Optimization-based state estimation for perimeter controlled large-scale urban road networks

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INTRODUCTION

Modeling, estimation, and control of large-scale urban traffic networks present considerable challenges. Inadequate infrastructure and coordination, low sensor coverage, spatiotemporal propagation of congestion, and the uncertainty in traveler choices contribute to the difficulties faced when creating realistic models and designing effective traffic estimation and control schemes for urban networks. Although considerable research has been directed towards designing efficient real-time traffic management schemes in the last decades, estimation and control of heterogeneously congested large-scale urban networks remains a challenging problem.

Traffic modeling and control studies for urban networks usually focus on microscopic models keeping track of link-level traffic dynamics with control strategies using local information. Based on the linear-quadratic regulator (LQR) problem, traffic-responsive urban control (TUC) (Diakaki et al., 2002) and its extensions (Aboudolas et al., 2010; Kouvelas et al., 2011) represent a multivariable feedback regulator approach for network-wide urban traffic control. Although TUC can deal with oversaturated conditions via minimizing and balancing the relative occupancies of network links, it may not be optimal for heterogeneous networks with multiple pockets of congestion. Inspired by the max pressure routing scheme for wireless networks, many local traffic control schemes have been proposed for networks of signalized intersections (see Kouvelas et al. (2014); Varaiya (2013); Wongpiromsarn et al. (2012); Zaidi et al. (2015)), which involve evaluations at each intersection requiring information exclusively from adjacent links. Although the high accuracy of microscopic traffic models is desirable for simulation purposes, the increased model complexity results in complications for control, whereas local control strategies might not be able to operate properly under heavily congested conditions and fast propagation, as they do not protect the congested regions upstream. Another disadvantage of sophisticated local controllers is that they might require detailed information on traffic states, which is difficult to estimate or measure.

An alternative to traditional local real-time traffic control methods is the two layer hierarchical control approach. At the upper layer, the network-level controller optimizes network performance via regulating macroscopic traffic flows through interregional actuation systems (e.g., perimeter control), whereas at the lower layer the local controllers regulate microscopic traffic movements through intraregional actuation systems (e.g., signalized intersections). The macroscopic fundamental diagram (MFD) of urban traffic is a modeling tool for developing low complexity aggregated dynamic models of urban networks, which are required for the design of efficient network-level control schemes for the upper layer. It is possible to model an urban region with roughly homogeneous accumulation (i.e., small spatial link density heterogeneity) with an MFD, which provides a unimodal, low-scatter, and demand-insensitive relationship between accumulation and trip completion flow (Geroliminis and Daganzo, 2008).

The concept of MFD with an optimum accumulation was first proposed by Godfrey (1969), and its existence was recently verified with dynamic features and real data by Geroliminis and Daganzo (2008). Control strategies based on MFD modeling and using perimeter control type actuation (i.e., manipulating transfer flows between neighboring regions) have been proposed by many researchers for single-region (Daganzo, 2007; Gayah et al., 2014; Haddad, 2017a; Haddad and Shraiber, 2014; Keyvan-Ekbatani et al., 2012) and multi-region (Aboudolas and Geroliminis, 2013; Ding et al., 2017; Fu et al., 2017; Haddad, 2017b; Haddad and Geroliminis, 2012; Haddad and Zheng, 2018; Kouvelas et al., 2017; Zhong et al., 2017) urban areas. Application of the MPC technique to the control of urban networks with MFD modeling also attracted recent interest. Geroliminis et al. (2013) design a nonlinear MPC for a simple two-region urban network equipped with a perimeter control system. Hajiahmadi et al. (2015) generalize the two-region MFD network model of Geroliminis et al. (2013) to that of an R-region network, and propose hybrid MPC schemes for an urban network equipped with both perimeter control systems and switching signal timing plans. Ramezani et al. (2015) develop a model capturing the dynamics of heterogeneity and design a hierarchical control system with MPC on the upper level. An integration of perimeter control and route guidance type actuation within an MPC framework is proposed in (Sirmatel and Geroliminis, 2018). More detailed literature reviews of MFD-based modeling and control can be found in Saberi and Mahmassani (2012) and Yildirimoglu et al. (2015).

Although there is considerable literature on traffic state estimation (especially for freeway networks), combined estimation and control for heterogeneously congested large-scale urban networks remains an open problem. In this paper we propose integrated traffic management schemes involving optimization-based estimation and control for perimeter controlled urban networks with MFD-based modeling.

