A Continuum approach for modeling signalized nodes in dynamic traffic assignment

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1- State of the art

In order to calculate flow propagation through a traffic network by simulation, first-order macroscopic dynamic network loading (DNL) models are widely being used nowadays. In DNL models, submodels for traffic behavior at links and nodes interact. Link models describe flow and congestion propagation through links, whereas node models bring consistency among flows between incoming and outgoing links and may impose additional constraints to these flows.

The first node models proposed in literature considered simple merges with multiple incoming (sending) links and one outgoing (receiving) link, or diverges with one incoming and multiple outgoing links (Daganzo, 1995). Tampère et al. (2011) developed a general class of first order node model that is applicable in any unsignalized intersections for dynamic macroscopic simulation (i.e. regardless of the number of incoming and outgoing links). They proposed a convergent solution algorithm in which receiving link supply constraints and internal constraints representing conflict points inside intersections were included. The paper also suggests a method to deal with signalized intersections, however, it was not a comprehensive approach considering only the green time fraction of the cycle time as an additional constraint.

Flötteröd & Rohde (2011) proposed two alternative algorithms for solving a node model based on assumptions that are essentially very similar to (Tampère et al., 2011). They do not address specifically the problem of signalized intersections. Whereas the approaches mentioned so far apply constraints expressed macroscopically in terms of capacities, Smits et al. (2015) reformulated node model constraints based on the consideration of headway and turning delay per vehicle, herewith linking node models closer to behavior theories like priority taking or gap acceptance models. However, neither did these authors address the problem of signalized intersections.

Explicit consideration of signalized intersections in macroscopic node models was given by (Jabari, 2016). In this study, the traffic signal cycle is broken into phases, each of which has only a subset of the flows active. Then, and after making additional simplifying assumptions on the interactions of the active flows per phase, the node model is essentially split into a set of disjoint diverge models that can be solved explicitly. The approach however has the following drawbacks: (i) it requires more simplifying assumptions, (ii) the FIFO assumption of traffic in the links needs to be relaxed in a rather arbitrary way, and (iii) simulation time discretization cannot be larger than the shortest (inter)phase duration. The latter issue may not be prohibitive for link models using short time increments anyhow (e.g. Cell Transmission Model, (Daganzo, 1994)), but may be prohibitive in other network loadings that allow larger time steps like Himpe et al. (2016).

We conclude that so far it remained an unresolved issue to formulate and solve signalized node models that neither restrict the modeler to additional simplifying behavioral assumptions nor to short time increments. This study presents two different approaches for macroscopic modeling of signalized intersections without these restrictions.

2- Methodology

The most straightforward way of modeling signal control cycles is to explicitly consider their phase sequence like Jabari proposed. However, the duration of phases ranges from ~2s (short all-red time) to >60s (rather long green time). The time step of the traffic simulation should then be small enough (e.g. 2s) to sample these time-varying boundary conditions adequately. We adopt in this paper the continuum signal cycle approach (Han et al., 2015), which approximates *average* flow conditions *during the cycle*. As a result, the average conditions can be sampled at any time increment, regardless of the duration of the phases.

Combining the idea of average flow conditions with the generic node model (Tampère et al., 2011) requires additional specifications of how to deal with competition between turn flows. Such competition emerges when two types of constraints activate. First, internal conflict points may impose upper limits to the conflicting flows (=internal supply constraints). Second, turn flows towards the same outgoing link, may find insufficient space there due to limited inflow capacity or congestion spillback in that link (=receiving link supply constraint). In contrast to priority junctions where each turn can exert its competitive power at any time, competition is now affected by the signal phases that determine which turns compete simultaneously during the same phase, or sequentially in different phases. In the latter case, the sequence in which the turns can exert their competitive power may play a role as well.

We propose two approaches: a simultaneous continuum approach that neglects the fact that turns are active in different phases, and a sequential continuum approach that explicitly considers the phase sequence. For both cases, we will define:

- (i) how strong the flow interruption during red reduces the capacity of a turning flow,
- (ii) which turning flows can be constrained by which internal supply conflicts (as traffic signals separate in time certain conflicting flows that thus may never activate), and by which receiving link supply constraints; moreover the upper bound of the constraints needs to be defined, and
- (iii) how the reduced capacity of (i) affects a turn's competitive power in active internal or receiving link supply constraints.

