A complex networks approach for dynamical efficiency in multimodal transportation systems.

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Abstract

In this work, we propose a new measure for spatial networks and, in particular, for multimodal transportation systems. We give the definition of dynamical efficiency for single layer and for multilayer networks. This measure is based on the concept of "reachability" of a location in the city according to the road traffic and to the available modal choice. The dynamical efficiency captures the topological changes in a multilayer network due to the congestion like the average loss in travel time from and to each link and the centrality of intermodal stations. There are several advantages in using this measure on a point of view of the complex network theory and on the practical engineering applications and they are briefly illustrated in the conclusion. A detailed application on a real network with real traffic data is also presented as example.

1 Introduction

Analyzing the urban mobility of a city is one of the fundamental tasks of traffic engineering. This is due to the enormous consequences that an efficient or inefficient transportation urban system can have on all citizens' life not only in terms of travel time but also of pollution, stress, safety, public expenses. In modern cities, traffic is not only represented by private cars but also by public transportation like the underground, buses, car sharing, bike sharing, etc. Our research proposal is to define a universal and appropriate measure for integrating multimodal and complex transportation urban system. For this scope, inspired by classical measures from complex networks theory ([1, 2, 3]), we defined the *dynamical efficiency* for a multilayer network and, in particular, for time-varying transportation networks. This study reveals not only a consistent analysis of the entire transportation system but, also, it suggests new applications and optimized solutions for the multimodal urban mobility. Results based on real data from the Chinese city of Shenzhen, in Guangdong province, are provided as an explanatory example. In particular, we study the interaction between private cars traffic and public transportation (metro and bus) during a working day with detailed information.

2 Multilayered networks

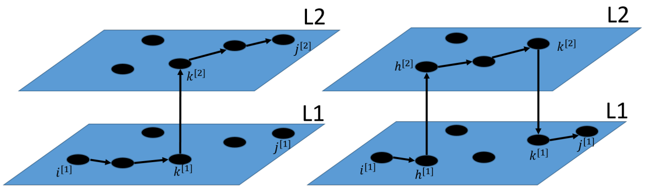
We describe the urban transportation system with the complex network theory. A network is represented by a graph $\mathcal{G}(N, E)$ composed by N nodes and E edges. An edge is a ordered couple of nodes $(i, j) \in E \subseteq N \times N$ and, for the sake of simplicity, we indicate also $l_{(i,j)}$ or simply $l \in E$. In ([4]) has been introduced the definition and several applications of the multilayer networks. If M is the number of layers, we will denote $\mathcal{G}^{\{M\}}$ the multilayer network and $\mathcal{G}^{[m]}(N^{[m]}, E^{[m]})$ the network relative to layer $m \leq M$. The edges in $E^{[m]}$ are called intralayers links, because they all belong to the same layer. The links $l_{i,j}^{[m,n]}$ that connect a pair of nodes $i \in N^{[m]}$ and $j \in N^{[n]}$ of two different layers $m, n \leq M$ are called interlayers links.

The multilayer networks have been used in several domains since their first appearance in literature. In particular, an urban multimodal transportation systems can be easily represented by a multilayered network where each layer represents the topology and the characteristics of each individual system (private cars, metro, bus, car sharing, etc.) and where the interlayer links stand for the interchange stations that bridge one system to another. In this sense, we consider the multilayered networks as appropriate mathematical tools to study and measure the performance and topological characteristics of multimodal urban mobility systems.

2.1 H-shortest path

Computing shortest paths in multilayered networks required, in general, a huge effort due to the size and the higher complexity respect to a single layer network. Nevertheless, the fact that a traveller changes from a layer to another, with no limitation, to follow the fastest path to his/her destination is not very realistic. Therefore, we assume that in transportation environment the most realistic paths are of 2 types: (α) with only a change between one layer to another; or (β) with two opposite changes from a layer m to n and, then, return to m passing through two different interchange stations (interlayer links) (as shown in Figure 1). We call paths with these characteristics the *H-shortest paths.* This strong but consistent assumption, not only takes in consideration the peculiar property of an urban spatial path but also, thanks to a matricial algorithm, that we explain in the full version of the paper, the computational cost is drastically reduced. We show that for computing the H-shortest paths between a node i and a node j using two layers $L1$ and $L2$, we need just to run the all shortest paths algorithm individually for each layer and then sum the i-row of the distance matrix of $L1$ and the j-column of the distance matrix of $L2$ to obtain the vector $\mathbf{v} = \{v_k\}_k$ of all the travel time from $i \in L1$ to $j \in L2$ changing in station k. The minimum value $v_{k^*} = min\{v_k\}_k$ will be the travel time of the H-shortest path that we were looking for, and $k^*(\neq i, j)$ the most convenient interchange station. An example of the type (α) is when a trip is made by car for a part

