While the former enlarges network coverage the latter enhances network connectivity and reduces the need to perform detours. Moreover, investments might be better directed at increasing the capacity of already existing infrastructure. In this study we address the following question: how do transport networks in metropolitan areas evolve over time and how can we effectively model this growth as function of demand and cost function? The goal of this study is to determine the influence various demand distributions and operational cost functions have on the evolution and topology of a monocentric urban transport network using topological indicators. To this end, an investment model network analysis framework is developed and applied to an idealised radio-centric network considered as a simplification of metropolitan urban public transport networks. The main contributions pertain to the examination of prevailing network structures for alternative modal supply properties and the relation between the underlying growth process and the evolution of key topological characteristics. Results provide insights into the circumstances under which various network structures prevail for different rapid transit modes and demand patterns.

Keywords: Public Transport; Network Structure; Network Topology; Network Evolution; Rapid Transit.

Introduction

The evolution of transport networks is determined by an interaction between changes in technology, society and the economy. Urban public transport networks worldwide are undergoing substantial expansions. Development decisions often involve choosing between investing in extending radial lines and by which enlarging network coverage to constructing ring connections to enhance network connectivity and reduce the need to perform detours. The following question thus arises: how do transport networks in metropolitan areas evolve over time and how can we effectively model this growth as function of demand and cost function?

The design of radio-centric networks has been subject of several studies. For example, Vaughan (1986) and Chen et al. (2015) who found the optimal spacing of radial and ring lines. Tirachini et al. (2010) developed a framework to compare alternatives for a radio-centric urban network with radial lines from the borders to a central business district attempting to minimise total costs. Their results provide insight into the characteristics of each of the modes competing in the radial network structure. Badia et al. (2014) study the formation of an ideal fully radial network based on the modal technology used and assuming a uniform demand. Those results provide insight into the mechanics and relationships between cost functions and physical characteristics of a mode (such as stop spacing) on the network topology. Saidi et al. (2016, 2017) developed a model that takes various cost effects into account and computes generalised transit passenger cost for a given network. However the model relies on a detailed and static evaluation of the network. The model can thus only be used to compare a limited number of scenarios on the generalised passenger costs and does not provide any insight into network growth patterns.

The modelling of transportation network growth and its history has been thoroughly reviewed by Xie and Levinson (2009). In their review they show progress within the field of modelling and analysing growth of transportation networks In the years after this review by Xie and Levinson (2009) there have been new studies related to principles behind network growth. Louf et al. (2013) studied the emergence of hierarchical structures in cost driven growth models for spatial models.

These studies are only completed for tree shaped random node networks, and do not provide insight in networks where either hierarchy is not a factor or where the points are not randomly distributed over the space, like the metropolitan networks studied in this research. The recent work of Saidi (2017) mentions the severe lack of models for ring-radial structures. The effects of transport modality and demand functions has only been researched in comparative scenarios or in research that assumes a fixed network not for growing (evolving) networks.

The process in which networks evolve over time is an important aspect in the growing interest in network science. Notwithstanding, according to Dupuy (2013), graph theory-based studies often result in a static representation of the network, hindering the analysis of network evolution. Furthermore Ducruet and Beauguitte (2014) concluded that research concerning the evolution and dynamics of networks using network science concepts and methods has remained surprisingly unexplored as most studies adopt a static approach.

The goal of this study is to determine the influence various demand distributions and operational cost functions have on the evolution and topology of a monocentric urban transport network using network indicators. To this end, an investment model network analysis framework is developed and applied to an idealised radio-centric network considered as a simplification of metropolitan urban public transport networks that exist in real life. The main contributions pertain to the examination of prevailing network structures for alternative modal supply properties and the relation between the underlying growth process and the evolution of key topological characteristics.

Methodology

Figure 1 depicts the modelling process of network growth decisions. The process starts with an initial network state and an origin-destination matrix which describes the latent travel demand. The core of the model consists of the following successive modules: (i) Input Parameters; (ii) Initialisation of the model by performing Network Generation, Trip Generation and Trip Distribution; (iii) Investment Candidate Generation; (iv) Investment Candidate Evaluation, and; (v) Scoring and Building. These steps generate a choice-set of investment alternatives, evaluate the consequences of each alternative investment and then execute the selected investment. These modules are detailed in the following sub-sections. Network growth terminates when none of the alternative investments yields a positive evaluation in the Cost Benefit Analysis decision as shown in the figure below.