2-1- Simultaneous continuum signal cycle

In a signalized intersection, each flow is active during a fraction of the entire cycle, with full capacity. In contrast, we assume here that all flows are active throughout the cycle simultaneously, but with only a fraction of their competition power. These assumptions allow us to make relatively simple modifications to the existing generic node model, but at the cost of some inconsistencies with the real process at the signalized nodes. In order to adopt the current generic node model, the following modifications are required:

- (i) An additional internal supply constraint is introduced for each incoming flow; this constraint represents the reduced time during which this incoming flow can discharge (because of the red signal); the constraint therefore equals saturation flow times the green percentage.

(ii) We distinguish here between receiving link constraints and internal supply constraints.
For receiving links, we inherit from the generic node model the assumption that all competitors demand simultaneously their share of the receiving supply (however with reduced competitive power (see (iii)), unless they are forced to withdraw from this competition because another (demand, internal or receiving supply) constraint is more stringent.
For internal supply, we neglect conflicts between turn flows that in reality are separated in time by the signal phases. The remaining partial conflicts (i.e. where turns from the same phase compete) are treated just like receiving link constraints.

(iii) We assume that turns competing for active (internal or receiving) supply constraints receive a share of the supply that is proportional to their saturation flow times the green fraction.

Modifications (i), (ii) and (iii) are easy to include in the generic node model; since they are merely configuration changes, the structure and solution algorithm of the generic node model remains unaltered. However, assumptions (ii) and (iii) are clear simplifications of the real process during a cycle. They may be justifiable for the sake of simplicity or whenever detailed information on the exact phase structure and order is missing. Otherwise, the sequential approach presented next is an enhanced alternative.

2-2- Sequential continuum signal cycle

In contrast with the previous method, we now consider explicitly the structure and order of phases. Each flow only competes with other active flows in the same phase, however they now compete only for the share of receiving link supply that comes available during their own phase. The phase order matters whenever flows in a phase do not exhaust their receiving flow constraint; unused space in this receiving link can then be consumed by active flows in next phase in the phase sequence. To illustrate the effects on unsignalized node model, some issues should be considered:

(i) Just like in the simultaneous approach, an additional internal supply constraint is introduced for each incoming flow, equal to saturation flow times the green percentage.

(ii) Again we distinguish between receiving link constraints and internal supply constraints.
We assume that supply in receiving links comes available homogeneously in time over the entire cycle[1]. E.g. a receiving link constraint of 10 veh/cycle of 80 sec, creates space for one entering vehicle every 8 seconds; hence turns during a phase of 20 seconds compete for a constraint of 20/8=2.5 veh. This number may be increased by adding unused supply from the previous phase in the sequence.

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[1] This is a first approximation, of which Han et al. (2015) showed that it is not always true, and that this affects the accuracy of continuum signal cycle approaches. The concept presented here can however be extended to cases where the modeler knows in finer time resolution the receiving flow profile (e.g. fig 2 in Han et al. (2015) where supply is affected by a downstream traffic signal, with offset between the signals known).

Internal constraints are only considered between active flows within the same phase. The total time that such conflict may be occupied by vehicles of all competing turns of a phase combined, is constrained to the phase duration.

(iii) The assumptions (i) and (ii) of this section, essentially mean that the generic node model can now be applied within each phase separately with only the active turns and constraints. Since by definition, the involved turns have a green signal, their competitive power is not constraint by the signal, and hence is equal to the turn saturation flow.

With the configurations of (i), ii) and (iii) of this section, the turn flows per phase can be solved for each phase separately using the solution algorithm of the existing generic node model; the average flows within a cycle are then found as green-percentage-weighted averages of the phase turn flows. However, mind that before averaging, one needs to check whether during any phase some receiving link supply remained unused, while turns in one of the next phases are constrained by that same receiving link. Whenever (and as long as) that is true, unused supply is transferred to the first next phase (see (ii)) and the turn flows for that phase need to be recomputed.

