An application of shock wave theory to urban traffic control via dynamic speed advisory

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Summary

In this work, shock wave theory is used to model traffic evolution within urban road segments between signalized intersections. The shock wave evolution depends algebraically on the boundary conditions at the signalized intersections, and on the fundamental diagram (FD) of each road segment, whereby e.g. the FD free flow speed can be seen as a control variable. Thus, the vehicular velocities, densities and queue evolutions can be described without the need for differential equations and their solution, noting that the traffic is characterized by cells, each with constant velocity and density, whose boundaries are the shock waves.

The free flow speeds may be set by dynamic speed advisory control with the aim to optimize vehicles energy consumption and total time spent in the road network. In future work other variables such as green splits and phase differences between the intersection signals will be used for control and optimization purposes.

State of the Art

The principal theoretical work on kinematic waves was done by Lighthill and Whitham [1955]. Given flow and density upstream and downstream from the shock wave, its propagation velocity was calculated analytically. Furthermore, it was stated that such a velocity represents the slope of the chord joining the two points on the flow-density curve (i.e. fundamental diagram) which correspond to the states ahead of and behind the shock wave. The importance of this finding is that if the traffic states are known, then their future evolution can be easily predicted by describing the boundary (i.e. shock wave speed) between them. This insight was used by Richards [1956] for the study of shock waves on highways, and by Daganzo [1994] in the Cell Transmission Model (CTM) derivation and analysis.

The first attempt to use shock wave theory for traffic control was presented by Hegyi et al. [2008] in order to propose a quick control scheme to resolve jams on highways by means of variable speed limits.

Shock waves principles were also used for alternative highway traffic modeling approaches. In Canudas de Wit [2011] the proposed variable-length CTM-like model aims to capture the traffic conditions on an arbitrarily long road segment by describing the dynamics of the shock wave and its upstream and downstream traffic states. An adaptation of this model to the energy optimization by variable speed advisories in an urban traffic framework, with boundary flows enabled by cyclic traffic signals, was proposed by De Nunzio et al. [2014]. However only the evolution of upstream congestion boundaries was considered, and downstream rarefaction waves were neglected. In Canudas de Wit and Ferrara [2016] an improved variable-length model with downstream rarefaction was used for speed control on a ring road.

Contribution of this work

An algebraic model of the traffic evolution within urban road sections, i.e. modeling of traffic light induced phenomena, is proposed. The model is able to provide a solution of traffic density and flow distributions without solving differential equations, and therefore the computational burden is drastically reduced. This is particularly desirable for large-scale traffic optimization. An energy consumption model and a travel time model have been adapted to the proposed traffic model and used for performance optimization in an urban scenario.

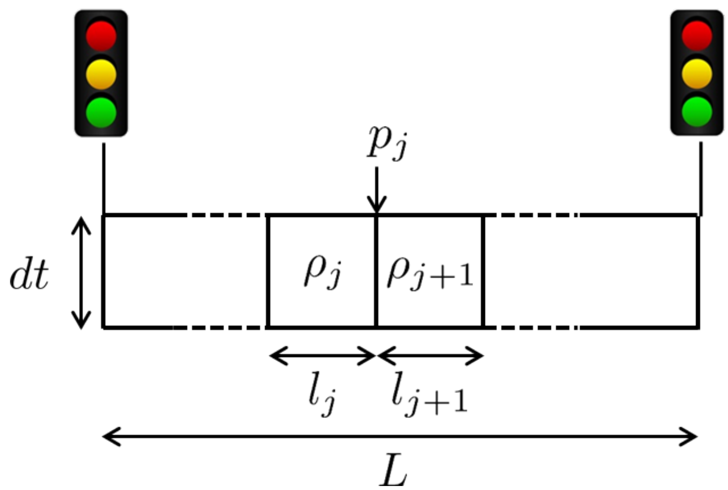

Figure 1: Urban road section and model notation.

Model Description

In urban traffic, it is reasonable to assume that flows are generated by traffic lights or intersections, and hence inflows and outflows of a road segment are pulses starting at discrete time instants. Then, under such conditions, with the road segment decomposed into cells, see Fig. 1, during a finite time interval, each cell state will be defined by one point on the fundamental diagram (FD), see Fig. 2. Hence the slope between the FD points of two neighboring cells will be constant over the said finite time interval. According to the shock wave theory, this slope gives the propagation speed of the front or boundary between the cells. Hence, there is no need to solve differential equations to find neither the value of cell states which remain constant during the existence of the specific cell, nor the evolution of the cell boundaries which move at a constant speed determined algebraically.

