

# Cost Function for urban rail transport systems

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## 1. Introduction

Empirical analysis of transportation costs is indispensable both for transport operators and policy makers. Although the relevant literature is large and the theoretical relationship between costs and output well-understood, endogeneity issues in cost model estimation have received insufficient attention. This has led to inconsistencies between observed industry behaviour and the obtained estimates of traditional industry descriptors, (1) returns to scale (RTS) and (2) returns to density (RTD) (Basso et al., 2011).

In this research we extend developments in the complimentary production function literature to cost analysis to improve estimation of cost models for the urban rail transit industry. Recently considerable progress has been made in production function estimation techniques, which account for the fact that decisions taken by firms are based on their current level of productivity. This productivity term is partly endogenous and can create a downward bias in RTS estimates if not given due consideration (Collard-Wexler, 2012).

The aim of this paper is to use the state-of-the-art econometric methods to control for these unobserved productivities in cost function estimation and thus produce unbiased estimates of RTS and RTD for the urban rail transit industry.

## 2. Data and Methods

Because the cost function is the dual characterisation of the firm's production technology implies that all economically relevant variables embodied in a firm's production function should also be present in its cost function. For a two factor Cobb Douglas production technology, the cost function is the solution to the following cost minimization problem:

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$$\begin{aligned}
C(y, \mathbf{p}) &= \min_{x_1, x_2} p_1 x_1 + p_2 x_2 \\
&\text{such that} \\
y &= e^\omega x_1^\alpha x_2^\beta
\end{aligned} \tag{1}$$

where,  $y$  represents production technology and  $C$  represents cost function;  $x_1$  and  $x_2$  are the factors of production and  $p_1$  and  $p_2$  are corresponding factor prices.  $\alpha$  and  $\beta$  are constants.  $\omega$  stands for the unobserved productivity differences between firms.

The solution to equation (1) has the following form:

$$C(y, \mathbf{p}) = \alpha_0 + \frac{\alpha}{\alpha + \beta} \log p_1 + \frac{\beta}{\alpha + \beta} \log p_2 + \frac{1}{\alpha + \beta} \log y - \frac{1}{\alpha + \beta} \log \omega$$

We see that the unobserved productivity term ' $\omega$ ' ends up in the cost function. Since more productive firms are more likely to produce more output,  $C$  and  $\omega$  are negatively correlated. Thus in absence of  $\omega$ , the RTS estimates obtained from empirical cost analysis will have a downward bias. We use the following models to account for this unobserved productivity term in our cost analysis:

1. Fixed Effects Model: A fixed anticipated component of productivity is assumed for each firm.

$$\omega_{it} = \omega_i$$

2. Random Trend Model: Productivity is defined as a random walk with drift.

$$\omega_{it} = \omega_i + \delta_i t$$

3. Instrumenting Endogenous Variables: Lagged values of endogenous variables are used as their instruments.
4. Quasi- Differencing: Unobserved productivity is assumed to follow an auto-regressive AR(1) process.

$$\omega_{it} = \rho \omega_i^{t-1} + \sigma_{it}$$

We compare results from these different models. Some researchers argue that the use of firm level dummy variables as in (1) and (2) can bias the estimates of scale economies by capturing a part of the variation from key variables of the cost function.

For our empirical analysis, we make use of a unique panel dataset that has been collected by the Railway and Transport Strategy Centre (RTSC) at Imperial College London since 1994. The data relate to 34 urban rail operations around the world. This extensive panel data allows us to use better estimation techniques like system Generalised Method of Moments to estimate models in (3) and (4). This helps us overcome the econometric challenges associated with complete elimination of cross-sectional variation as encountered in previous research. We model the short-run variable costs of operation of transit using the widely adopted translog functional form in the transport cost analysis literature.

### 3. Results

The key results that emerge from our analysis are as follows:

1. A comparison of our models with the traditional cost model using pooled ordinary least squares estimation confirms that failure to account for unobserved productivity differences between firms in empirical cost analysis creates a downward bias in the obtained estimates of RTS and RTD.
2. We find evidence of very increasing RTD as found in the urban rail transit literature (Savage, 1997). This is because a range of fixed and semi-fixed costs are prevalent in the railway industry that do not vary proportionally with output.
3. We find evidence of increasing RTS which justifies the presence of large size firms in urban rail transit industry. The weight of evidence in the railway literature indicates that the industry is characterised by constant RTS. The observed industry behaviour with respect to network expansions has been argued to be an attempt to exploit economies of density. However, we find that controlling for endogeneity issues in empirical cost analysis gives RTS estimates that are consistent with observed industry behaviour. Recent railway cost studies also suggest cost complementarities between operational and way and track cost components. These studies find that scale economies are associated with vertical integration of operations and infrastructure maintenance (Basso et al., 2011). The RTSC dataset indicates that around eighty-percent of way and structure maintenance costs comprise of labour and electricity costs, which can be varied in the short-run. Since our analysis includes track maintenance costs as a component of variable costs in the short-run, this could be another reason why we find economies of scale.

### 4. Summary and Relevance

Overall, our results suggest that the urban rail transit industry is characterised by both increasing returns to scale and density. We also present evidence to support the existence of cost complementarities between way and structure maintenance and operational costs in railway industry. By controlling for endogeneity issues in traditional cost analysis we provide more reliable estimates of industry indices for transport investment appraisal and guiding decisions on pricing rules.

The presence of network size economies may be particularly relevant from a policy point of view, in case of the economic appraisal of large infrastructure projects that lead to network expansion. Returns to network size implies that such investments may generate external benefits in the form of a network-wide reduction in operational costs. As part of this research our aim is to quantify this external benefit and assess whether it could have significant impact on the outcome of traditional cost-benefit analyses.

## References

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