

## Fixing the number of vehicles and/or setting a tax for ride-hailing? Insights from a social welfare maximisation approach

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### Abstract

A social welfare maximisation problem is developed, in order to determine the optimal number of vehicles and the optimal fare of a ride-hailing service. The solution is compared with the one that maximises private profit only. External costs of traffic are included into the social welfare framework. It is found that the optimal solution depends on two colliding forces: users waiting time, that pushes to increase driver supply in order to for the users of the service to wait less time, and the existence of traffic externalities such that congestion, pollution and crashes, which pushes to reduce driver supply and increase fares. Optimality rules are analysed on first-best ground and second-best grounds, after introducing a constraint on the minimum wage that drivers could received. The problem is numerically solved with data from Germany and Chile, in order to estimate the relative weight of the aforementioned effects in a range of alternative scenarios regarding input parameters.

**Keywords:** sharing economy, ride-hailing, traffic externalities, optimal pricing, social welfare

## 1. Introduction

The regulation of ride-hailing (ridesourcing) services has been a topic of heated debate between policy makers, practitioners and researchers around the world. A perceived increase in traffic or the need to raise funds to support sustainable mobility initiatives have usually been put forward by city officials as a way to justify special taxes for ride-hailing or caps to the number of ride-hailing licences operating in a city. As examples, we can mention Washington D.C. with a special 6% tax on Uber and similar apps, in order to support public transport improvements; Sao Paulo, which since 2017 charges a fee per kilometre to ride-hailing companies, with discounts for female drivers, off-peak periods and pooled rides (Alonso Ferreira *et al.*, 2018); and New York, which apart from setting a tax, has decided to temporarily fix the number of ride-hailing licences during 2018.

In this context, a relevant question is understanding if ride-hailing supply controls, by quantity and/or price interventions, are sensible ways to proceed by policy makers. In this paper, we present a social welfare maximisation model in which we study the problem of finding an optimal value for the number of ride-hailing vehicles and/or the optimal tax per trip. The solution is compared with a setting in which private profit only is maximised, to mimic an unregulated situation in which the ride-hailing operator is free to set the price with any special tax being imposed on them.

## 2. Model

A demand function  $Q$  that is sensitive to ride-hailing in-vehicle time, waiting time and fare is presented in expression (1), from which the consumer surplus is obtained.

$$Q = q[1 - e_w t_w - e_v t_v - e_p P] \quad (1)$$

In (1),  $q$  is a potential demand,  $t_w$  is the waiting time,  $t_v$  is the in-vehicle time,  $P$  is the fare and  $e_w$ ,  $e_v$ , and  $e_p$  are demand sensitivities to waiting time, in-vehicle time and fare, respectively. The ratios of  $e_w$  and  $e_v$  to  $e_p$  are the values of waiting time savings and in-vehicle time savings, respectively.

Following Castillo *et al.* (2018), driver supply  $L$  comprises three stages: drivers idle (waiting to be assigned a passenger), drivers on their way to pick up an assigned passenger and drivers with passengers:

$$L = \underbrace{I}_{\text{idle}} + \underbrace{\frac{t_w Q}{\text{en route to pick up a rider}}}_{\text{en route to pick up a rider}} + \underbrace{\frac{t_v Q}{\text{with riders}}}_{\text{with riders}} \quad (2)$$

Waiting time is modelled as an inverse function of the number of idle drivers (Castillo *et al.*, 2018):

$$t_w = \frac{\theta}{I} \quad (3)$$

Where  $\theta$  is a positive parameter that has to be calibrated with empirical data. Equation (3) assumes a Poisson process for the arrival rate of idle taxis. With a demand function given by expression (1), consumer surplus  $G$  is found as follows (Chang and Schonfeld, 1991):

$$G = \frac{q}{2e_p} [1 - e_w t_w - e_v t_v - e_p P]^2 \quad (4)$$

Operators profit  $\pi$  (including the ride-hailing platform and the drivers) is formulated as follows:

$$\pi = QP - c_o L \quad (5)$$

where  $c_o$  is the operator cost [\$/veh-h]. Finally, we define an external cost  $C_e$  to account for traffic externalities such that pollution and crashes.

$$C_e = c_e L \quad (6)$$

where  $c_e$  is the cost of externalities [\$/veh-h].