Let us consider an urban road section as in Fig. 1. It is reasonable to assume that the considered road section represents an elementary segment of the traffic network, meaning that no exogenous flows are allowed within the segment. The inflow and outflow are only enabled by the two traffic lights at the two ends of the section.

The road section may be intuitively split up into cells, $j = 1, \ldots, k$, of variable length l_j [m] corresponding to different traffic states ρ_j [veh/m]. Each traffic state (or density) remains constant within its spatial domain,

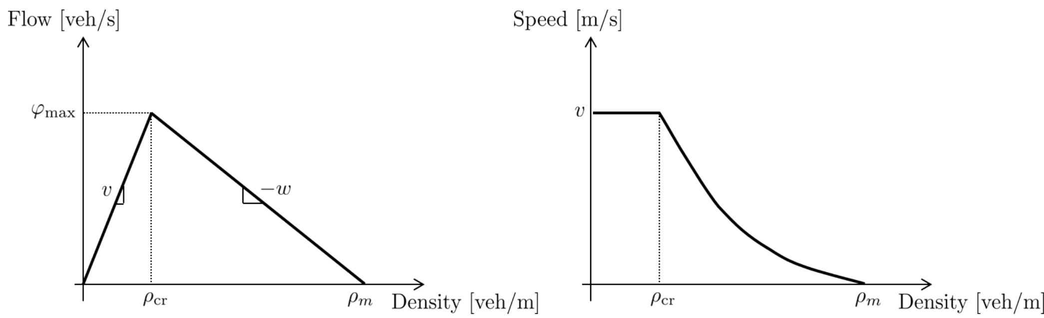

Figure 2: Fundamental diagram. ρ_m is the stand-still or jam density, ρ_{cr} is the critical, or sweet-spot density, where the state changes from free flow to congested flow. φ_{max} is the sweet-spot flow. v [m/s] stands for free flow velocity, and w [m/s] is the backward wave front propagation speed.

delimited by moving boundaries, e.g. boundary or front p_j [m] in Fig. 1. According to the shock wave theory, over an infinitesimal time step dt [s] the front p_j will move by

$$dp_j = \frac{\rho_{j+1}v(\rho_{j+1}) - \rho_j v(\rho_j)}{\rho_{j+1} - \rho_j} dt \tag{1}$$

where the symbols are defined in the caption of Fig. 2. Hence, the length l_j of each cell is calculated by the difference of continuously moving fronts. In Fig. 2, the fundamental relations between speed, flow and density are shown. Note that the speed of the front in equation (1) depends only on the upstream and downstream traffic densities which remain constant. Therefore equation (1) may be rewritten in its algebraic form as:

$$p_j(t) = p_{j0} + \frac{\rho_{j+1}v(\rho_{j+1}) - \rho_j v(\rho_j)}{\rho_{j+1} - \rho_j} \cdot (t - t_{j0}) \tag{2}$$

where p_{j0} and t_{j0} are the starting position and time of the moving front p_j, respectively. Typically, traffic dynamics and behavior present several shock waves generated for instance by traffic lights turning green or red, or by the intersection of two other shock waves. In order for these events to be predictable and for traffic dynamics to be described algebraically, the following assumptions must hold:

- the FD of the road section is known;

- the initial traffic conditions are known;
- the traffic signals timings are known.

Note that signalized intersections may present very complex phases and movements during a traffic light cycle, therefore the inflow to a particular road section may be determined by several simultaneous and/or consecutive movements. Thus, the green light duration enabling boundary flows may be thought of as an equivalent duration which comprises all the movements entering or leaving the section.

Performance metrics

Let us introduce two optimization criteria of interest: mean vehicle energy consumption and travel time. Based on the modeled traffic conditions, vehicle trajectories can be determined by taking into account: traffic signals timings which enable boundary flows, and traffic densities and cells boundaries which determine travel speed and acceleration. The performance metrics will be evaluated on the trajectory of the vehicles completing the trip in the considered road section. If the traffic conditions are at an equilibrium (i.e. equal boundary flows), the performance may be evaluated solely on the vehicles entering the section during a traffic light cycle time.