MODELING OF URBAN NETWORKS

Consider an urban network $\mathcal{R}$ with heterogeneously distributed accumulation consisting of R homogeneous regions, i.e., $\mathcal{R} = \{1, \ldots, R\}$, with each region $i \in \mathcal{R}$ having a well-defined production MFD $P_i(n_i(t))$ (veh.m/s) expressing the production at accumulation $n_i(t)$ (veh). A network consisting of 3 regions is schematically shown in figure 1. The demand for trips in region i with destination j is $q_{ij}(t)$ (veh/s), whereas $n_{ij}(t)$ (veh) is the accumulation in region i with destination region j, and $n_i(t)$ (veh) is the total accumulation in region i, at time t; $i, j \in \mathcal{R}$; $n_i(t) \triangleq \sum_{j\in\mathcal{R}} n_{ij}(t)$. Between each pair of neighboring regions i and h (with h belonging to the set of neighbors of i, i.e., $h \in \mathcal{N}_i$) there exists perimeter control actuators, modeled via the control input $u_{ih}(t) \in [u_{\text{min}}, u_{\text{max}}]$, with $0 \leq u_{\text{min}} < u_{\text{max}} \leq 1$, that can restrict the vehicle flow transferring from region i to region h. Moreover, the regional route choice decisions of the drivers are modeled via route choice terms $\theta_{ihj}(t) \in [0, 1]$, expressing the percentage of drivers in region i with destionation j deciding to transfer to region h, with $\sum_{j\in\mathcal{R}\setminus\{i\}} \theta_{ihj}(t) = 1$. The dynamics of

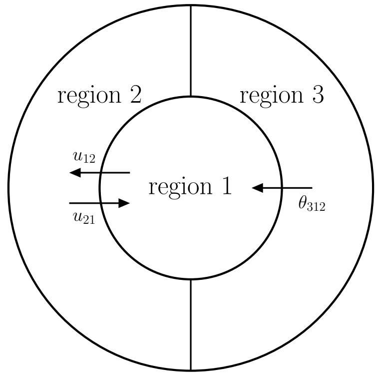

FIGURE 1: Schematic of an urban network with 3 regions.

the R-region MFDs network is (Ramezani et al., 2015):

$$\dot{n}_{ii}(t) = q_{ii}(t) + \sum_{h\in\mathcal{N}_i} u_{hi}(t)M_{hii}(t) - M_{ii}(t) \tag{1a}$$

$$\dot{n}_{ij}(t) = q_{ij}(t) + \sum_{h\in\mathcal{N}_i\setminus\{j\}} u_{hi}(t)M_{hij}(t) - \sum_{h\in\mathcal{N}_i} u_{ih}(t)M_{ihj}(t), \tag{1b}$$

where $M_{ii}(t)$ (veh/s) is the exit (i.e., internal trip completion) flow that is defined as:

$$M_{ii}(t) = \frac{n_{ii}(t)}{n_i(t)}\frac{P_i(n_i(t))}{l_{ii}} \quad \forall i \in \mathcal{R}, \tag{2}$$

while $M_{ihj}(t)$ (veh/s) is the flow transferring from region i to h with destination j:

$$M_{ihj}(t) = \theta_{ihj}\frac{n_{ij}(t)}{n_i(t)}\frac{P_i(n_i(t))}{l_{ij}} \quad \forall i \in \mathcal{R},\ \forall h \in \mathcal{N}_i,\ \forall j \in \mathcal{R}\setminus\{i\}. \tag{3}$$

Trips inside a region are assumed to have similar trip lengths (i.e., the origin and destination of the trip does not affect the distance traveled by a vehicle). Simulation and empirical results (Geroliminis and Daganzo, 2008) suggest the possibility of approximating the MFD by an asymmetric unimodal curve skewed to the right (i.e., the critical accumulation $n_{i,\text{cr}}$, for which the production $P_i(n_i(t))$ is at its maximum, is less than half of the jam accumulation $n_{i,\text{jam}}$ that puts the region in gridlock). Thus, $P_i(n_i(t))$ can be expressed using a third-degree polynomial in $n_i(t)$:

$$P_i(n_i(t)) = a_i n_i^3(t) + b_i n_i^2(t) + c_i n_i(t), \tag{4}$$

where a_i, b_i, and c_i are MFD parameters that can be extracted from data.