Figure 1: The two general cases of H-paths using 2 different transportation modes considered in this paper. On the left, the user starting the trip from layer 1 arrives at his destination in layer 2 changing at station k. On the right, the user travels layer 2 only for the intermediate line $h^{[2]} - k^{[2]}$ and then continues his trip until the destination $j^{[1]}$. This can be the case when both nodes i and j do not belong to layer 2.



and then, in some parking spot change to public transportation, because of the congestion in the city centre. Type (β) represents, for example, the case when a traveller walk from a point to a metro station, take the metro and arrive at his/her destination by walking again.

3 Dynamical efficiency

In [5], the authors define the dynamical efficiency for a single layer. It is a measure for each link $(i, j) = l_{(i,j)} = l$ calculate as $E(l_{(i,j)}) = \frac{e_i+e_j}{2}$, where $e_i = \frac{1}{|N|-1} \sum_{k \in N \setminus \{i\}} \frac{d^{i,k}}{\tau^{i,k}(t)}$ and $d^{i,k}$ is the time of the shortest time path in free flow condition between node i and k (for all $k \in \setminus\{i\}$) while $\tau^{i,k}(t)$ is the travel time of the shortest time path at time t between the same pair of nodes. The dynamical efficiency for each link measures the 'reachability' of the link that change with the congestion growing. It also has the characteristic to be spatially smoothed and this is useful for traffic engineering to visualize immediately the congested zone, without the help of any clustering algorithms, in order to apply strategies to mitigate the congestion (e.g. perimeter control as shown in [6, 7]). The average of all the link $l \in E$ is the Network Dynamical Efficiency and it is defined as $E(\mathcal{G}(t)) = \frac{1}{|E|} \sum_{(l \in E)} E(l(t))$.

3.1 Multi-layers case

In a multilayer transportation network, we use the definition of H-shortest paths to compute the multi dynamical efficiency. Because of its definition, we can estimate the dynamical efficiency always between a couple of layers. We define $d_{i,j}$ the H-shortest path in a free flow condition in all the multi-layer network and $\tau_{i,j}^{[m,n]}(t)$ the H-shortest path between i and j using layer m and n (in this order). The couple $([m^*, n^*])$ that minimizes the shortest time path at time t is denoted $s_{i,j}(t) = \tau_{i,j}^{[m^*,n^*]}(t)$. The multilayer efficiency of a couple of layers, in this case, will be $E^{[m,n]}(l_{(i,j)}) = \frac{e_i^{[m,n]}+e_j^{[m,n]}}{2}$, where $e_i^{[m,n]} = \frac{1}{|N^{[n,m]}|-1} \sum_{k \in N^{[n,m]} \setminus \{i\}} \frac{d_{i,k}}{\tau_{i,k}^{[m,n]}(t)}$, and $N^{[n,m]} = N^{[n]} \cup N^{[m]}$.

3.2 Layer performance analysis, dilemma factor and station centrality

We can also estimate the gain that a layer brings to the transportation system comparing the efficiency of the entire network $\mathcal{G}^M(t)$ with the efficiency of $\mathcal{G}^{M\setminus\{m\}}(t)$. This is already a powerful analysis tool to quantify the contribution of each layer to the whole mobility and a first step to calculate the cost/benefits of a transportation network component by component. With our algorithm for H-shortest path, we obtain also the difference in term of travel time of all the alternatives from the fastest H-shortest path and, in particular, the 'rank' of convenience, in terms of shorter travel time, of each interchange station between two layers $[m, n]$. Thanks to the estimation of the travel times from an origin to a destination changing layer in k, we can establish a measure of "substitutability" of each station k. This factor, that we called *dilemma factor*, tell us if some other alternative paths, respect to the best, are still convenient or not. The dilemma factor can have application not only to study the multiplicity of the travel choices of each zone of a city but also can be used to quantify the feasibility for path recommendation strategies and route guidance. The other important measure that we can extract from our algorithm for dynamical efficiency is the station centralities. This measure is an indicator of how many times a station k has been used by the all H-shortest paths algorithm for changing the layer. If k corresponds to the origin or the destination of the path we do not count it as interchange station. The station centrality is strongly dependent on our original definition of H-shortest path and it can be seen, in a more general point of view, as the betweenness of the interlayer links considering one (α type) or two opposite change (β).