Let us consider the following simple expression for the power request $P(t)$ [W] at the wheels

$$P(t) = F_t(t)v(t) \tag{3}$$

where $v(t)$ [m/s] is the vehicle speed. The traction force $F_t(t)$ [N] at the wheels is

$$F_t(t) = m\frac{\mathrm{d}v(t)}{\mathrm{d}t} + a_2 v(t)^2 + a_1 v(t) + a_0 + mg\sin(\alpha) \tag{4}$$

where m [kg] is the vehicle mass, a_i, $i = 0, 1, 2$ are given constants, g [m/s/s] is the acceleration due to gravity, and α [rad] is the slope of the road. In each cell j, traffic density and speed are constant, therefore the power request within each cell becomes a constant $\bar{P}(v(\rho_j))$, with no acceleration term. At the interface between adjacent cells, the speed difference triggers additional energy consumption which can be expressed as:

$$E_{trans} = \int_0^{\frac{v(\rho_{j+1})-v(\rho_j)}{a}} P(\tau)\,\mathrm{d}\tau \tag{5}$$

where the interval of integration represents the time necessary to complete the speed transition at constant acceleration a. Finally the single vehicle energy consumption over the necessary travel time T to leave the section, is expressed as:

$$E_k = \int_0^T \sum_{j=1}^{c} \bar{P}(v(\rho_j))\,\mathrm{d}t + \sum_{j=1}^{c-1} E_{trans} \tag{6}$$

where c is the number of cells in the road section at each time instant along the vehicle trajectory.

Hence, the mean energy consumption of the N vehicles entering the road section during a cycle time according to the current inflow is calculated as:

$$\bar{E} = \frac{1}{N}\sum_{k=1}^{N} E_k \tag{7}$$

The number of vehicles N is known and equal to:

$$N = \varphi_{in} \cdot T_g \tag{8}$$

where φ_{in} is the flow entering the road section, and T_g is the upstream green light duration.

The mean travel time is defined as the average of the time required to complete the trip over the road section by all vehicles $k = 1, \ldots, N$. It can be expressed as:

$$\bar{TT} = \frac{1}{N}\sum_{k=1}^{N}(t_{f,k} - t_{0,k}) \tag{9}$$

where $t_{0,k}$ is the time at which vehicle k enters the road section, $t_{f,k}$ is the time at which vehicle k leaves the road section. Such an exit time $t_{f,k}$ can be calculated by tracking the vehicle trajectory until its position x_k reaches the end of the road section. The vehicle trajectory may be computed as follows:

$$x_k(t + dt) = x_k(t) + v(\rho(x_k(t))) \cdot dt \tag{10}$$

or algebraically by computing the intersection points with the equations of the moving fronts. The intuition is that the speed is constant within a certain cell and it changes when the vehicle trajectory crosses the closest downstream moving front. The pattern repeats itself until the vehicle leaves the road section. An illustration of some vehicle trajectories is given in Fig. 3.

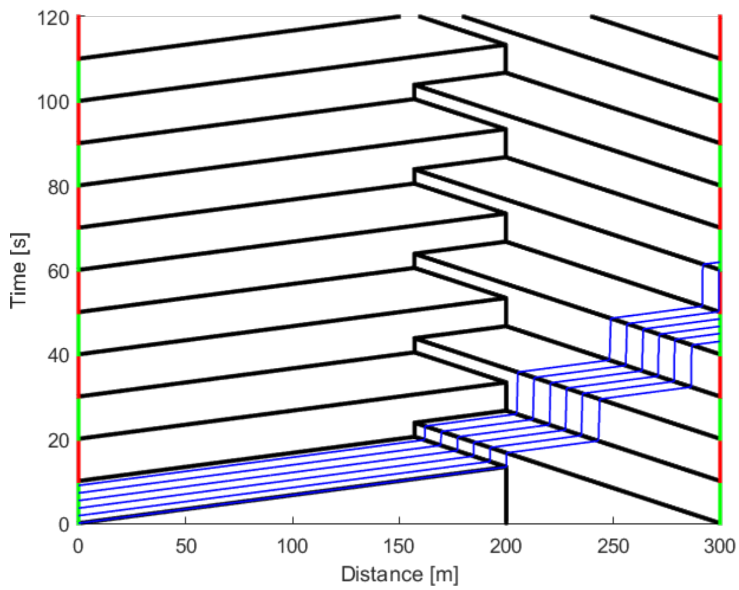

Figure 3: Vehicles trajectories within the road section. Stops and idling time are also captured by the proposed model.

