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A unified modeling and solution framework for stochastic routing problems

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1 Introduction

In this work, we develop a generalized framework for solving various classes of rich vehicle and inventory routing problems with stochastic demand as well as other probability-based routing problems with a time-horizon dimension. Demand is the only stochastic parameter. It is non-stationary and is forecast using any model that provides expected demands over the planning horizon, with error terms from any empirical distribution that can be simulated. The logistic setting includes a heterogeneous fixed fleet to service demand points from supply points in a distribution or collection context. Vehicles can perform open tours with multiple supply point visits per tour when and as needed. We can have time windows, maximum tour durations, visit periodicities and various other practically relevant constraints. Based on the way we construct the routing graph, this conceptual framework can be reduced to many classical types of routing problems that appear in the literature, such as the vehicle routing problem (VRP), the inventory routing problem (IRP), the periodic VRP, the pickup and delivery problem, routing problems based on event probabilities, etc., subject to various operational and decision policies. The solution methodology is based on Adaptive Large Neighborhood Search (ALNS), which exhibits excellent performance on selected benchmark instances and appears to be very stable on randomly generated instances for the problems we consider.

2 Formulation

Given a set of demand points $\mathcal{P}$ and a planning horizon $\mathcal{T}$, let ρ_{it} denote the random demand of point $i \in \mathcal{P}$ on day $t \in \mathcal{T}$. We decompose ρ_{it} as:

$$\rho_{it} = \mathbb{E}\left(\rho_{it}\right) + \varepsilon_{it}, \tag{1}$$

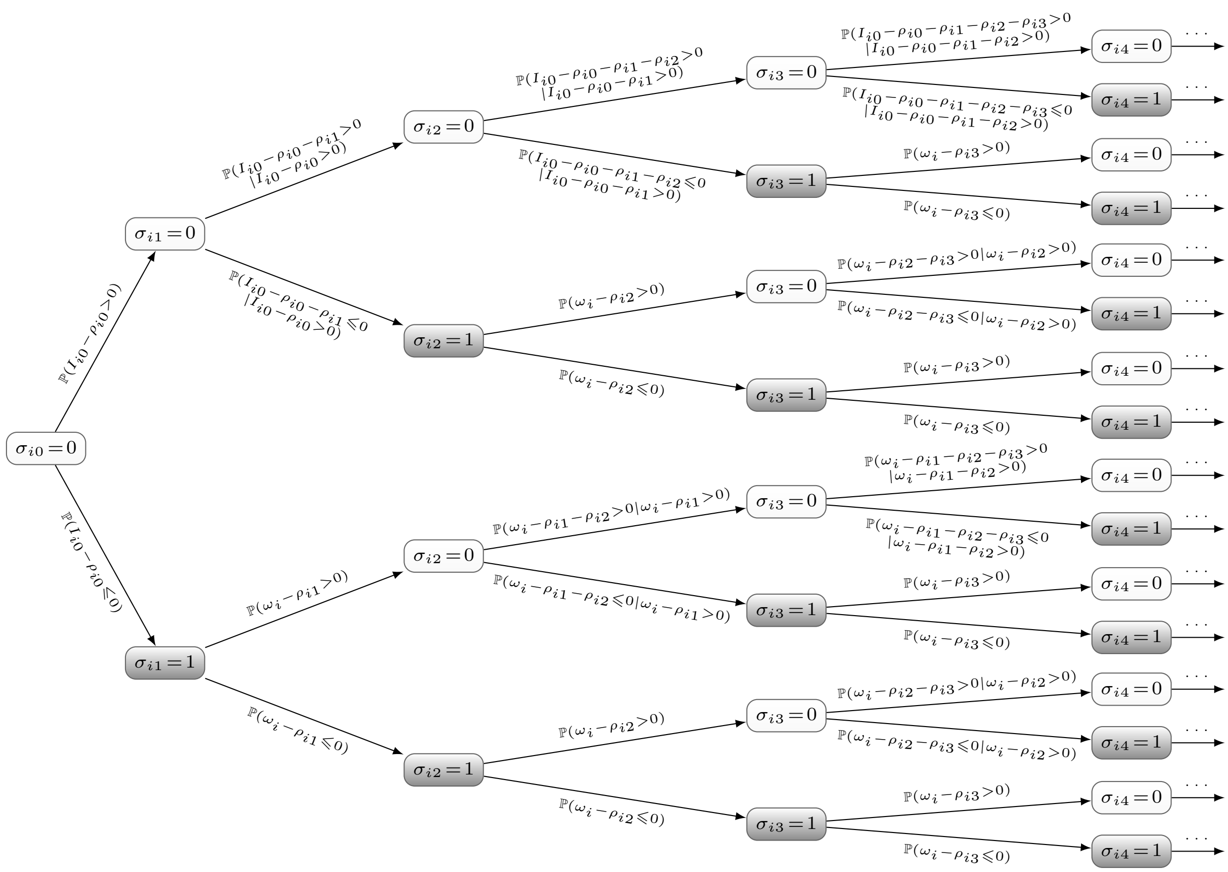
where $\mathbb{E}\left(\rho_{it}\right)$ is a point forecast and ε_{it} is an uncertainty component. We can use any forecasting model that provides $\mathbb{E}\left(\rho_{it}\right), \forall i \in \mathcal{P}, t \in \mathcal{T}$ and the statistics necessary to define the probability distribution of ε_{it}.