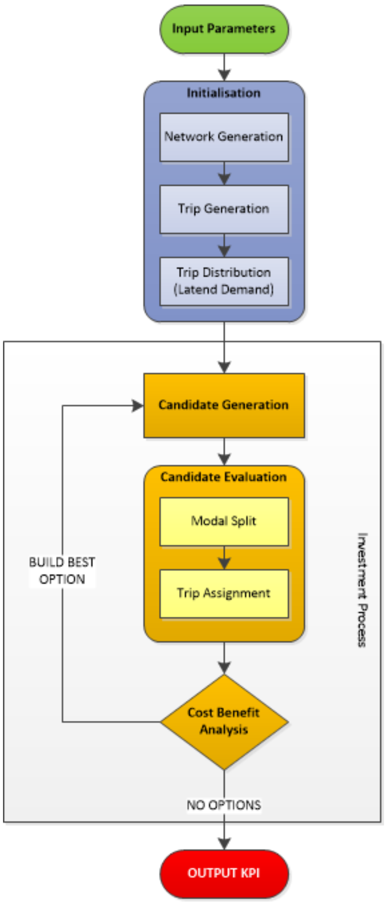

Figure 1: Iterative network growth model workflow

Network and Trip Generation

Let us consider a metropolitan urban area which can be abstracted in the form of an idealized radio-polar form. The area can then be represented using an undirected polar-grid graph $G(N, E)$ where the set of nodes, N, denotes population centres which serve as potential locations for (interchange) stations and the set of links, $E \subseteq N \times N$, representing direct connections between stations. Networks are built within a pre-defined limited area containing a finite number of nodes. The area boundaries are based on the maximum radius r_{max} from the centre of the idealized monocentric agglomeration to its outer edges. The symmetric polar-grid area is defined using the abovementioned set of geometrical parameters. The location of the nodes $i \in N$ is defined by the distance and angle $r_i = [0, r_{max}]$ $\forall n$ and $\varphi_i = [0,2\pi]$ $\forall n$, respectively. Furthermore, each of the nodes is assigned with a travel demand x_i that is generated from and attracted to this node.

Trip Distribution

Trip distribution is performed using a doubly constraint gravity model. We approximate the travel cost by the network travel distance when using the polar distance. The generated Origin-Destination matrix constitutes thus the number of trips performed in a fully connected graph. Hence, it reflects thus the latent demand or in other words the maximum potential travel demand that can be realised in case all nodes are connected and all possible links have been constructed.

Candidate Generation

Each iteration of the Investment Process (see Figure 1) involves the generation of a set of candidate investments from which in the evaluation at most a single investment is selected. Options for investments are either expanding the network by adding a new connection or an enhancement of the network by means of increasing the capacity of an already existing link. New links must result with a single connected planar graph. Barabasi and Albert (1999) showed that the benefits of connecting an additional node to an existing network given equal conditions is preferred over connecting any two other nodes that are detached from the original network. Arguably, providing an additional node access to all other connected nodes is likely to attract a larger share of the latent demand than connecting a single OD pair. Given the polar grid assumed, only radial links (constant angle from origin) and ring links (constant radius from origin) are allowed.

Let E^q denote the investment candidates considered in iteration q. Element $e_{ij}^q \in E^q$ is one if the link has not been constructed yet and zero if the edge has been constructed. Therefore elements that are non-zero correspond to links that can be considered for investment. The matrix is determined by taking all potential connections and subtracting the current network adjacency matrix, i.e. $E^q = \hat{A} - A^q$ while ensuring that $e_{ij}^q = 0$ if $\sum_{i \in N} a_{ij}^q + \sum_{j \in N} a_{ij}^q = 0$. The latter constraint ensures that the network consists of a single connected component. In addition to these new connections, a fixed capacity increase is considered for each existing links. A maximum capacity per link can be set as an input parameter.

Candidate evaluation
In order to evaluate the impacts of a candidate investment using a Cost Benefit Analysis later on, the benefits of the candidates have to be first assessed. This is performed in two steps: (i) the new modal split is calculated to find out how many public transport trips will be performed given the new network state, and; (ii) an non-capacitated assignment is performed to assess network saturation and the associated travel costs. These two steps are detailed in the following sections.