Optimization problem

The objective of this work is to find the optimal speed limit that, given some traffic conditions, minimizes a combination of the aforementioned performance criteria: energy consumption and travel time. The optimization problem could be formulated as follows:

$$\begin{aligned} \min_{v} \quad & J = \lambda \frac{\bar{E}}{\bar{E}_{max}} + (1-\lambda)\frac{\bar{TT}}{\bar{TT}_{max}} \\ \text{s.t.} \quad & \dot{\rho} = 0 \\ & \dot{p} = \frac{\rho_{j+1} v(\rho_{j+1}) - \rho_j v(\rho_j)}{\rho_{j+1} - \rho_j} \\ & v_{\min} \leq v \leq v_{\max} \end{aligned} \tag{11}$$

where λ is the optimization weight setting the trade-off between the objectives, and $\bar{E}_{max}$ and $\bar{TT}_{max}$ are the normalization factors that ensure comparability of the objectives. The dynamic equations of density and moving boundaries, to be calculated for every cell and for every boundary, can be reduced to their algebraic equivalent form as previously described.

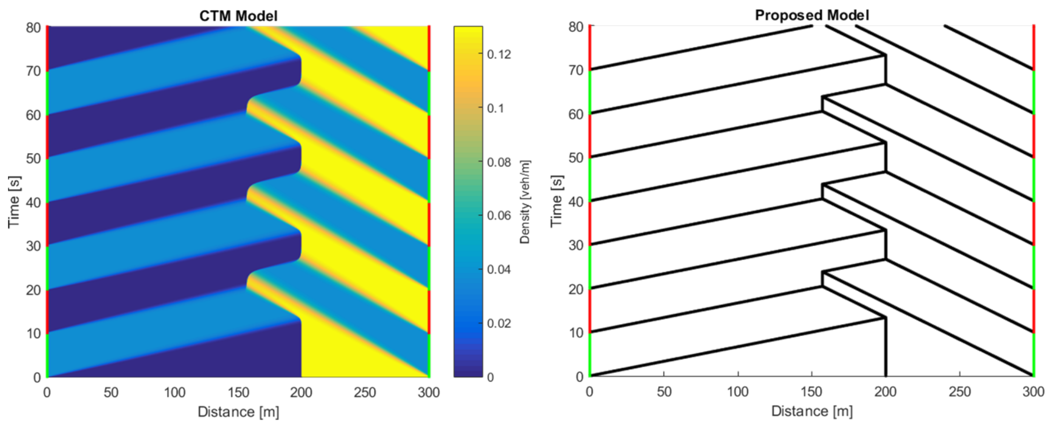

Figure 4: Comparison of the CTM solution and the algebraic fronts solution of the proposed model in a distance-time diagram. The densities are color coded.

Results

Let us consider the following FD parameters: $\rho_m = 0.13\,\text{veh/m}$, $w = 6\,\text{m/s}$, $v = 15\,\text{m/s}$. An illustrative comparison between the CTM solution and the proposed model, for an urban road section with upstream and downstream traffic lights equally timed, and for a given initial traffic condition, is shown in Fig. 4 in a distance-time diagram. The initial traffic state was set to have an empty cell upstream, a fully congested cell downstream, and an initial queue length of 100 m (i.e. the initial leftmost point of the congested cell in yellow in Fig. 4). It is noted in the CTM solution that the density, and hence velocity, are constant in each cell in the following way: a red light causes stand still, a stand still density upstream, and zero velocity and density downstream. A green light causes a rarefaction wave at ρ_{cr}. These state values are implicit in the proposed shock wave solution.

Let us assume that the free flow speed v equals the given but variable speed limit. We find that changing the speed limit within the road section has significant impact on both objectives, depending on the particular traffic density distribution and traffic light timings. The value of the objectives for different values of the speed limit, from 5 to 15 [m/s], and for different initial queue lengths, from 150 to 0 [m], is shown in Fig. 5.