Each demand point can be in one of two states. Given a set of vehicles $\mathcal{K}$, $\sigma_{it} = 1$ denotes that demand point i is in a state of stock-out in period t, while $\sigma_{it} = 0$ denotes otherwise. A regular delivery to a demand point is one which is executed by a vehicle $k \in \mathcal{K}$. Contrarily, an emergency delivery occurs when the demand point is in a state $\sigma_{it} = 1$ and there is no vehicle $k \in \mathcal{K}$ that visits demand point i in period t. An emergency delivery incurs a high cost.

For a demand point i with initial inventory I_{i0} and initial state $\sigma_{i0} = 0$, if there are no regular deliveries over the planning horizon, its state probability tree develops as illustrated in Figure 1. Branches starting from a state $\sigma_{it} = 0$ involve the calculation of conditional probabilities, while those starting from a state $\sigma_{it} = 1$ involve unconditional probabilities because the demand point state is reset by the emergency delivery. Imposing a regular delivery in period t requires starting a new tree of the same type to calculate the stock-out probabilities for subsequent periods.

While the example is presented for a distribution context, the same reasoning holds in a collection context, where we consider the probabilities of demand point overflows during the planning horizon. Emergency collections are the counterpart of emergency deliveries. The conditional and unconditional probabilities are

Figure 1: Demand point state probability tree



calculated in an identical way, that being numerical integration or simulation. Given that they require the summation of random variables, an obvious candidate for modeling the error terms ε_{it} would be the normal distribution for its simple convolution property. If this assumption is relaxed, the state probabilities can be calculated by simulation. Under an order-up-to (OU) level inventory policy, all probabilities can be precomputed. The same does not hold for a maximum level (ML) policy, in which trees started by regular deliveries cannot be precomputed as the starting inventory is not known in advance, but depends on how much was delivered. A compromise may be a discretized ML policy, which is still more general than the OU policy, and in which the resulting inventory after a regular delivery is chosen from a set of discrete values. For all practical purposes, the emergency deliveries should still apply an OU policy, otherwise the combinatorial dimension becomes intractable.

The objective function contains various components reflecting the generality of the approach, including a) inventory holding cost b) demand point visit cost c) continuity of service by the same vehicle d) routing cost e) demand point stock-out and emergency delivery cost f) and route failure cost, the latter reflecting the probability of running out of capacity during delivery before reaching the next supply point. The constraints work with the deterministic parts of the demand and enforce the operational rules and policies.

