Auction-based Implementation of Traffic Services to Maximize Activity-based Social Welfare
Extended abstract

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1 Introduction

Activity-based approaches [Axhausen and Gärling, 1992, Kitamura, 1988, Recker, 2001] have recently begun to be applied to modeling user trip-chains that consider multimodal travel and multiple activities [Arentze and Timmermans, 2004, Liao et al., 2013]. If traffic resources are appropriately allocated to users based on this approach, the value of mobility and of cities implementing such mobility systems will greatly increase. In Fig. 1, we summarize the research on traffic assignment. Several works have considered traffic assignment without capacity constraints; for instance, stochastic traffic assignment in static situations is often considered using logit models, which can be extended to dynamic situations by considering users’ sequential decision-making with a recursive logit model [Fosgerau et al., 2013]. By contrast, Nie et al. [2004] proposed a traffic assignment problem with capacity constraints that considers the user equilibrium state on networks with link capacity constraints. Here, we investigate another approach for traffic assignment in such situations, in which a traffic operator can control users with incentives. In this approach, incentives are designed to lead to the user equilibrium state to coincide with the socially optimal state, meaning that the socially optimal state can be achieved by self-interested user behaviors. Based on this approach, Akamatsu and Wada [2017] proposed demand management based on a Vickrey–Clarke–Groves (VCG) mechanism [Clarke, 1971, Groves, 1973, Vickrey, 1961] for static conditions. The fundamental concept of this mechanism is the allocation of traffic resources to users based of system optimality, and charges users corresponding to the externalities the users provide to society.

In this paper, we apply this framework to activity-based traffic analysis in dynamic situations, examining sequential decision-making by traffic managers and users, and

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focusing on the time–space prism constraints of agents. A key motivating example of this setting is mobility as a service (MaaS), in which the operator allocates traffic resources to users with individual preferences and constraints. Specifically, this study makes the following contributions.

- We formalize an activity-chain auction and introduce an online mechanism that maximizes expected social welfare and offers an appropriate incentive design by which agents voluntarily reveal their preferences and constraints to the traffic operator.

- We propose a trajectory-based algorithm that computes an allocation that maximizes expected social welfare considering the time–space constraints of agents and resources’ capacity constraints.

- We numerically show that our proposed mechanism is superior to the common benchmark mechanism in terms of the efficiency of traffic resources’ allocation.

2 Activity-chain Auction

In this section, we introduce an activity-chain auction system implemented by a MaaS operator. In this system, agents must obtain a moving permit from the operator to make trips in the network. An illustrative example of agents considered in this section is CPUs implemented in personal automated vehicles or devices that people use to book a trip. To request the permit, each agent reports its preferences and constraints to the operator. Collecting these reports, the operator allocates traffic resources to each agent and collects payment from each agent. We consider a dynamic setting in which
the operator continuously accepts requests from agents and sequentially allocates traffic resources to each agent.

### 2.1 Activity-State Network with Normalized Time

We first introduce an activity-state network, which is a space–time expanded network with normalized time. All spatial links are divided into multiple segments depending on the link cost. All nodes in the activity-state network express the state includes location, mode of transportation, and so on, and all links in the network express the state transition at each time. Each agent takes a series of actions in the network, keeping its time–space prism constraint, as shown in Figure 2 and receives rewards from the actions. Using this network, we can consider behaviors such as detours and stopovers during the trip with considering the time constraints $T_i$ of agents, as shown in Figure 3. Although this network cannot treat congestion, it is based on a general transition system and can appropriately express utilities over users’ entire trip-chain. In this paper, we focus on the sequential state transition rather than the congestion, by introducing auction-based systems.

### 2.2 System model of the activity-chain auction

We consider a set of user agents $i \in I$ and discrete time step $t \in T = \{0, 1, \ldots, \bar{T}\}$. We consider a spatial network $(N, E)$ with a set of nodes $n \in N = \{0, 1, \ldots, \bar{N}\}$ and a set of links $e \in E$. For some nodes, we consider an active time for nodes $b_n \subset T$, meaning that a facility on node $n \in N$ is available only in the time-window expressed by $b_n$.