The results suggest that the energy consumption is very high when the road section is congested and the speed limit is kept at the standard value of

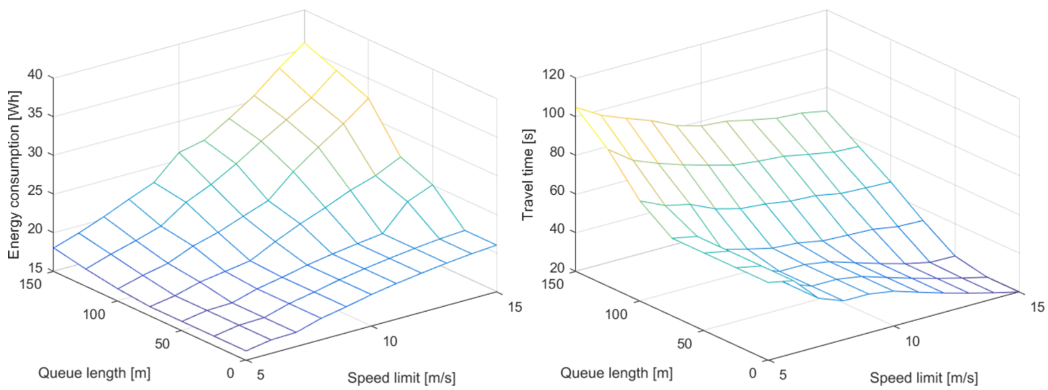

Figure 5: Mean energy consumption and travel time over a cycle time as a function of the speed limit and the initial length of the queue in the considered road section.

approximately 15 m/s. The long queue induces several accelerations towards a high speed limit, which proves inefficient in terms of energy expenditure. Energy consumption decreases when the queue is reduced and, trivially, when the speed limit is decreased.

Travel time is a somewhat competing performance metric, showing that travel time is minimized for high values of the speed limit. However, the results surface is not fully monotone due to the traffic lights offset effect. In order to have an insight into the solution of the optimization problem in (11), the evaluation of the objective function for $\lambda = 0.5$ is displayed in Fig. 6. It is interesting how the overall performance is minimized for different speed limits depending on the initial queue length inside the road section. For instance, for a highly congested section with an initial queue length of $l = 150$ m, the performance is optimized for a speed limit of $v = 9$ m/s, showing a performance gain of about 14% with respect to the case of a standard speed limit of $v = 15$ m/s.

Conclusions

In this work, an adaptation of the shock wave theory to urban traffic modeling was proposed. The dynamic traffic model may be defined by simple algebraic equations which describe the evolution of the boundaries between adjacent traffic states (i.e. cells). The model proves to effectively reproduce

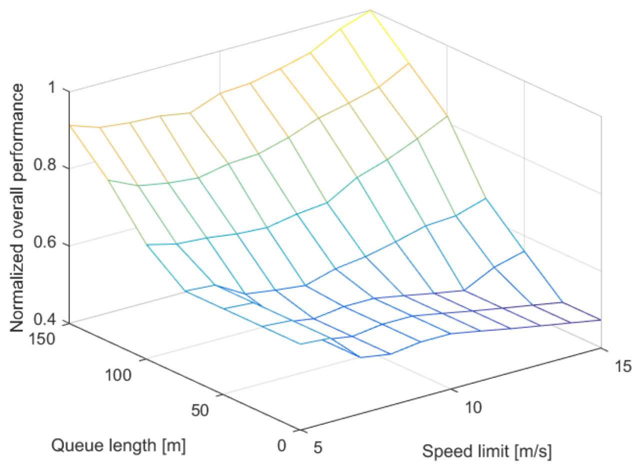

Figure 6: Overall traffic performance over a cycle time as a normalized weighted sum of mean energy consumption and travel time. The performance is represented as a function of the speed limit and the initial length of the queue in the considered road section. The graph represents the evaluation of the objective function in (11) for $\lambda = 0.5$.

the traffic evolution in urban environment with the same precision shown by well-established and reference macroscopic models, such as the CTM. The proposed modeling approach is able to describe traffic not only in highly-congested cases but also when the queue at the traffic light is completely dissipated. Furthermore, the underlying assumption of regular traffic demand during the upstream green phases can be easily relaxed by introducing additional moving fronts whenever the density of the entering vehicles changes.

The algebraic formulation of the model presents significant advantages in terms of computation time and scalability, which will favor the application to large-scale modeling and optimization. In future work, the impact of varying traffic light timings on the performance will be assessed, with the goal of suggesting an optimization based on two decision variables: speed limit and green lights offset or duration. Furthermore, multiple sections concatenation effects are expected to have an impact on the network overall performance and need to be studied.

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