4 A case study in a megacity

4.1 Network description

Here, we show the results of the application of the multilayer dynamical efficiency definition in the case of the central part of Shenzhen, a large and important metropolis of China in Guangdong province, where we have an extensive dataset of taxis GPS data that gives us an accurate estimation of links speeds for the road networks every 5 minutes. The road network that we consider is composed of 2013 links and 1858 nodes and the metro network with 11 lines, 75 stations and 163 links (Figure 2). We note that many of the nodes in the road network are intermediate nodes and not all of them represent an intersection. Moreover, we extracted from OpenStreetMap detailed information of the metro system of the studied area about speed, frequency, location, distances. For the sake of simplicity, we located every metro station to the closest node of the road network. We also took into account the walking time needed to go from a node to the closest metro station and added to the travel time via public transport the average waiting time.

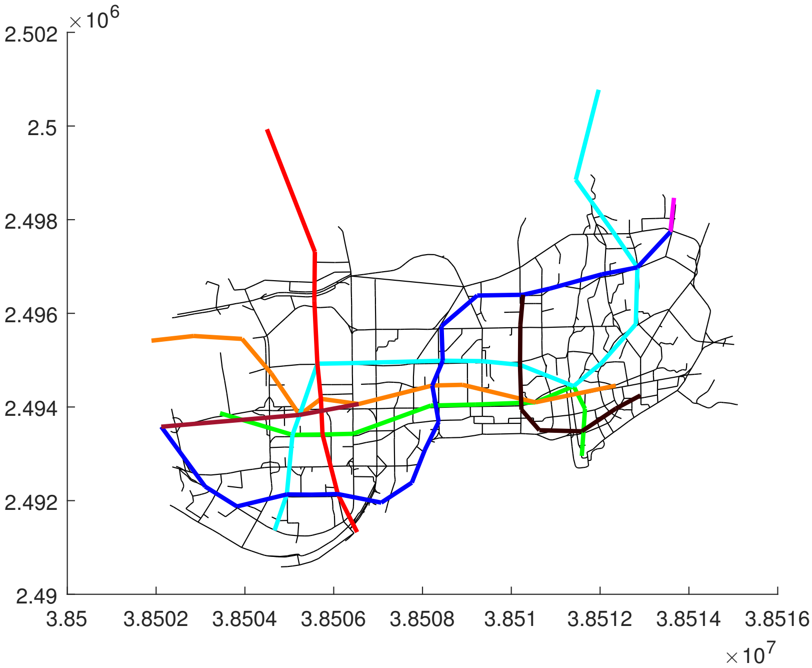
4.2 Results

In Figure 3, we plot the average among all the links in the network of the road dynamical efficiency (private cars) during a whole day, the 7th September 2011 in Shenzhen. As free flow condition, we set 30mph the link speed of all the network. For the definition of dynamical efficiency, we compare the travel time in free flow condition with the effective travel time calculated from the available speed data. The results show the presence of two distinct peak hours: one at 8am (morning PH) and the other, more severe, around 6pm (evening PH). The spatial distribution of the local dynamical efficiency is shown in the snapshots of the network performances in Figure 4. Here, we notice the smoothed distribution of the dynamical efficiency values, and also that it is possible to follow the spreading of the congestion over the network and visualize the perimeter and the level of congested zones. This is one of the main difference with the links speeds data where there is no clear spatial correlation with the link values. In a transportation network, we believe, it certainly is more useful a parameter path-based rather a myopic link speeds values map, especially for applications and traffic estimations. In Figure 5, we plot the metro system of Shenzhen, composed of 11 lines and 75 stations. Here, it has been computed the dynamical efficiency of the metro system, considering the walking time distance from every node to the closest station, on the Shenzhen network. We notice that along the metro lines the values of efficiency is surely higher than far from the metro station. This is because, here, the dynamical efficiency has been compared with the max road speed (30 mph) in all network. Finally, in Figure 6, we compute the H-shortest paths between all pair of node considering private car layer and the metro system. Every time a change of type (α) or (β) is more convenient in a certain station k, we increase the value of the centrality of k. The size of the red spots in Figure 6 are proportional to the station centrality at different time during the morning peak hour, every 20 minutes, from 6am. We notice as the congestion grows in the central area the station centralities becomes more and more important. This is because in severe traffic condition the presence of alternatives like metro or bus is more suitable, especially to protect the central zone of a city. Moreover, the station centrality can be used to regulate the distribution of users in a network and avoid unexpected accumulation and congestion.