3- Numerical Example

To illustrate the differences between the two presented concepts, the same example will be solved by the two concepts. In order to show the effect of traffic signal on flow through a node, based on the methodology of this study, first the example is solved by using Tampère et al. (2011) unsignalized node model.

Consider an intersection with three incoming links and two outgoing links (Fig. 1). Link capacities (per link) and partial demands are represented in Fig. 1 and Table 1, respectively. Link 1 and 3 have one lane, while link 2 has one lane for each turn (two lanes in total).

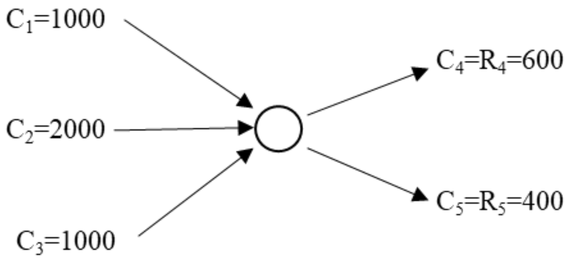

Fig. 1. Sample intersection

Table 1: Partial demands S_{ij}

S_{ij}	4	5	S_i
1	100	0	*100*
2	600	600	*1200*
3	0	300	*300*
$\sum_i S_{ij}$	*700*	*900*	*1800*
Rj	600	400	

notations in Table 1 are as below:

i: incoming links

j: outgoing links

S_{ij}: Sending flow from incoming link i to outgoing link j (similarly S_i stands tor total sending flow of incoming link i)

R_j: total available receiving flow of outgoing link j

The general unsignalized node model would give the following partial sending flows as a result:

Table 2: Flows result of general node model

S_{ij}	4	5
1	100	0
2	200	200
3	0	200

In this solution, incoming link one is demand constrained (S_{14}=100 which is less than its available share based on capacity oriented principle) and all the others are supply constrained (since their q_{ij} is less than what they demand, i.e. q_{25}= 200 while its demand is 600).

Now, let us calculate the same example with the two presented concepts. The signalization at this intersection includes three phases. In phase 1, which has a green time percentage of 25%, incoming link 1 is active. Likewise, link 2 is active in the second phase with 45% green. This leaves phase 3 for link 3 with a green time of 25% as 5% of simulation time considered as all red.

In order to solve this signalized intersection, both presented algorithms are applied. For *simultaneous continuum signal cycle* results are:

Table 3. Results of implementing simultaneous continuum signal cycle method

S_{ij}	4	5
1	100	0
2	257	257
3	0	143

As a consequence of different green time fractions, sending flow proportion of competitors would be different. Capacity oriented logic of general node model is still correct, just capacities have been replaced with reduced capacities equal to capacity multiply by green time fraction. In the other words, receiving flow is distributed among all competitors based on: $\frac{R_j}{\sum_i C_{ij} \times g_i\%}$. For instance, incoming links 2 and 3, which are competing to send flow toward the same outgoing link (5), have a different share of sending flow. More precisely, due to larger green times of link 3 (1.8 times green time percentage of link 2), it has greater proportion of available supply, which was the same in the unsignalized intersection. These two links are still supply constrained.

As a consequence of the same behavior between link 1 and 2, link 1 proportion of empty spaces in link 4 would be 1.8 times share of link 1. But due to internal conflict, link 2 is not capable to send more than 257 vehicle (the minimum share of available receiving flow for q_{24} and q_{25} is equal to share of q_{25} which will prevent movements toward link 4) also it cannot use remained sending flow proportion of link 1 which was not able to serve all its own share of supply.

Although redistribution of remained available receiving flow among competitors for one outgoing link in the absence of other internal conflicts can be considered, the allocation of extra available receiving flow among phases is only possible by implementing the *sequential continuum signal cycle* approach which determines flows as it is shown in Table 4.

Table 4. Results of implementing sequential continuum signal cycle method

S_{ij}	4	5
1	100	0
2	500	257
3	0	143

In this case, interactions of all links are different since green time effect is not considered as capacity reduction but as different available share of receiving flow which is identified based on incoming links green time proportion: $R_j = \frac{R_j}{\sum_i g_{ij}} \times g_{ij}$ (plus unused share of previous phase). Table 5 shows available supply for all movements.

