An integer programming formulation for the vehicle routing problem with cargo stability and multi-drop constraints

Hatice Çalık[1], Jordi D'hondt[2], Marc Juwet[2], Greet Vanden Berghe[1]

[1] KU Leuven, Department of Computer Science, CODeS, Belgium
[2] KU Leuven, Department of Mechanical Engineering, Mechanical Engineering Technology, Belgium
hatice.calik@kuleuven.be, jordi.dhondt@kuleuven.be, marc.juwet@kuleuven.be, greet.vandenberghe@cs.kuleuven.be

Abstract

In this paper, we consider the vehicle routing problem with cargo safety and multi-drop restrictions. The demand of each customer should be delivered by a single visit. As unloading and reloading of packages is not allowed, the placement of the cargo inside the vehicle should be arranged considering the order of visits to the customers. Moreover, the loading strategy should take into account the stability restrictions which must be ensured at every leg of the route for each vehicle. We propose an integrated mathematical programming approach that tackles the routing and loading decisions simultaneously to come up with an optimal strategy. We aim to minimize the total cost of routing.

Keywords: Vehicle routing problem, container loading, cargo stability, axle weight constraints, integer programming

1 Introduction

The distribution companies serving multiple customers often do not dedicate different vehicles to different customers but rather combine their cargo (packages) inside the vehicles. This requires partial unloading of the packages at multiple drop-off points which might result in weight unbalances leading to security issues on the road. In order to achieve safer road transport, it is essential to ensure the cargo stability inside the vehicle at every point of the delivery trips [7, 9, 10].

Pollaris et al. [10] introduce the capacitated vehicle routing problem with sequence-based pallet loading and axle weight constraints. They consider pallets of identical sizes and two rows to place the pallets inside the vehicle. The weights of the pallets are the same for an arbitrary customer but they may be different for different customers. The computational study conducted by [10] reveals the importance of considering axle weight restrictions to ensure the feasibility of the vehicle routes in real-life. In addition to the flow-based formulation of [10], Pollaris [9] proposes a set-partitioning based formulation for the same problem, which outperforms the flow based model on the benchmark instances.

Our work generalizes the problem of [9] by allowing non-identical rectangular shaped pallets that can be placed in two different orientations. We propose an integrated integer programming formulation that tackles the routing and loading decisions simultaneously to minimize the total cost of routing.

In Section 2, we give a very brief overview of the most related studies in the literature. In Section 3, we give the notation and describe our problem in detail. In Section 4, we present our mathematical formulation and in Section 5, we conclude with future research directions.

2 Related work

The three-dimensional loading capacitated vehicle routing problem (3L-CVRP) is first introduced by Gendreau et al. [4] who present a tabu search method to solve this problem. Bortfeldt [1] proposes a hybrid algorithm for the same problem. This algorithm combines a tabu search method for the routing decisions and a tree search algorithm for the loading constraints. Junqueira et al. [6] propose mixed integer programming approaches to solve the three-dimensional loading problem with multi-drop constraints. However, the multi-drop constraints are not integrated in the formulations but they are handled by heuristic approaches.

Iori et al. [5] present a branch-and-cut approach to the vehicle routing problem with loading constraints including sequential loading but with fixed orientations of the packages. An integrated mathematical formulation to solve the vehicle routing problem with container loading constraints is proposed by Moura and Oliveira [8]. Ceschia and Schaerf [3] introduce extensions of the 3L-CVRP with additional real-world constraints such as box rotation limitations, weight bearing constraints, and heterogeneous vehicles. To the best of our knowledge, the axle weight constraints are considered only by [9, 10] for the vehicle routing problem with loading constraints. A review of the studies considering container loading constraints can be found in [2]. Additionally, a more recent paper by Ramos et al. [11] proposes new performance indicators to evaluate the dynamic stability of cargo arrangements and analytical models of these indicators based on multiple regression analysis to be incorporated in container loading problem. We also refer to [9] for a broad overview of the vehicle routing problems with loading constraints. The problem we study in this paper is more general than that of [9] and it corresponds better with reality.

3 Notation and problem description

Given a network $G = (N^0, E)$ with edge set E and node set $N^0 = N \cup \{0\}$, where node '0' represents the depot location and $N = \{1, \ldots, n\}$ denotes the set of customer locations, we are required to deliver a set of packages $P_i = \{p_1, \ldots, p_{n_i}\}$ to each customer $i \in N$ by using a subset of vehicles available with limited freight capacities (volume and weight) and tour lengths (time). In doing so, we need to obey certain loading and cargo safety restrictions. In the remaining of the paper, we refer to the part of a vehicle to load packages as its 'container'.