#### 2.2.1 Agent model

Each agent $i$ takes a series of actions within time–space prism constraints $\{O_i, D_i, T_i\}$, where $O_i$ and $D_i$ denote the origin and destination respectively, and $T_i = \{t_i^B, \ldots, t_i^E\} \subset T$. Agents receive neither positive nor negative rewards by moving, and receive non-negative rewards by staying at any node during its active time window. The type of agent $i$ is expressed by $\theta_i = \{O_i, D_i, T_i, v_i\}$, where $v_i = \{v_i^0, \ldots, v_i^N\}$ and $v_i^n$ denotes the reward that agent $i$ receives by staying at node $n \in N$ during the active time window.
b_n of the node. In this paper, we consider agent type information to be private. At departing time t_i^R, each agent reports its type, including its preferences and constraints, to the operator to obtain a permit. We assume that agents are self-interested and can misreport their type. Obtaining the permit, each agent moves as directed by the operator and makes a payment to the operator at the end of its active time duration T_i. If it cannot obtain the permit, the agent cancels the trip and is not charged.

2.2.2 Operator model
Collecting the agents’ reports, the operator dynamically allocates moving permits to each agent and determines the agent payments, aiming to maximize social welfare. We assume that the operator has a predicted demand model \( \tilde{D} \) that represents stochastic information about agents reporting in future. However, the operator does not know the accurate agent types until receiving their report.

At each time, the operator decides joint action \( \pi_t \) of all agents reported by time \( t \) as follows,

\[
\pi_t = \pi(\theta_t, S_t, \tilde{D}),
\]

where \( \theta_t \) denote all reports that the operator has received from agents until time \( t \) and \( S_t \) denote the joint state of all agents reported by time \( t \). This decision is made by considering time–space constraints of each agent and capacity constraints for each spatial link. In addition, the operator also determines the payment \( x_i \) to each agent at the end of its time constraint \( t_i^E \) such that

\[
x_i = x_i(\theta_{t^E_i}, S_{t^E_i}, \tilde{D}),
\]

and charge it to the agent.

2.3 Mechanism
We adopt dynamic pivot [Bergemann and Välimäki, 2010] mechanism that extend a static VCG mechanism to dynamic situations. The mechanism consists of model-based dynamic allocation that maximizes social welfare and the payment that coincides with externalities that agent gives to the market. Under this mechanism, the truthful report coincides with agents’ best-response strategy, and thus Bayesian–Nash equilibrium is spontaneously achieved by truthful reports from all agent.

2.4 Solution algorithm
An overview of the proposed solution is shown in Algorithm [1].

The initial input to the algorithm is spatial network \((N, E)\), control time duration \( T = \{0, 1, \ldots, \bar{T}\} \), and demand-predictive model \( D \). In addition, when receiving reports from agents at each time, the information about a set of agents \( I \) and their type \( \theta \) is dynamically updated.

The algorithm first enumerates all possible allocation plans in control duration \( T \) and saves the set of plans to \( Z \) using ZDD [Minato, 1993] (Line 2). The enumerated plans \( Z \) are updated (Line 5) with new agents or those finishing their trip, whereas the plans
Algorithm 1 Main

Require: \((N, E), T, \tilde{D}, I, \theta\)
Ensure: \(\pi, x\)

1: \(t \leftarrow 0\)
2: \(Z \leftarrow \text{Enumerate}((N, E), T, \tilde{D})\)
3: for \(t \in T\) do
4: \(I_t \leftarrow \text{RenewAgent}(I, t)\)
5: \(Z \leftarrow \text{Update}(Z, \theta_t)\)
6: \(\theta_t \leftarrow \text{RenewType}(\theta, t)\)
7: \(S_t \leftarrow \text{GetState}\)
8: \(\pi_t \leftarrow \text{EfficientAllocation}(I_t, \theta_t, S_t, \tilde{D}, Z)\)
9: \(Z \leftarrow \text{NarrowDown}(Z, \pi_t)\)
10: for \(i \in I_t\) do
11: \(x_{i,t} \leftarrow \text{CalculatePayment}(\pi_t, \theta_t, Z)\)
end for
end for

are narrowed down (Line 9) with the determination of allocation at each time. Based on agent reports (Line 6) and the state (Line 7), the algorithm determines an efficient allocation (Line 8) and calculates agent payments (Line 11) at each time. In these steps, a multi-scenario approach [Chang et al. 2000] is applied to calculate the allocation that maximizes the discounted social welfare. They are efficiently calculated by using the enumerated set of plans \(Z\) using ZDD.

