# Regional dynamic traffic assignment framework for multi-reservoir MFD models

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## Extended Abstract

The first ideas of an aggregated traffic modeling were introduced by Godfrey (1969) and later revisited by Daganzo (2007) and Geroliminis & Daganzo (2008). For this type of modeling, the city is divided into reservoirs (Fig. 1) where the traffic conditions are approximately homogeneous and are given by a well-defined low scatter Macroscopic Fundamental Diagram (MFD). The MFD is a relationship between the aggregated flow $q$ and aggregated density $k$. The traffic dynamic inside a reservoir is governed by the evolution of the vehicles accumulation $n$ at a given time instant $t$ Daganzo (2007):

$$\frac{dn}{dt} = Q_{in} - Q_{out}, t > 0 \tag{1}$$

where $Q_{in}$ and $Q_{out}$ are the inflow and outflow functions, respectively.

Depending on the assumptions made on $Q_{out}$, two MFD-based models can be distinguished in the literature: the accumulation-based (Daganzo, 2007); and the trip-based (Arnott, 2013, Lamotte & Geroliminis, 2016, Mariotte et al., 2017). In this work, we focus our attention on the trip-based model, where the MFD dynamics is entirely described by the vehicle trip length $L$:

$$L = \int_{t-T(t)}^{t} V(n(s))ds \tag{2}$$

where $T(t)$ is the travel time of the vehicle exiting at $t$. Our numerical implementation of this model is based on Mariotte et al. (2017).

One key ingredient for a proper trip-based MFD simulation is a decent estimation of trip lengths. In fact, different vehicles might travel different distances inside the same reservoir. In Fig. 1 (a), we show an example of a microscopic network where there are represented three microscopic trips. A microscopic trip is

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defined by a sequence of links. Two microscopic trips are correlated if they share links in common. This is represented by the red links shown in Fig. 1 (a). But, when we scale from microscopic to the regional network, we no longer have access to the topology information of the real network (Fig. 1 (b)). We focus instead on the set of reservoirs that define the regional network as shown in Fig. 1 (b). In the regional network, microscopic origin-destination (od) pairs correspond to macroscopic Origin-Destination ($OD$) reservoirs; and each microscopic trip corresponds to a regional path. A regional path is defined by the sequence of crossed reservoirs from O to D. In Fig. 1 (a), the green microscopic trips define the same regional path. But, they have different trip lengths inside each reservoir that is crossed. In Fig. 1 (b), the green and blue regional paths are correlated since they cross two reservoirs in common. In Fig. 1 (c), we show a zoom of the gray reservoir with a well-defined MFD function. We denote $L_1$ and $L_2$ as the trip lengths of the green and blue regional paths inside the gray reservoir, respectively. There are two main differences between the correlation on the microscopic network and the one on the regional network. First, the correlation between the green and blue regional paths inside the gray reservoir are captured by the homogeneous speed assumption of the MFD-based models. That is, a vehicle that enters the gray reservoir and is traveling on the blue regional path, will automatically affect the travel times of all vehicles inside this reservoir, independent of the regional path they are traveling. Second, a regional path is characterized by a distribution of trip lengths inside each reservoir that it crosses. This is because one regional path can be defined by several microscopic trips with distinct trip lengths.

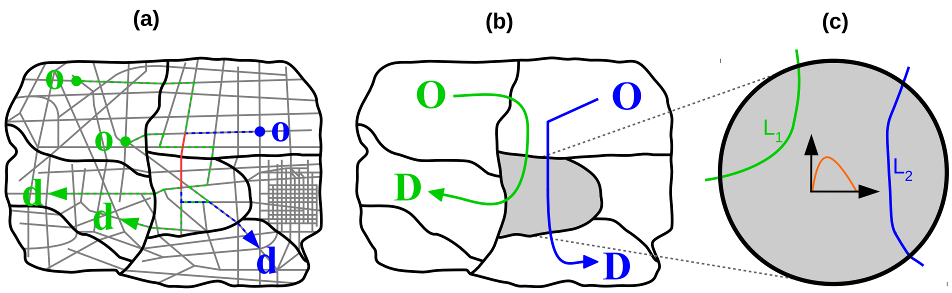


Fig. 1 – *(a) Microscopic network with three microscopic routes. Two of these microscopic routes are correlated as shown by the red links. The macroscopic network is also represented. (b) Macroscopic network with the corresponding macro-paths to the green and blue microscopic routes. (c) Gray reservoir with a well-defined MFD function, that is crossed by the green and blue macro-paths, each with a corresponding trip length* $L_1$ *and* $L_2$*.*

Few attention has been paid to the dynamic traffic assignment on the MFD-based models context, i.e. the characterization of regional paths and the calculation of the path flow distributions. Up to the authors best knowledge, the most advanced dynamic traffic assignment framework for the MFD-based models is discussed by Yildirimoglu & Geroliminis (2014) (for the Stochastic User Equilibrium) and Yildirimoglu et al. (2015) (for the System Optimum). Yildirimoglu & Geroliminis (2014) discuss a regional dynamic traffic assignment framework that considers trip lengths that are explicitly calculated in an iterative way. The stochastic network loading is based on the Multinomial Logit formulation.