The first group of restrictions that we need to satisfy can be considered as multi-drop constraints. We assume that split deliveries are not allowed and each customer will be visited exactly once by a single vehicle. Moreover, the additional unloading and loading operations at the customer points are not desirable as they bring additional workload to the operators and the customers might have a limited space for the unloading operations. Therefore, the placement of the cargo inside the vehicle containers should take into account the order of visits to the customers to make sure that the packages of a customer are not blocked by the packages of another customer that will be visited afterwards.

The second group of constraints are the cargo stability restrictions which ensure the total weight on any axle of a vehicle (coupling and the central axle of the semi-trailer) is within the limits at every leg of the delivery routes. These restrictions are of high importance for the feasibility of a safe cargo loading.

We assume that the packages are grouped on pallets of rectangular shapes. Any customer might have multiple pallets and they are not necessarily identical neither in size nor in weight. But we assume that the weight of each pallet is homogeneously distributed. It is possible to locate an arbitrary pallet in two different orientations: (i) the long edge of the pallet is parallel to the long edge of the container (ii) the short edge of the pallet is parallel to the long edge of the container. The pallets cannot be located on top of each other. We further need to make sure that the packages do not overlap and all the packages of the customers assigned to the same vehicle lie entirely inside the container.

We aim to minimize the total routing cost of the vehicles.

We utilize set partitioning type of constraints to model the routing restrictions of our problem. In order to do so, we generate all feasible vehicle routes (tours) that satisfy the capacity and tour length restrictions. We present our formulation for a homogeneous fleet of vehicles but it can easily be adapted to heterogeneous (mixed) fleets by generating the feasible routes for each vehicle type. Let T be the set of all such feasible tours generated, then, we assign exactly one tour from T to each customer $i \in N$.

For the loading and stability restrictions, we divide each container into identical grid squares of largest possible size (e.g. the largest common divisor of the edge lengths of the pallets and the container).

Below we define our parameters:

Pallets:

P_i: the set of pallets to be delivered to customer $i \in N$ and $P = \bigcup_{i \in N} P_i$.

$I_p \in N$: the customer of pallet $p \in P$ i.e. $p \in P_{I_p}$.

w_p: the weight of pallet $p \in P$.

w_p^G: the per grid square weight of pallet $p \in P$.

l_p^S: the length of the short edge of pallet $p \in P$.

l_p^L: the length of the long edge of pallet $p \in P$.

Grid squares:

G: the set of (identical) grid squares in the container.

R_g^S: the coordinate value of the reference point of grid square $g \in G$ on the short axis of the container.

R_g^L: the coordinate value of the reference point of grid square $g \in G$ on the long axis of the container.

C_g^L: the coordinate value of the center point of grid square $g \in G$ on the long axis of the container.

Tours:

T_t: the ordered set of nodes visited via tour $t \in T$.

$T_t^{fol}(i)$: the ordered set of nodes visited after node $i \in T_t$ via tour $t \in T$.

$W_{it} = \sum_{j \in T_t^{fol}(i)} \sum_{p \in P_j} w_p$: the total weight in the container of $t \in T$ after leaving node $i \in T_t$.

$pre_t(i)$: the preceding node of $i \in T_t$, $t \in T$.

D_1: the distance from the front of the container to the coupling.

D_2: the distance between the coupling and the central axle of the semi-trailer.

F_1^{max}: maximum force allowed on the coupling.

F_2^{max}: maximum force allowed on the central axle of the semi-trailer.

F_1^{min}: minimum force required on the coupling.

F_2^{min}: minimum force required on the central axle of the semi-trailer.

c_t^r: the routing cost of tour $t \in T$.

4 An integer programming (IP) formulation

In our formulation, we utilize four sets of binary decision variables defined as follows:

$z_t = 1$ if tour $t \in T$ is selected, 0 otherwise.

$x_{pgt} = 1$ if grid square $g \in G$ of $t \in T$ is occupied by pallet $p \in P$, 0 otherwise.

$y^S_{pgt} = 1$ if the short edge reference point of pallet $p \in P$ is located on grid square $g \in G$ of $t \in T$, 0 otherwise.

$y^L_{pgt} = 1$ if the long edge reference point of pallet $p \in P$ is located on grid square $g \in G$ of $t \in T$, 0 otherwise.

We initially present our constraints in four groups: set partitioning constraints, assignment of pallets to grid squares, blocking constraints, and stability constraints. We provide the complete formulation after the description of the constraints.