5 Conclusion

In this work, we exposed briefly the concept and some formula of the multilayer dynamical efficiency. We believe that this measure has the ambition to become a fundamental feature for all networks, single or multilayered, that experience smoothed connectivity changes in time like a transportation network that experience traffic and congestion propagation. This is because it is a path-based measure instead of a local measure as the average link speed. The advantages are that the efficiency of a network can be weighted also based on the real or estimated demand between each couple of nodes. Moreover, the values of the dynamical efficiency is smoothed among the network and this is useful to visualized the congested zone, for example, where and when to apply strategies that involves clustering algorithm and congestion perimeter delimitation. From the point of view of the computational cost, we want to remark the advantage that the above-mentioned H-shortest path algorithm brought to the evaluation of the efficiency. This algorithm requires the all-shortest path algorithm applied to each layer individually and not for the whole multilayer network, and after

Figure 2: **Shenzhen metro line system.**



some trivial sum of two vectors for each couple of nodes we have the travel time that we need to go from a origin node to another through each intermediate station $k \in N^{[m,n]}$. This allowed us to define other important measures like the station centrality and the dilemma factor. The first quantify how much is convenient to change in a station from a layer to another at each time t and can be applied for parking spot management, for station capacity regulation and optimized distribution of users in a control traffic domain. The second metric, the dilemma factor, measures the richness of equivalent transportation offers divided par zones and can be used, also in traffic assignment management and control to quantify the feasibility of some path recommendation and route pricing strategies.

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Figure 3: **Average of the road dynamical efficiency for 1 day.** We plot the average value of dynamical efficiency that we measure with the links speeds data of Shenzhen the 7th September 2011. The spatial distribution of efficiency for link is reported in Figure 4. We notice the two peaks hours, in the morning around 8am and in the evening at 6pm.

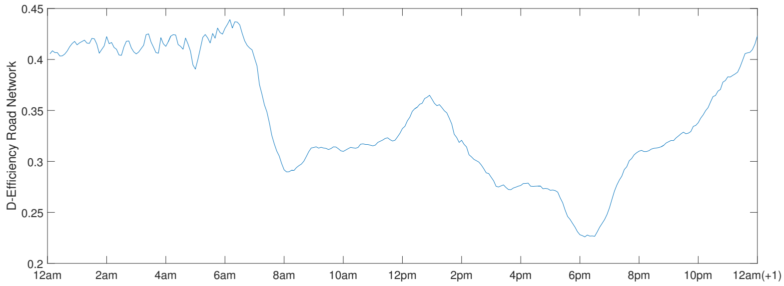

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Figure 4: **Road network efficiency for 24h.** Here we plot the screenshot every 2 hours of the road network dynamical efficiency for the 7th September 2011 in Shenzhen. The warm (red) colours stand for lower efficiency while the cold (blue) colors for high efficiency. We notice two peak-hours period, the morning and the afternoon. The congestion seen under the dynamical efficiency measure results connected and smoothed area. In particular, we can see two separate zones that experienced congestion in the west part and in the south-east part of the network.