Modal Split and Assignment
The latent demand resulting from the trip generation and distribution steps is to be divided into public transport and non-public transport demand matrices. The share of demand that will travel using public transport is calculated for each candidate network investment and is determined using a Binary Logit model for choosing between public transport and alternative modes. We assume that all users travel by the alternative modes (e.g. car, bike) if no public transport connection is available. The utilities for PT and Alternative mode are computed as a function of the impedance imposed by the travel time along the shortest path.

Cost Benefit Analysis
Each candidate investment is evaluated by weighting the costs against the benefits as assessed by a planning authority. Both the benefits and costs are discounted for using the investment time-horizon of 30 years. Each candidate investment in the set E^q is thus assessed using the following scoring function which translates both costs and benefits into monetary terms:

$$z(e) = \frac{\omega \cdot [f(A^{\hat{q}}) - f(A^q)]}{\sum_{mode \in M} \delta_e^{mode} \beta^{mode} c_e} \quad \forall e \in E^q \tag{1}$$

The nominator reflects the total benefits stemming from the investment corresponding to travel time savings and the denominator is the cost associated with the investment under consideration. The benefits are the product of changes in total travel costs and the value of time, ω. The network-wide travel impedance, f, for a given network state matrix, A^q, is calculated as follows:

$$f(A^q) = \sum_{i \in N} \sum_{j \in N \setminus i} \left[p_{ij}^{PT,A^q} \cdot x_{ij} \cdot c_{ij}^{PT,A^q} + (1 - p_{ij}^{PT,A^q}) \cdot x_{ij} \cdot c_{ij}^{ALT,\hat{A}} \right] \tag{2}$$

f comprises of the total travel impedance by public transport and by the alternative mode, each of which consists of the product of the respective share (p_{ij}^{PT} or $1 - p_{ij}^{PT}$) of the demand flow (x_{ij}) and travel impedance, c_{ij}, when travelling between these nodes using a certain mode for the respective network. $A^{\hat{q}}$ in Eq. 1 is the network state matrix prior to iteration q that is extended by the candidate investment on link e.

The costs are determined by the length of the link under consideration, c_e, and the cost per length unit for the respective mode, β^{mode}, representing both the investment in cost units per kilometre of infrastructure and the costs of offering a given capacity per hour as service costs. δ_e^{mode} is a dummy variable indicating whether the link investment considered is of a certain mode or not for each of the possible modes in set M.

Among the candidate solutions, the one attaining the highest score while ensuring that the benefits exceed the costs is selected in each iteration, $e_q^* = \max_{e \in E^q} z(e)$. If $z(e_q^*) \leq 1$, i.e. no candidate solution yields a non-zero net benefit, then the network development process terminates. The network evolution process is greedy since it considers the short-term benefits in relation to the current network state and is not designed to obtain the optimal network solution. Instead, it reflects the iterative decision making process that evolves over a long time span.

EXPERIMENTAL SET-UP

We test the growth of a monocentric urban public transport network for a series of combinations of public transport service specifications and population distributions. The growth patterns and network structure of BRT, LRT and Metro which often serve as the backbone of public transport systems are investigated. While all these modes operate using an exclusive right of way, their operations differ in terms of speed and capacity and there are noticeable differences in their cost functions.

Table 1 Summary of Modal specifications

ID	Mode	Operational Speed υ [km/hour]	Cost β [M€/km]	Capacity increase step κ	Maximum capacity κ^{max}
1	BRT	35	6	2000	20.000
2	LRT	45	20	3500	35.000
3	Metro	60	300	8000	80.000

We investigate three distributions: (i) Uniform; (ii) Linear decay, and; (iii) Exponential (power-law) decay. Scenarios are designed by considering all possible combinations of public transport modalities and population distribution.

RESULTS

A summary of key topological indicators of the network attained at the final growth iteration is provided in Table 2. Evidently, different combinations of population distribution and mode characteristics result in final network states with distinctly different properties. In particular, one can observe that given a certain mode, the indicators vary considerably for different demand distributions. A more peaked travel demand distribution results in a greater spatial disparity as the most lucrative investments are to be found in the core area while service provision at the periphery might not fulfil the cost-benefit criterion. This is reflected in the topology indicators for connectivity, i.e. beta and gamma. The more concentrated population distribution is the lower network connectivity becomes due to the fewer links constructed. The total system length and ringness indicator confirm that the system is less expansive for more peaked distributions.