3 Numerical Analysis

To evaluate our proposed mechanism, we ran a numerical analysis focusing on the total social welfare achieved by the allocation in our proposed mechanism compared with the common first-come first-served (FCFS) mechanism.

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Figure 4: Sample network
3.1 Experimental setup

The sample network considered in this analysis is shown in Fig. 4. We consider Nodes A and D to be residential and office areas, and Nodes B and C to be amusement areas. The numbers in brackets show the required time to move on each link. In this analysis, we consider passing and cruising agents. The main objective of passing agents is to move between places; the origin and destination of passing agents are different. By contrast, the main objective of cruising agents is spending time at some places, enriching their experience by cruising around; the origin and destination of cruising agents are the same. A key motivating example of this setting is the area pricing in tourist sites. Passing agents express the background traffic demand represented by morning or evening commuters and the cruising agents express tourists cruising multiple areas in the network.

3.2 Preliminary Results

First, we evaluate the efficiency of the proposed and FCFS mechanisms. Here, we define efficiency as the total social welfare achieved by each mechanism as a proportion of the offline-optimal welfare. We consider two cases, with 2 or 4 passing agents, and vary the number of cruising agents from 1 to 7. The experiment was repeated over 30 trials for each setting. The plots in Fig. 5 show the mean value of the trials and the error bars express 95% confidence. As seen in Fig. 5, the efficiency of FCFS decreases rapidly with agent interference, whereas our proposed mechanism retains its quality.

We show the calculation time of the proposed mechanism in Fig. 6. In this figure, we call the process in Line 2 of Algorithm 1 in Section 2.4 as the enumeration time and call the process in Line 3 as the allocation time. As we can see in this figure, by the nature of ZDD, the calculation time required for the allocation is less than that for the enumeration, despite the many optimization processes it includes for each sample scenario and joint action.

4 Conclusions

In this paper, we proposed activity-chain auction mechanism for mobility services that aims to maximize social welfare, considering the time–space prism constraints of agents and the capacity constraints of traffic resources. To this end, we proposed a space–time extended network with normalized time in which agent behaviors are defined as a general state-transition model. We introduce dynamic pivot mechanism that require the sequential decision-making of the service operator to achieve dynamic and socially optimal allocation to agents. We then proposed an algorithm based on ZDD that can calculate the allocation efficiently. After numerically evaluating our proposed mechanism, we show that it is superior to the common FCFS mechanism in allocation efficiency, especially if competition for traffic resources is strong. Although the computational time increases with the number of agents, the computational time for each time-step is less than what was required in advance. Therefore, our proposed algorithm can react immediately to dynamically varying situations if the initial process has been prepared in advance. Overall, our proposed mechanism can realize a floating booking system by
Figure 5: Comparing the Efficiency of the Proposed and FCFS Mechanisms

Figure 6: Calculation Time
which priority temporarily assigned to low-value, early-coming users is rationally trans-
ferred to high-value, late-coming users while guaranteeing time–space constraints for all
users whose requests are once accepted. Because the mechanism grantees that users’
best response strategy is truth-telling, users voluntary report their true type and the
operator can correct the true demand to run services efficiently.

There are several limitations in this paper. In the very near future, we will be extend-
ing the numerical analysis and offering parameter studies with respect to the variety of
variables, such as the variance in agent utility, the distribution of departure and arrival
times, and the length of agents’ time constraints.

Further works will be needed to implement our proposed mechanism in real-world con-
ditions. For instance, an algorithm must be able to estimate users’ reward functions and
a method for revealing the correlation between multiple states is needed. By combining
these efforts, we establish a new norm for urban planning in the coming age.

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