In this work, we discuss a methodology for a regional-based dynamic traffic assignment that accounts for explicitly calculated trip lengths, to solve the Deterministic and Stochastic User Equilibrium (DUE and SUE). For this, we consider the utility function defined as:

$$\begin{aligned} U_p^{OD} &= \sum_{r \in Y} \left( \frac{L_{rp}}{V_r(n_r)} \right) \delta_{rp} \\ &\approx \sum_{r \in Y} \left( \frac{\overline{L_{rp}}}{\overline{V_r(n_r)}} + \epsilon_r(L_{rp}, V_r(n_r)) \right) \delta_{rp}, \forall p \in \Omega^{OD} \land \forall (O,D) \in W \end{aligned} \tag{3}$$

where $L_{rp}$ is the set of trip lengths of regional path $p$ inside reservoir $r$; $V_r(n_r)$ is the speed-MFD function of reservoir $r$; $\delta_{rp}$ is a dummy variable that equals 1 if regional path $p$ crosses reservoir $r$; $Y$ is the set of reservoirs that define the regional network; $\Omega^{OD}$ is the macroscopic choice set for the macroscopic origin-destination (OD) pair; and $W$ is the set of all macroscopic $OD$ pairs of the regional network. In Eq. 3, the term $\frac{\overline{L_{rp}}}{\overline{V_r(n_r)}}$ defines the deterministic part of the utility function. While, the term $\epsilon_r(L_{rp}, V_r(n_r))$ is the error term that depends on:

- $L_{rp}$ that defines the distribution of trip lengths of regional path $p$ inside reservoir $r$. Batista et al. (2018) investigates four approaches to properly define $L_{rp}$ based on a set $\Gamma$ of microscopic trips. The authors propose to filter these microscopic trips based on different levels of information of the regional network. We follow one of the approaches discussed in Batista et al. (2018), that considers a level of filtering at the regional path level. That is, all microscopic routes that travel inside a given reservoir and that define the same regional path $p$ are considered to define $L_{rp}$.

- $v_r(n(t))$ is the speed-MFD function and yields the users the different perception of the traffic states inside reservoir $r$.

Note that, for the DUE, $\epsilon_r(L_{rp}, V_r(n_r))$ is set to 0.

To calculate the network equilibrium, accounting for these two uncertainty terms, we consider Monte Carlo simulations (Sheffi, 1985). Both the DUE and SUE are formulated as fixed-point problems and are solved based on the Method of Successive Averages. We choose a $\frac{1}{k}$ descent step to ensure the good algorithm convergence. The idea of the Monte Carlo simulations is to sample trip lengths directly from $L_{rp}$ and mean speeds from $v_r(n(t))$. We perform a large number $(M)$ of samples. For each one, we solve deterministic problems where users minimize their perceived utility. The final choices, for each MSA descent loop, correspond to the averaging of all these local deterministic choices. As convergence criterion, we consider the Gap function and the number of violations (Sbayti et al., 2007). A maximum number of descent steps $N_{max}$ is also fixed.

We discuss some preliminary results based on one implementation on a 1 reservoir network with entry queuing. The network is composed by 1 $OD$ and two regional paths (Fig. 2). For the MFD-trip based simulation, we consider a parabolic MFD function with: critical jam $n_{jam} = 1000$ veh; critical production

$P_{critical} = 3000$ veh.m/s; and free-flow speed $u = 15$ m/s. The demand level is: 0.3 veh/s between 0 and 1000 s; 1.0 veh/s between 1000 and 6000 s; and 0.3 veh/s between 6000 and 10000 s. For the convergence, we fix 0 for the maximum number of violations, a Gap tolerance of 0.01 and $N_{max} = 100$. For this test, we fix $L_2 = 1500$ m and vary $L_1 \in [1300, 1700]$ as shown in Table 1. For the SUE calculations, the trip lengths are sampled for the two regional paths following a normal distribution. For the sampling of trip lengths for regional path 2, we consider a normal distribution with mean 1500 m and standard deviation $\sigma_L$. While, for the case of regional path 1, we consider a normal distribution with mean value as listed in Table 1 and standard deviation $\sigma_L$. We consider three different values for $\sigma_L = 50, 100, 200$ m. The calculated regional path flows for the DUE and three SUE cases are listed in Table 1.