Note here that the route choice behaviour of the drivers can also be considered with this dynamical model. In simulations, routing can be captured by a logit model (see (Ben-Akiva and Bierlaire, 1999)), where the route choice terms θ_{ihj} are calculated based on the current travel times from region i to destination j through a fixed number of shortest regional paths (i.e., sequences of regions) connecting the two. In general, the set of regional paths can be calculated using Dijkstra's algorithm for K-shortest paths for complicated network topologies (see, e.g., (Sirmatel and

Geroliminis, 2018)). The θ_{ihj} values are updated at each time step of the simulation by the logit model, to model real-time driver adaptation to traffic conditions. The assumption that the physical shortest path is always chosen by the drivers is relaxed through the logit model. Thus, more realistic simulations are obtained, as in reality drivers rarely have access to perfect information and do not always behave rationally. The information available to drivers or their sensitivity to travel time differences between routes can be reflected in simulations by adjusting the parameters of the logit model.

Although there exists empirical evidence about its validity via aggregated data (e.g., Geroliminis and Daganzo (2008)), the MFD should not be considered as a universal law. Strong demand fluctuations, for example, can trigger fast evolving transients, affecting the trip length distribution in a region at a specific time. This can potentially lead to inaccuracies in the approximation of outflow as the ratio of production over trip length. Although, for a range of cases, this can be a valid assumption, further research is required to examine the conditions requiring modeling via more complex dynamics with delays (see, e.g., some analysis in Lamotte and Geroliminis (2017)), which can be a research priority.

OPTIMAL ESTIMATION AND CONTROL OF LARGE-SCALE URBAN NETWORKS

Moving Horizon Estimation

We formulate the problem of finding the w_{ij} values that yield a state estimate striking a trade-off between measurements and the prediction model, for a moving time horizon extending a fixed length into the past, as the following discrete time nonlinear MHE problem:

$$
\begin{aligned}
\underset{w_k}{\text{minimize}} \quad & \sum_{k=0}^{N_\mathrm{e}-1} \|w_k\|_Q^2 + \sum_{k=0}^{N_\mathrm{e}} \|v_k\|_R^2 \\
\text{subject to} \quad & \text{for } k = 0, \ldots, N_\mathrm{e}: \\
& \quad v_k \triangleq y(t - N_\mathrm{e} + k) - H(n_k, u_k) \\
& \text{for } k = 0, \ldots, N_\mathrm{e} - 1: \\
& \quad n_{k+1} = F(n_k, u(t - N_\mathrm{e} + k), \hat{\theta}(t - N_\mathrm{e} + k), d(t - N_\mathrm{e} + k), w_k, T) \\
& \text{for } k = 1, \ldots, N_\mathrm{e}: \\
& \quad 0 \leq n_{i,k} \leq n_{i,\mathrm{jam}},
\end{aligned}
\tag{5}
$$

where k is the time interval counter internal to the MHE, N_e is the horizon of the MHE, t is the current time step, Q and R are weighting matrices on the demand and measurement noise, respectively, w_k, v_k, and n_k are vectors containing the variables $w_{ij,k}$, $v_{ij,k}$, and $n_{ij,k}$ for all $i, j \in \mathcal{R}$, expressing the demand noise, measurement noise, and accumulation state variables internal to the MHE, respectively, H is the measurement equation (i.e., a function expressing how the measurements depend on the state variables and control inputs), F is the discrete-time version of the dynamics given in equation (1) with sampling time T, whereas $y(t)$, $u(t)$, $\hat{\theta}(t)$, and $d(t)$ are vectors containing the variables $y_{ij}(t)$, $u_{ih}(t)$, $\hat{\theta}_{ihj}(t)$, and $d_{ij}(t)$ for all $i, j \in \mathcal{R}$, $h \in \mathcal{N}_i$, expressing the measurement, perimeter control input, route choice measurement, and known inflow demand terms recorded at time step t, respectively, while $n_{i,k}$ is the accumulation state of region i internal to the MHE, with $n_{i,k} \triangleq \sum_{j \in \mathcal{R}} n_{ij,k}$. Note that the arguments of F reflect the information available to the MHE, which is assumed to have access to the recorded values of $y(t)$, $u(t)$, $\hat{\theta}(t)$, and $d(t)$ for a finite horizon of length N_e into the past.