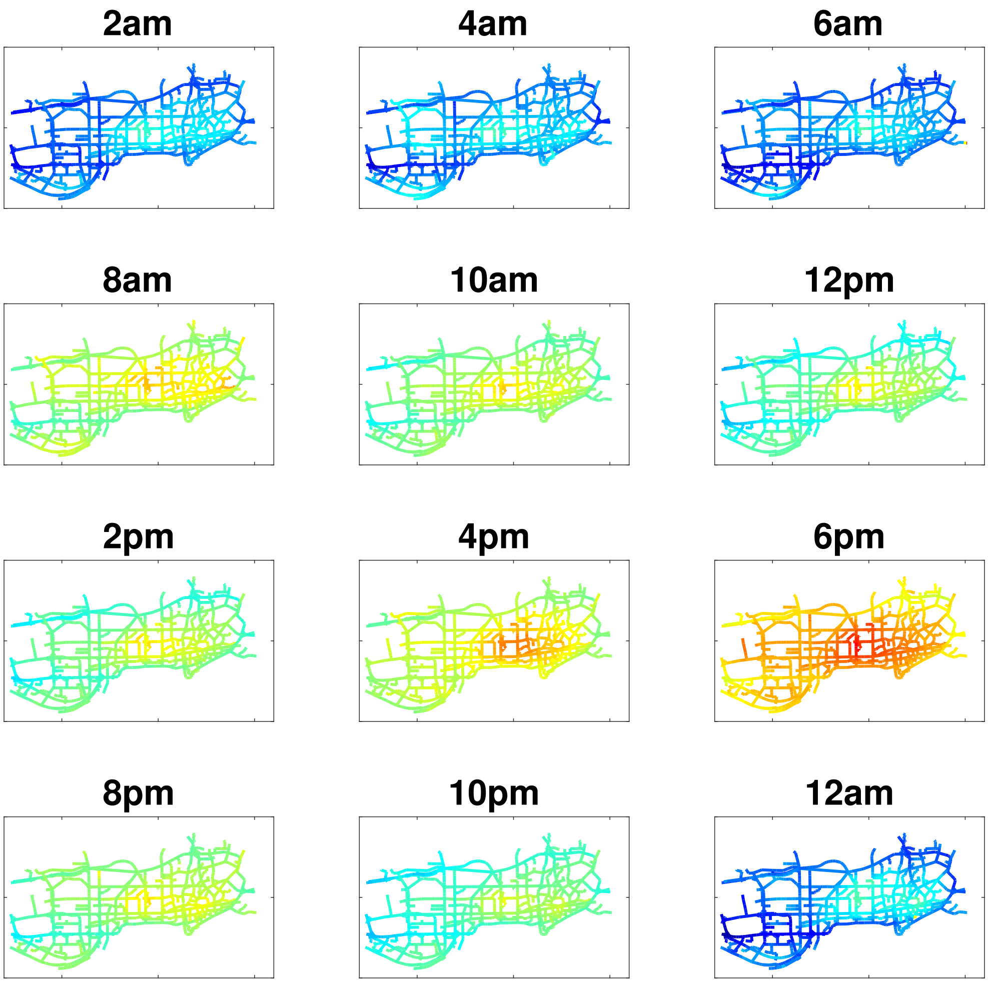

Figure 5: **Metro network efficiency.** We plot here the efficiency of the metro network when the only H-shortest paths considered are between the walking network (coincident with the road network with speed of 4 miles/hours) and the metro network. This network does not depend on traffic but just on the frequency of the metro lines. The colder colors (blue) stand for higher efficiency rather than warm colors (red) for bad efficiency compared to the network with free flow road speed (30 mph).

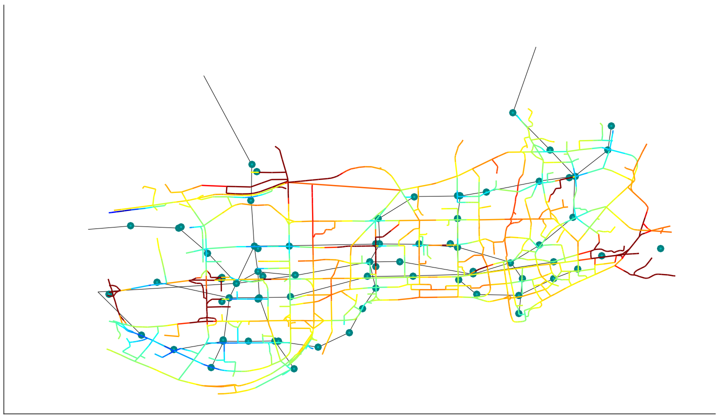

Figure 6: **Station centrality during peak-hour period.** In this plot we can observe how the station centrality change. The red spot on the maps are proportional to the number of interchange from the road network to the metro network. We show here the snapshot every 20 minutes while the congestion is growing.