Then, social welfare  $SW$  is obtained as the summation of the consumer surplus and the private profit, minus the external cost of traffic:

$$SW = \underbrace{\frac{q}{2e_p} [1 - e_w t_w - e_v t_v - e_p P]^2}_{\text{Consumer surplus}} + \underbrace{QP - c_o L}_{\text{Profit}} - \underbrace{c_e L}_{\text{External cost}} \quad (7)$$

Social welfare (7) is maximised with respect to  $P$  and the number of idle drivers  $I$  (since  $I$  will determine the total driver supply  $L$  in equation 2, for a given demand). The social welfare maximisation problem is stated as follows:

$$\max_{P,I} SW = \frac{q}{2e_p} [1 - e_w t_w - e_v t_v - e_p P]^2 + QP - (c_o + c_e) \left( I + \frac{\theta}{I} Q + t_v Q \right) \quad (8)$$

The solution of (8) can be compared with the solution from maximising private profit only:

$$\max_{P,I} \pi = QP - c_o \left( I + \frac{\theta}{I} Q + t_v Q \right) \quad (9)$$

It is worth noting that in equation (9), there is no distinction between the profit of the ride-hailing company (e.g., Uber, Lyft) and the profit of the drivers. If  $\tau$  is the percentage of the fare that goes to the ride-hailing platform, average earnings  $E$  per driver are:

$$E = \frac{(1-\tau)PQ}{L} \quad (10)$$

One scenario to be studied includes a constraint of drivers' earnings, if the regulator wants to set a minimum wage  $E_0$  for drivers, as recently ruled in New York<sup>1</sup>,

$$E \geq E_0 \quad (11)$$

In a first stage, the problem is solved assuming no congestion effect of ride-hailing, i.e., that travel time  $t_v$  in equations (1) and (2) is a fixed parameter. Then, we will solve the case in which travel time is endogenous, as a function of the number of vehicles on the road, i.e.,  $t_v = t_v(L)$ , which introduces a fixed-point to the determination of drivers supply (2). This is a relevant extension given the growing number of empirical studies that attempt to assess the effects of ride-hailing on vehicle-kilometres travelled and congestion (Li *et al.*, 2016; Henao and Marshall, 2018; Schaller, 2018; Tirachini and Gómez-Lobo, 2019).

<sup>1</sup> [https://www.washingtonpost.com/technology/2018/12/04/new-rules-guarantee-minimum-wage-nyc-uber-lyft-drivers/?utm\\_term=.a592813064de](https://www.washingtonpost.com/technology/2018/12/04/new-rules-guarantee-minimum-wage-nyc-uber-lyft-drivers/?utm_term=.a592813064de), accessed 10/Feb/2019

We find the conditions under which it is optimal to set a price per trip larger than the price that maximises private profit, and also the conditions that would make beneficial to fix the number of vehicles in service. We will also analyse the effects of an active driver wage constraint (11).

Next, we show the rules for the optimal fares that maximise social welfare and private profit in an unconstrained environment. After applying first order conditions, we find the following rules for the optimal fare that maximises social welfare,  $P_{SW}^*$ , and private profit,  $P_{\pi}^*$ :

$$P_{SW}^* = (c_o + c_e) \left( \frac{\theta}{I_{SW}^*} + t_v \right) \quad (12)$$

$$P_{\pi}^* = \frac{1 - e_w \frac{\theta}{I_{\pi}^*} - e_v t_v}{2e_p} + c_o \left( \frac{\theta}{I_{\pi}^*} + t_v \right) \quad (13)$$

By inspecting (12) and (13), it follows that not necessarily the optimal fare to maximise welfare is larger as the optimal fare to maximise profit. Numerically, it will depend on the parameters of the problem. Theoretically, it depends on two colliding effects. On the one hand, reducing the ride-hailing fare increases demand, which in turn increases driver supply, which reduces waiting time for users. Following this reasoning, the existence of economies of density, Arnott (1996) finds that taxi travel should be subsidised with a model that ignores congestion due to taxi travelling. On the other hand, accounting for the external costs of traffic pushes for an increase in the optimal fare (12).

Equations (12) and (13) are not the final solutions for the optimal fares, because they depend on the optimal number of idle drivers,  $I_{SW}^*$  and  $I_{\pi}^*$ , which need to be found by applying first order conditions to (8) and (9) with respect to  $I$ . These equations are higher order polynomials, which imply that the full problem must be solved numerically for optimising both profit and welfare.

## 2. Outlook

Results are illustrated with numerical applications taken from empirical studies on ride-hailing and discussed under different scenarios regarding demand sensitivity to trip characteristics, social valuation of external costs and other relevant parameters.

The application of the model will be made with data from two different environments, Germany and Chile, which then can be compared. We will gather data on values of travel time, drivers wages, vehicle operating cost from secondary sources. External costs (congestion, pollution, traffic crashes and so on) are obtained from Korzhenevych *et al.* (2014) for Germany and Rizzi and De La Maza (2017) for Chile. We first solved the unconstrained version of the problem for increasing levels of demand. Then we introduce the minimum wage constraint, which implies moving into a second-best solution. Finally, we will introduce congestion in the problem.

In the numerical solutions, a comparison will be made between electric vehicles and standard gasoline vehicles, including that the former have lower running cost due to energy consumption (see, e.g., Bösch *et al.*, 2018) but a larger capital cost and, depending on the period of operation, need time out to recharge their batteries. All these trade-offs can be accommodated in our model. We plan to find a range of optimal fares and number of vehicles in different illustrative scenarios.

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