Set partitioning constraints:

$$\sum_{t \in T: i \in T_t} z_t = 1, \qquad \forall i \in N \tag{1}$$

$$z_t \in \{0,1\}, \qquad t \in T \tag{2}$$

Constraints (1) are the set partitioning constraints which assign each customer to exactly one of the tours generated.

Assignment of pallets to grid squares:

$$\sum_{g \in G} y^S_{pgt} + \sum_{g \in G} y^L_{pgt} = z_t, \qquad \forall p \in P_i, t \in T : i \in T_t \tag{3}$$

$$\sum_{p \in P} x_{pgt} \leq z_t, \qquad \forall g \in G, t \in T \tag{4}$$

$$x_{pg^2t} \geq y^L_{pg^1t}, \qquad \forall t \in T,\ p \in P_i,\ g^1, g^2 \in G : i \in T_t,\ R^S_{g^1} \leq R^S_{g^2} \leq R^S_{g^1} + l^S_p,\ R^L_{g^1} \leq R^L_{g^2} \leq R^L_{g^1} + l^L_p \tag{5}$$

$$x_{pg^2t} \geq y^S_{pg^1t}, \qquad \forall t \in T,\ p \in P_i,\ g^1, g^2 \in G : i \in T_t,\ R^S_{g^1} \leq R^S_{g^2} \leq R^S_{g^1} + l^L_p,\ R^L_{g^1} \leq R^L_{g^2} \leq R^L_{g^1} + l^S_p \tag{6}$$

$$y^S_{pgt}, y^L_{pgt} \in \{0,1\}, \qquad p \in P, g \in G, t \in T \tag{7}$$

$$x_{pgt} \in \{0,1\}, \qquad p \in P, g \in G, t \in T \tag{8}$$

Constraints (3) assign each pallet of a customer to a grid square in a selected tour visiting that customer and the orientation of the placement to the grid square should be selected among one of the two options mentioned in Section 3. Constraints (4) prevent assigning grid squares to multiple pallets or pallets to unselected tours. Constraints (5) and (6) ensure that once the location and orientation of the reference point of a pallet is decided, all the grid squares within the rectangle of that pallet are also assigned to the same pallet by taking into account its reference point and the orientation of placement.

Blocking constraints:

$$x_{pg^2t} + x_{p^1g^1t} \leq 1, \qquad \forall g^1, g^2 \in G,\ t \in T,\ i \in T_t,$$

$$x_{p^2 g^2 t} + x_{p^1 g^1 t} \leq 1, \qquad \begin{array}{l} p^1 \in P_i,\ p \notin P_i : pre_t(i) = 0, \\ R^S_{g^1} = R^S_{g^2}, R^L_{g^1} \leq R^L_{g^2} \end{array} \tag{9}$$

$$\begin{array}{l} \forall g^1, g^2 \in G,\ t \in T,\ i, j \in T^{fol}_t(0), \\ p^1 \in P_i,\ p^2 \in P_j : pre_t(j) = i, \\ R^S_{g^1} = R^S_{g^2}, R^L_{g^1} \leq R^L_{g^2} \end{array} \tag{10}$$

Constraints (9) and (10) make sure that the pallets of a customer are not blocked by the load of other customers that will be visited afterwards.

Stability constraints: We define the following intermediary decision variables to show how we construct our axle constraints; however, we will not need these variables in our model:

CG^L_{it}: the center of gravity on the long axis of the vehicle of $t \in T$ when leaving node $i \in N^0$.

f^1_{it}: the force on the axles of the tractor when leaving node $i \in N^0$ with tour $t \in T$.

f^2_{it}: the force on the axles of the semi-trailer when leaving node $i \in N^0$ with tour $t \in T$.

As also indicated by [9], CG^L_{it}, f^1_{it}, and f^2_{it} can be expressed as follows:

$$CG^L_{it} = \frac{\sum\limits_{g \in G} C^L_g \sum\limits_{j \in T^{fol}_t(i)} \sum\limits_{p \in P_j} w^G_p x_{gpt}}{W_{it}} \qquad \forall t \in T, i \in T_t \tag{11}$$

$$f^1_{it} = \frac{(CG^L_{it} - D_1)}{D_2} W_{it} \qquad \forall t \in T, i \in T_t \tag{12}$$

$$f^2_{it} = W_{it} - f^1_{it} \qquad \forall t \in T, i \in T_t \tag{13}$$

Then, the axle constraints, which ensure that the total weight on the axles are within the limits, can be written as in (14) and (15).