Model Predictive Control

We formulate the problem of finding the $u_{ih}(t)$ values that minimize total time spent (TTS) for a finite horizon as the following discrete-time economic nonlinear MPC problem:

$$
\begin{aligned}
\underset{u_k}{\text{minimize}} \quad & \sum_{k=0}^{N_\text{c}} \sum_{j\in\mathcal{R}} n_{i,k} \\
\text{subject to} \quad & n_0 = \tilde{n}(t) \\
& \text{for } k = 0, \dots, N_\text{c} - 1 : \\
& \quad n_{k+1} = F(n_k, u_k, \hat{\theta}(t), d(t+k), 0, T) \\
& \quad u_{\min} \le u_k \le u_{\max} \\
& \quad |u_0 - u(t-1)| \le \Delta_u \\
& \text{for } k = 1, \dots, N_\text{c} : \\
& \quad 0 \le n_{i,k} \le n_{i,\text{jam}},
\end{aligned}
\tag{6}
$$

where k is the time interval counter internal to the MPC, N_c is the horizon of the MPC (i.e., the prediction horizon), $\tilde{n}(t)$ is the available information (e.g., either the measured or estimated value) on the accumulation state $n(t)$ at time t (with t being the current time step), u_k and n_k are vectors containing the variables $u_{ih,k}$ and $n_{ij,k}$ for all $i, j \in \mathcal{R}$, $h \in \mathcal{N}_i$, expressing the perimeter control inputs and accumulation state variables internal to the MPC, respectively, F is the discrete-time version of the dynamics given in equation (1) with sampling time T, $\hat{\theta}(t)$ and $d(t)$ are vectors containing the variables $\hat{\theta}_{ihj}(t)$ and $d_{ij}(t)$ for all $i, j \in \mathcal{R}$, $h \in \mathcal{N}_i$, expressing the route choice measurement and known inflow demand terms recorded at time step t, respectively, Δ_u is the rate limiting parameter on perimeter control inputs, whereas $n_{i,k}$ is the accumulation state of region i internal to the MPC, with $n_{i,k} \triangleq \sum_{j\in\mathcal{R}} n_{ij,k}$. Note that the arguments of F are different for MPC and MHE: In MPC the prediction is from the current time t into the future (while for MHE the horizon extends from t into the past), thus the MPC uses future values of known inflow demands $d(t)$, while the uncertainty present in these demands (i.e., the demand noise $w(t)$) is unknown for the future and is thus fixed to zero. The route choice terms $\theta(t)$ are assumed to be fixed to their value measured at the current time step t for the horizon. This can be viewed as a crude approximation as the route choice decisions of the drivers are expected to change within this horizon. Nevertheless, modeling the dynamics of route choice are considered outside the scope of this paper, and it might further complicate the already non-linear formulation of MFD dynamics.

The optimization problems given in equation (5) and equation (6) are nonconvex nonlinear programs (NLPs), which can be solved efficiently using solvers based on sequential quadratic programming or interior point methods (for details, see (Diehl et al., 2009)).

Integrated Moving Horizon Estimation and Model Predictive Control

For the combined accumulation state estimation and perimeter control of large-scale urban networks with regional route choice, we propose a structure integrating the MHE and MPC schemes given in equations (5) and (6) (see figure 2 and figure 3). This combined scheme operates in the following manner: Given the past information the MHE computes the accumulation state estimate $\tilde{n}_{ij}(t)$, which is communicated to the MPC. Then, given the estimate $\tilde{n}_{ij}(t)$, the MPC computes the optimal perimeter control inputs $u_{ih}(t)$, which is then applied to the urban network. The whole procedure (in receding horizon fashion) is repeated in the next time step. Operation of the combined MHE-MPC scheme can be formalized as given in algorithm 1.

Algorithm 1 Operation of the combined MHE-MPC scheme.