3 Methodology

The solution methodology relies on ALNS. Some of the destroy and repair operators are adapted from the literature (Ropke and Pisinger, 2006; Pisinger and Ropke, 2007; Coelho et al., 2012; Buhrkal et al., 2012), while others are specifically designed for the framework, in particular taking into consideration the stochastic demand decomposition given by formula (1) and the stock-out probabilities depicted in Figure 1. The search admits intermediate infeasible solutions with a penalty for each type of constraint violation. These penalties are added to the objective function and are dynamically adjusted during the search so as to encourage diversification but also to avoid staying infeasible for too long. The ALNS shows excellent performance on several sets of classical benchmark instances. The IRP benchmarks of Archetti et al. (2007) test the ALNS in an IRP context with OU policy. Crevier et al.'s (2007) instances test the ALNS in a VRP setting with intermediate facilities. Finally, Taillard's (1999) instance set tests the ALNS in a VRP context with a rich heterogeneous fixed fleet.

4 Applications

The applications of the framework are numerous. Markov et al. (2016) use a specific version of this approach to evaluate the benefit of including uncertainty in the objective function on rich IRP instances derived from real data in Geneva, Switzerland. The result is a significant reduction in the occurrence of container overflows for the same routing cost compared to alternative policies. A particularly interesting application can be found in maritime inventory routing, which is characterized by open and multi-period tours and this problem is usually modeled on a time-expanded graph. Our generalized framework implicitly allows for tours to last more than one period. We generate a dummy origin and destination point with zero service time at each demand and supply terminal. The model can then build open tours that end at a demand or a supply terminal in one period and start from there the next period, using the dummy origin and destination points as inter-period connecting points. Another more particular application is facility maintenance, which is not an inventory routing problem, but which can be seen through the same lens, with the probability of breakdown of a facility accumulating as would inventory. Numerical testing shows that capturing this breakdown probability in our framework leads to significantly fewer realized breakdowns after simulation, while having only a moderate effect on the routing cost.

5 Conclusion

The proposed generalized framework can model various types of rich stochastic routing problems and relies on a sophisticated and efficient solution methodology, which can be applied in an operational setting. Future work will involve further testing on benchmark instances and instances derived from real data as well as comparison to the state of practice.

References

Archetti, C., Bertazzi, L., Laporte, G., and Speranza, M. G. (2007). A branch-and-cut algorithm for a vendor-managed inventory-routing problem. *Transportation Science*, 41(3):382–391.

Buhrkal, K., Larsen, A., and Ropke, S. (2012). The waste collection vehicle routing problem with time windows in a city logistics context. *Procedia-Social and Behavioral Sciences*, 39:241–254.

Coelho, L. C., Cordeau, J.-F., and Laporte, G. (2012). Consistency in multi-vehicle inventory-routing. *Transportation Research Part C: Emerging Technologies*, 24:270–287.

Crevier, B., Cordeau, J.-F., and Laporte, G. (2007). The multi-depot vehicle routing problem with inter-depot routes. *European Journal of Operational Research*, 176(2):756–773.

Markov, I., Bierlaire, M., Cordeau, J.-F., Maknoon, Y., and Sacha (2016). Inventory routing with non-stationary stochastic demands. Technical Report TRANSP-OR 160825, École Polytechnique Fédérale de Lausanne, Switzerland.

Pisinger, D. and Ropke, S. (2007). A general heuristic for vehicle routing problems. *Computers & Operations Research*, 34(8):2403–2435.

Ropke, S. and Pisinger, D. (2006). A unified heuristic for a large class of vehicle routing problems with backhauls. *European Journal of Operational Research*, 171(3):750–775.

Taillard, É. D. (1999). A heuristic column generation method for the heterogeneous fleet VRP. *RAIRO - Operations Research*, 33(1):1–14.