Table 2 Summary of Final Network State Indicators

	Scenario								
	BRT			LRT			Metro		
Demand distribution	**Uniform**	**Linear Decay**	**Exp. Decay**	**Uniform**	**Linear Decay**	**Exp. Decay**	**Uniform**	**Linear Decay**	**Exp. Decay**
Connectivity β index	1.98	1.86	1.24	1.98	1.86	1.73	1.48	1.48	1.36
Connectivity γ index	1.00	0.94	0.63	1.00	0.94	0.88	0.75	0.75	0.69
Average node degree	3.96	3.71	2.47	3.96	3.71	3.46	2.97	2.97	2.72
Total network Length [km]	1611	1360	925	1611	1360	1140	888	794	668
Ringness Φ_{ring}	0.70	0.65	0.61	0.70	0.65	0.58	0.46	0.40	0.28
Total Travel Time [hr]	2699	2786	2929	2396	2475	2406	2170	2222	2181

Various mode characteristics also obtain a different set of topological indicators for a given population distribution. This can be explained when examining the underlying mechanisms driving the model: an investment in BRT offers a relatively low speed connection at a lower cost compared to LRT and Metro, yet its investment costs are low enough to warrant investment in the network. For LRT the relatively higher speed means that travel time savings are larger. This is partially because of a lower travel time between nodes due to increased speed, resulting in a higher modal share for public transport and thus assigning more of the latent demand to the network. Given the marginal increase in costs for LRT this means that for most scenarios almost all possible links are built since the cost-benefit ratio is consistently positive. For Metro the relatively high costs mean that not all of the potential system benefits for users can be obtained. This is because the benefits of adding a new connection are not always sufficient to cover the larger investment costs that metro require, consequently making the investment not worthwhile. This particularly can be observed in the Gamma connectivity index and Network length indicators that are systematically lower for Metro when comparing them to the respective demand distribution scenarios for LRT or BRT.

In order to visualise the evolution of network states under various scenarios, intermediate stages are plotted in Figure 2. Each graph displays the network state for a given public transport mode after 100, 250, 500, 750 and 1000 iterations as well as the final stage, for each population distribution scenario. These graphs therefore help to visualise the process of link addition or investing in capacity addition. Each link in the graph is coloured to show the travel volume in relation to the available capacity (high load is red, low loads in blue) effectively a heat map showing network saturation and potential capacity bottlenecks. In the case of BRT (above), the network quickly expands to the edges along the radials for the uniform case (radial elements are shorter and thus cheaper for same benefits) and then in a later stage some shortcuts and ring elements are constructed. The higher cost, capacity and speed for the LRT mode evidently results in a different network development process (middle). As in the BRT scenarios a more peaked population distribution strongly affects the network shape and the evolutionary path but due to the higher speeds and costs of LRT this effect is more distinct showing that the modal cost parameter also affect network evolution by limiting outer edges where the combination of lower population density and higher modal costs result in a more compact network. The Metro mode (below) is one of the few modes where capacity is not limited in the final states. Given the high costs per km and higher speeds it is clear that a different balance is held throughout the iterations as can be seen. Network development focusses on the core. Like previous scenarios the more peaked population distributions also result in a very core centric network with ring elements being constructed in the central area before stretching out towards the edges.