At time $t = 1$, initialize simulation from initial accumulation state $n(1)$. Then, at each time step t:

1) Given the accumulation state $n(t)$, calculate the route choice terms $\theta(t)$ using the k-shortest path algorithm together with the logit model.
2) Given the records (for a finite horizon of length N_{e} into the past) of measurements $\{y(t - N_{\mathrm{e}}), \ldots, y(t)\}$, perimeter control inputs $\{u(t-N_{\mathrm{e}}), \ldots, u(t-1)\}$, measured route choice terms $\{\hat{\theta}(t - N_{\mathrm{e}}), \ldots, \hat{\theta}(t - 1)\}$, and the known average inflow demands $\{d(t - N_{\mathrm{e}}), \ldots, d(t - 1)\}$, solve the MHE problem (5) to obtain the state estimates $\{\tilde{n}(t - N_{\mathrm{e}}), \ldots, \tilde{n}(t)\}$ for the finite horizon into the past.
3) Given the last element $\tilde{n}(t)$ of the state estimates (computed by MHE), measured route choice term $\hat{\theta}(t)$, and the known average inflow demands $\{d(t), \ldots, d(t + N_{\mathrm{c}} - 1)\}$, solve the MPC problem (6) to obtain the perimeter control inputs $\{u(t), \ldots, u(t+N_{\mathrm{c}}-1)\}$ (for a finite horizon of length N_{c} into the future).
4) Given current state $n(t)$, inflow demand $d(t)$, route choice terms $\theta(t)$, and the first element $u(t)$ of the perimeter control inputs (computed by MPC), evolve system dynamics by evaluating the discrete-time versions of the differential equations given in equation (1) to obtain $n(t + 1)$.

Repeat steps 1, 2, 3 and 4 for $t = 1, \ldots, t_{\text{final}}$.

CONCLUSION

In this paper we proposed the formulations of a combined MHE-MPC scheme for real-time optimization based estimation and control of large-scale urban road networks using dynamical models based on the macroscopic fundamental diagram of urban traffic.

The full paper will include two measurement configurations, one with measuring directly the accumulations $n_{ij}(t)$, the other one with measuring regional accumulations and transfer flows, which is easier to achieve in practice. Furthermore, extensive simulation experiments will be provided to test whether the MHE scheme is able to accurately estimate the accumulation states, even in the face of high levels of measurement noise. The results will shed light on the applicability of the proposed combined MHE-MPC scheme for practical applications where keeping good control performance in the face of highly noisy measurements is of critical importance to the success of the large-scale urban traffic management systems.

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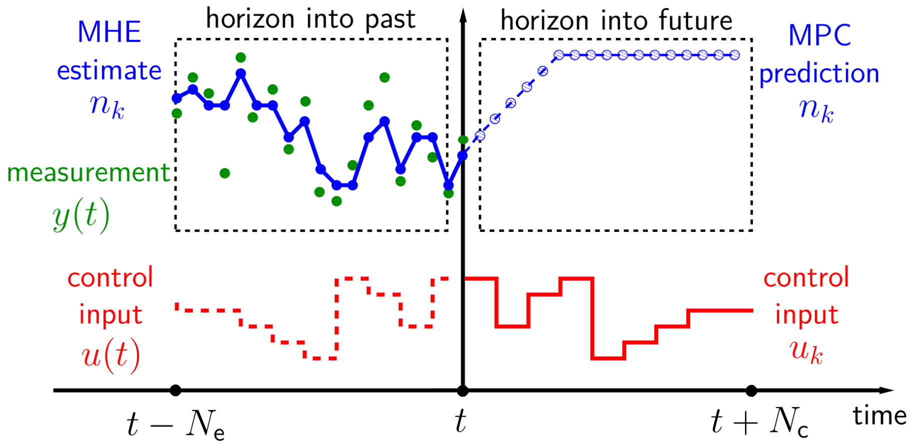

FIGURE 2: Operation procedure of combined MHE-MPC scheme.

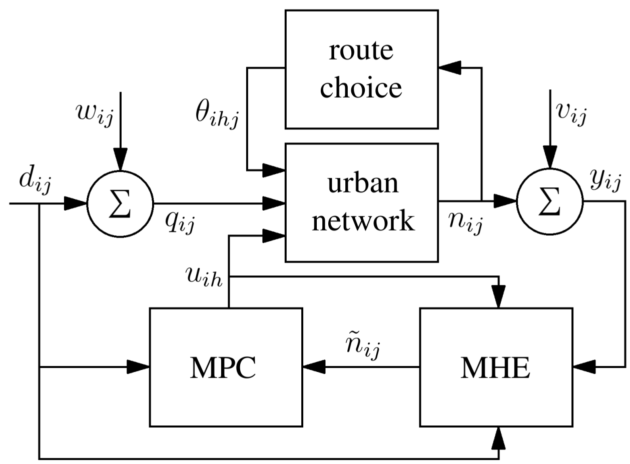

FIGURE 3: Architecture of combined MHE-MPC scheme.

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