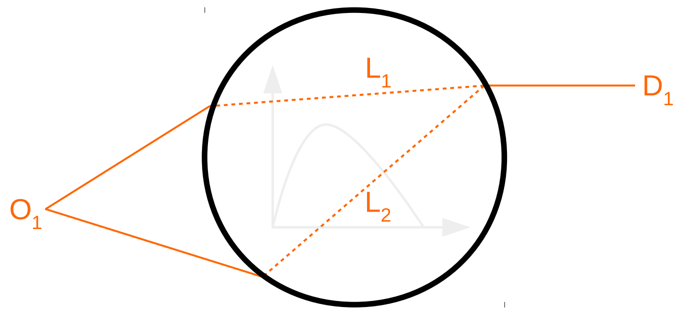


Fig. 2 – *One reservoir test network with one OD and two regional paths.*

We first analyze the DUE results, where $\epsilon_r(L_{rp}, V_r(n_r))$ is set to 0 (see Eq. 3). For $L_1 < L_2$, all users choose the minimum utility that corresponds to regional path 1, since the trip length is smaller. Note that the MFD dynamics is the same for both regional paths and the only term that affects the utility function are the trip lengths. For $L_1 = L_2 = 1500$ m, users equally choose regional paths 1 and 2. For $L_1 > L_2$, all users choose regional path 2, as expected.

We now analyze the SUE case, considering first $\sigma_L = 50$. Initially, for $L_1 = 1300$ m, all users choose the regional path 1. Note that, the distribution of trip lengths for regional path 1 follows a $N(1300, 50)$ and for regional path 2 follows a $N(1500, 50)$. Since the standard deviation is small, the values that are sampled for the trip length distribution of regional path 1 are always inferior to the ones for regional path 2. Thus, all users choose regional path 1. But, as $L_1$ increases, users start to also choose regional path 2. For $L_1 = L_2 = 1500$ m, users also equally choose regional paths 1 and 2, as expected. For $L_1 > 1500$ m, the fraction of users that choose regional path 2 increases. This happens until $L_1 = 1650$, when all users choose regional path 1. A similar trend for switching from regional path 1 to 2 as $L_1$ increases is also observed.

In the extended version of this work, we will consider this one reservoir MFD model with entry queuing to: (i) analyze the algorithm convergence for both DUE and SUE; (ii) compare the SUE results against the Multinomial Logit model; (iii) analyze the DUE and SUE results considering 2 $OD$ pairs. We will also analyze the implementation of this framework for both DUE and SUE calculations on a real network and considering a re-assignment procedure per periods during the

| $L_1$ | DUE | | SUE ($\sigma_L = 50$) | | SUE ($\sigma_L = 100$) | | SUE ($\sigma_L = 200$) | |
|---|---|---|---|---|---|---|---|---|
| | $Q_1$ | $Q_2$ | $Q_1$ | $Q_2$ | $Q_1$ | $Q_2$ | $Q_1$ | $Q_2$ |
| 1300 | 1.00 | 0.00 | 1.00 | 0.00 | 1.00 | 0.00 | 0.92 | 0.08 |
| 1350 | 1.00 | 0.00 | 1.00 | 0.00 | 0.98 | 0.02 | 0.85 | 0.15 |
| 1400 | 1.00 | 0.00 | 0.98 | 0.02 | 0.91 | 0.09 | 0.76 | 0.24 |
| 1425 | 1.00 | 0.00 | 0.94 | 0.06 | 0.84 | 0.16 | 0.69 | 0.31 |
| 1450 | 1.00 | 0.00 | 0.87 | 0.13 | 0.74 | 0.26 | 0.64 | 0.36 |
| 1475 | 1.00 | 0.00 | 0.73 | 0.27 | 0.64 | 0.36 | 0.57 | 0.43 |
| 1500 | 0.50 | 0.50 | 0.51 | 0.49 | 0.50 | 0.50 | 0.50 | 0.50 |
| 1525 | 0.00 | 1.00 | 0.28 | 0.72 | 0.38 | 0.62 | 0.43 | 0.57 |
| 1550 | 0.00 | 1.00 | 0.14 | 0.86 | 0.24 | 0.76 | 0.36 | 0.64 |
| 1575 | 0.00 | 1.00 | 0.07 | 0.93 | 0.17 | 0.83 | 0.30 | 0.70 |
| 1600 | 0.00 | 1.00 | 0.03 | 0.97 | 0.10 | 0.90 | 0.25 | 0.75 |
| 1650 | 0.00 | 1.00 | 0.00 | 1.00 | 0.03 | 0.97 | 0.15 | 0.85 |
| 1700 | 0.00 | 1.00 | 0.00 | 1.00 | 0.00 | 1.00 | 0.08 | 0.92 |

Tab. 1 – *Regional path flows for the DUE and SUE results and for the one reservoir MFD model with entry queuing. $Q_1$ and $Q_2$ are the flows of regional paths 1 and 2, respectively. Three values of $\sigma_L$ are considered. The trip length $L_2 = 1500$ m is kept constant, while $L_1$ is increased from 1300 to 1700 m.*

simulation. For this test, we will simulate a morning peak.


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