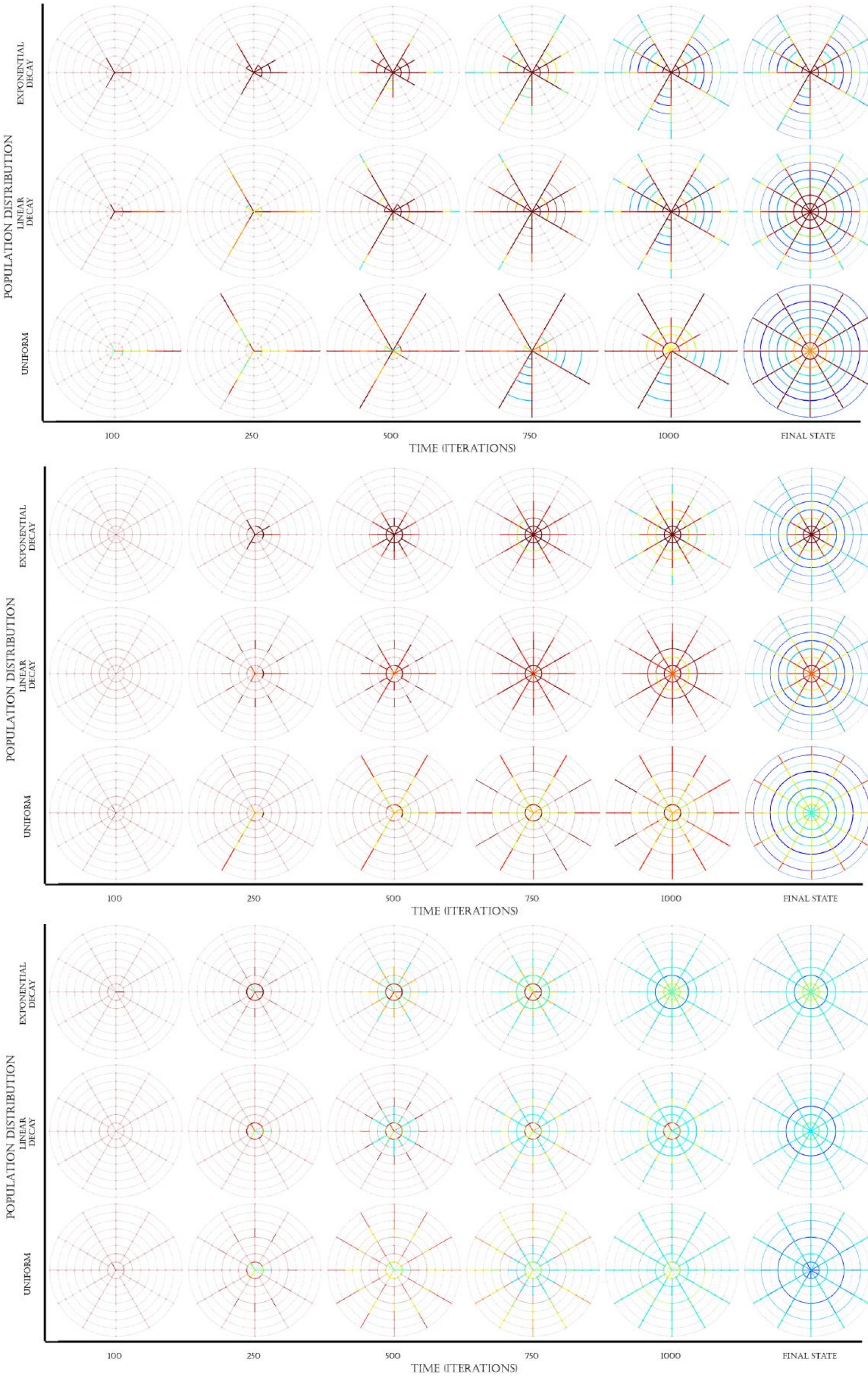

Figure 2: Network Evolution for Uniform, Linear and Exponential decaying population distributions for BRT (above), LRT (middle) and Metro (below)

Conclusion and Outlook

Results provide insight into the relationships between demand, costs and the network evolution of monocentric metropolitan networks. The results from experiments support the suggestion of a relationship between the population distribution and the final topological evolution of a network. The actual network metrics do not account for capacity and thus do not really show these effects as clearly in the evolution of a network as expected. The scenarios affected by a decaying population show limited connections in the outer periphery and a limitation of capacity for links that are constructed in these peripheral areas. The full paper includes a complete set of results in relation to network properties, utilization and their evolution. In addition, results from a sensitivity analysis for the impact of operational costs and modal speeds on the obtained network properties are discussed.

Future studies may relax some of the assumptions made in this study by introducing a capacitated assignment or a feedback loop from network state to trip generation to allow for induced demand. Further advancements may introduce the notion of service lines and model the evolution of their alignments or represent the investment decision as a budget constraint.

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