$$F^{min}_1 z_t \leq f^1_{it} \leq F^{max}_1 z_t \qquad \forall t \in T, i \in T_t \tag{14}$$

$$F^{min}_2 z_t \leq f^2_{it} \leq F^{max}_2 z_t \qquad \forall t \in T, i \in T_t \tag{15}$$

By Constraints (11) and (12), we obtain Constraints (16) below:

$$f^1_{it} = \frac{\sum\limits_{g \in G} C^L_g \sum\limits_{j \in T^{fol}_t(i)} \sum\limits_{p \in P_j} w^G_p x_{pgt} - D_1 W_{it}}{D_2} \qquad \forall t \in T, i \in T_t \tag{16}$$

Finally, we can replace Constraints (14) and (15) with Constraints (17) and (18) below:

$$F^{min}_1 z_t \leq \frac{\sum\limits_{g \in G} C^L_g \sum\limits_{j \in T^{fol}_t(i)} \sum\limits_{p \in P_j} w^G_p x_{pgt} - D_1 W_{it}}{D_2} \leq F^{max}_1 z_t, \qquad \forall t \in T, i \in T_t \tag{17}$$

$$F^{min}_2 z_t \leq W_{it} - \frac{\sum\limits_{g \in G} C^L_g \sum\limits_{j \in T^{fol}_t(i)} \sum\limits_{p \in P_j} w^G_p x_{pgt} - D_1 W_{it}}{D_2} \leq F^{max}_2 z_t, \qquad \forall t \in T, i \in T_t \tag{18}$$

After all aforementioned modifications are done, we can present our compact formulation as follows:

$$\min \quad \sum_{t \in T} c^r_t z_t \tag{19}$$

$$\text{s.t.} \quad (1) - (10), (17), (18)$$

where the objective function (19) minimizes the total routing cost.

5 Conclusion

In this paper, we study a generalization of the problem introduced by [9] focusing on loading and cargo safety constraints in vehicle routing problems. We model this problem as an integer programming formulation by using the set-partitioning constraints for the routing restrictions which lead to further simplifications of the loading and cargo stability constraints.

Currently, we are working on an extension of this problem where additional security restrictions are integrated. These include the dynamic stability restrictions, which take into account the force applied to the walls of the vehicle and avoidance of empty spaces between the pallets and at the borders (walls) of the vehicle. The goal of our study is to provide a detailed analysis of how safety restrictions affect the routing and loading decisions through the experiments to be conducted on real data sets via exact and heuristic methods.

References

[1] Bortfeldt, A. (2012). A hybrid algorithm for the capacitated vehicle routing problem with three-dimensional loading constraints. *Computers & Operations Research*, 39(9):2248–2257.

[2] Bortfeldt, A. and Wäscher, G. (2013). Constraints in container loading – A state-of-the-art review. *European Journal of Operational Research*, 229(1):1–20.

[3] Ceschia, S. and Schaerf, A. (2013). Local search for a multi-drop multi-container loading problem. *Journal of Heuristics*, 19:275–294.

[4] Gendreau, M., Iori, M., Laporte, G., and Martello, S. (2006). A Tabu Search Algorithm for a Routing and Container Loading Problem. *Transportation Science*, 40(3):342–350.

[5] Iori, M., Salazar-González, J.-J., and Vigo, D. (2007). An exact approach for the vehicle routing problem with two-dimensional loading constraints. *Transportation Science*, 41(2):253–264.

[6] Junqueira, L., Morabito, R., and Sato Yamashita, D. (2012). MIP-based approaches for the container loading problem with multi-drop constraints. *Annals of Operations Research*, 199:51–75.

[7] Juwet, M., Esprit, E., and Vanden Berghe, G. (2018). On horizontal dynamic effects on palletized goods during road transport. *Packaging Technology and Science*, 31(5):310–330.

[8] Moura, A. and Oliveira, J. F. (2009). An integrated approach to the vehicle routing and container loading problems. *OR Spectrum*, 31:775–800.

[9] Pollaris, H. (2017). *Loading constraints in vehicle routing problems: a focus on axle weight limits*. PhD thesis, University of Hasselt.

[10] Pollaris, H., Braekers, K., Caris, A., Janssens, G. K., and Limbourg, S. (2016). Capacitated vehicle routing problem with sequence-based pallet loading and axle weight constraints. *EURO Journal on Transportation and Logistics*, 5(2):231–255.

[11] Ramos, a. G. a., Oliveira, J. F., Gonçalves, J. F., and Lopes, M. P. (2015). Dynamic stability metrics for the container loading problem. *Transportation Research Part C: Emerging Technologies*, 60:480 – 497.