Table 5. Receiving flow share based on sequential phases

S_{ij}	4	5
1	214	0
2	386	257
3	0	143

In addition, conflicts among different movements of the same link is eliminated[2]. For instance, in the absence of internal conflicts of S_{24} and S_{25}, link 2 is capable to send more flow towards link 4. Also it consumes remained proportion of receiving flow of S_{14} (114=214-100), which was active in the previous phase. Interaction of link 2 and 3 differs too. Since each one has a certain share of available receiving flow during different phases, there is no competition between them and each one consumes its own share.

Another remarkable point is that solution of both algorithms gives the same results for q_{25} and q_{35}. This happens since for these two links outgoing link 5 is the most strength constraint and they both are still in demand constrained regime. Therefore they will consume all their own proportion regretting how this amount is calculated.

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[2] One important assumption which is needed to be clarified is acceptance of First In First Out behavior of traffic flow which is going to be discussed in detail in full paper. Briefly, upon on the responsivity of modeler it can be accepted or not. Developed algorithm of sequential continuum signal cycle does not accept any FIFO behavior. Case of existing such behavior is in the scope of future research.

As a final remark, it cannot be concluded which approach is better to use. Only some sensitive analysis can be considered (in future research) and it is upon the modeler to consider which approach is the best based on each project goals and limitations as results differ.

4- Conclusion

Although signalized node models attract some attention recently, refined continuum signal cycle node models which can be combined with any time increment size of the DNL model were lacking. To tackle this problem, two different approaches with the aim of developing current generic node model to signalized node model have been presented. The formulas, algorithms and numerical results of implementing them will be available in full paper.

The last issue which still needs to be considered is about First In First Out behavior in the intersections. Although it is an important nature of traffic flow, it will not change presented algorithm and will impose pre-processing steps in order to make node model compatible with the level of accepted FIFO behavior. As it is not a part of signalized node model, but a separate pre-process, it will be discussed in full paper.

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6- Keywords

Macroscopic signalized node model, Phase order in node model, Dynamic network loading

7- References

Daganzo, C.F., 1995. The cell transmission model, part II: Network traffic. Transp. Res. Part B Methodol. 29, 79–93. https://doi.org/10.1016/0191-2615(94)00022-R

Daganzo, C.F., 1994. The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. Transp. Res. Part B Methodol. 28, 269–287. https://doi.org/10.1016/0191-2615(94)90002-7

Flötteröd, G., Rohde, J., 2011. Operational macroscopic modeling of complex urban road intersections. Transp. Res. Part B Methodol. https://doi.org/10.1016/j.trb.2011.04.001

Han, K., Sun, Y., Liu, H., Friesz, T.L., Yao, T., 2015. A bi-level model of dynamic traffic signal control with continuum approximation. Transp. Res. Part C Emerg. Technol., Engineering and Applied Sciences Optimization (OPT-i) - Professor Matthew G. Karlaftis Memorial Issue 55, 409–431. https://doi.org/10.1016/j.trc.2015.03.037

Himpe, W., Corthout, R., Tampère, M.J.C., 2016. An efficient iterative link transmission model. Transp. Res. Part B Methodol., Within-day Dynamics in Transportation Networks 92, Part B, 170–190. https://doi.org/10.1016/j.trb.2015.12.013

Jabari, S.E., 2016. Node modeling for congested urban road networks. Transp. Res. Part B Methodol. https://doi.org/10.1016/j.trb.2016.06.001

Smits, E.S., Bliemer, M.C.J., Pel, A.J., van Arem, B., 2015. A family of macroscopic node models. Transp. Res. Part B Methodol. https://doi.org/10.1016/j.trb.2015.01.002

Tampère, C.M.J., Corthout, R., Cattrysse, D., Immers, L.H., 2011. A generic class of first order node models for dynamic macroscopic simulation of traffic flows. Transp. Res. Part B Methodol. https://doi.org/10.1016/j.trb.2010.06.004