The extended abstract submitted to hEART 2015 Conference on April 10, 2015

Optimization of bus services in multimodal networks with dedicated bus corridors with network-level traffic models

Fangni Zhang[a], Nan Zheng[b], Nikolas Geroliminis[b], Hai Yang[a]

[a] *Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, P.R. China*

[b] *School of Architecture, Civil and Environmental Engineering, Laboratory of Urban Transport Systems, École Polytechnique Fédérale de Lausanne (EPFL), Switzerland*

Introduction

With the rapid social-economic development, the growing travel demand imposes increasing pressure on transport infrastructures and causes severe traffic congestion in urban areas. Constructing new infrastructure is expensive, while implementing road pricing is practically difficult due to user acceptability. Considerable efforts have been devoted to looking for smart planning and allocation of the existing road space among different mode usage, e.g., private cars, buses, taxis. The idea of "giving priority to public transport" is thus proposed to divert automobile users to higher occupied public transport. The design and efficiency of the dedicated bus lanes are discussed by Basso et al., (2011) and Tirachini and Hensher (2011), on the microscopic level. Recently, Zheng and Geroliminis (2013), on the aggregate level, carried out a study on the optimization of roadway space allocation among modes by introducing the multimodal macroscopic fundamental diagram (the "MFD") to represent the traffic dynamics of the multimodal transport system. Their study focuses on the optimal space allocation and its impact on congestion dynamics, linking public policy on multimodality to the realistic physical dynamic representation of traffic. However, the role of the optimal design of bus service in the multimodal system has not been analyzed.

Pertaining to the public transit operations and policy, there is a large pool of literature. The pioneering work by Mohring (1972) has identified that the optimal bus frequency for welfare maximization is proportionate to the square root of the ridership. This approach has been subsequently extended to jointly optimize frequency and other variables, such as bus size and fares (Basso and Jara-Díaz, 2012; Zhang et al., 2014). While most of the previous studies treat the bus running speed as an exogenous parameter, the congestion effect is often overlooked. However, the congestion induced by bus frequency and patronage slows down bus traffic and causes delays at bus stops when the demand is high. This aspect has been captured by the microeconomic model proposed by Ahn (2009), who discussed on the congestion interaction between the automobile and bus when sharing the same roadway capacity. Furthermore, most studies do not treat mode choice as an endogenous variable and integrate it in the optimization procedure.

Motivated by the discussion above, this paper aims to formulate the multimodal dynamic user equilibrium problem for separated roadway space and explore how the bus operator adjusts the bus service at peak and off-peak periods by considering both space allocation and congestion effect. The dynamics of the automobile traffic are captured by the MFD model. The delay of a bus trip depends decisively on the running speed, which is affected by the space allocated for bus lanes, the travel demand for bus, and the service frequency. The impact of congestion externality on the bus optimal design is firstly examined for given demand in the steady state, and then the model is applied to the dynamic framework where the space allocation change with the varying demand. Besides, to understand the impact of economic regime on transit policy, different settings are considered for the transit operation, in which the transit operator aims to maximize profit or minimize total travel cost while keeping break-even.

Model framework

Consider a downtown area where the total roadway capacity is divided for private automobiles and buses. The fraction assigned for the dedicated bus lanes is λ. Due to practical difficulty, the change of space allocation is not supposed to appear too frequently on the urban roadway; but it is indeed manageable to give one or more lanes for buses during the peak period. In this paper we consider a two-period dynamic space allocation strategy:

$$\lambda(t) = \begin{cases} \lambda_1, & \text{if } 0<t \le t_s, \text{or } t_e < t < T \\ \lambda_2, & \text{if } t_s < t \le t_e \end{cases} \tag{1}$$

λ_1: fraction of space allocation to dedicated bus lanes at off-peak time.

λ_2: fraction of space allocation to dedicated bus lanes at peak time, and $\lambda_1 \ge \lambda_1$.

t_s: start time of peak allocation.

t_e: end time of peak allocation.

T : the time horizon.

Users choose between the two alternatives, i.e., automobile and bus, based on their generalized travel costs. Denote $x_a(t)$ and $x_b(t)$ as the travel demand for automobile and bus at time t, respectively, and the total demand is $D(t) = x_a(t) + x_b(t)$.

The congestion of the automobile traffic is homogeneously distributed over space and exhibits an MFD with low scatter (Geroliminis and Daganzo, 2008). Denote $n_a(t)$ the accumulation (number of the vehicles in the system). The average running speed v_a of all the vehicles in the area decreases with the accumulation n_a, while increases with the capacity allocated for automobile use, i.e., $v_a = v_a\left(n_a;\lambda\right)$, $\partial v_a/\partial n_a < 0$, $\partial v_a/\partial\lambda < 0$. For given space allocation, the production (vehicle kilometers traveled per unit time) of the system is $P_a\left(n_a\right) = n_a v_a\left(n_a\right)$, and the outflow (rate vehicles reach their destination) of the system is $O_a\left(n_a\right) = P_a\left(n_a\right)/l = n_a v_a\left(n_a\right)/l$. The dynamics of the automobile traffic is captured by the differential equation:

$$\frac{dn_a\left(t\right)}{dt} = x_a\left(t\right) - O_a\left(t\right),\ n_a\left(0\right) = n_{a0}. \tag{2}$$

The travel time for an automobile user is $tt_a\left(n_a\right) = l/v_a\left(n_a\right)$, where l is the average trip length. Then the generalized travel cost of the automobile user who enters the system at time t is:

$$c_a\left(t\right) = \tau_a + \beta \cdot tt_a\left(n_a\left(t\right)\right), \tag{3}$$

where τ_a is the monetary cost of the automobile user, β is the value of in-vehicle travel time, and $n_a\left(t\right)$ is the instantaneous accumulation at time t.

For the bus system, it is assumed that the running speed of buses decreases when either the service frequency or demand goes up, but the free-flow speed is larger if more space is allocated for the bus lanes.

$$v_b\left(t\right) = v_{b0}\left(\lambda\right) - b_1 f\left(t\right) - b_2 x_b\left(t\right),$$

where the free-flow speed of buses v_{b0} increases with λ, and the parameters $b_1, b_2 > 0$.

The generalized travel cost of a bus rider is the summation of the bus fare, the in-vehicle travel time, and the waiting time at the bus stops or stations. The travel time is $tt_b\left(t\right) = l/v_b\left(t\right)$, and the average waiting time for a bus is approximated by the half of the headway $w\left(f\right) = 1/2f$. Thus the generalized travel cost of the rider who enters the system at time t is

$$c_b\left(t\right) = \tau_b\left(t\right) + \beta \cdot tt_b\left(t\right) + \alpha \cdot w\left(f\right), \tag{4}$$

where α is the value of waiting time.

Given this, the total travel cost is

$$TC = \int_0^T \left[x_a\left(t\right)c_a\left(t\right) + x_b\left(t\right)c_b\left(t\right)\right]dt. \tag{5}$$

The multimodal user equilibrium is defined where the generalized travel costs of the used modes are equal, i.e.,

$$\left[c_i(t)-\delta\right]\cdot x_i(t)=0,\ i=a,b,$$

where $\delta=\min\{c_a(t),c_b(t)\}$.

A constant marginal operational cost is assumed for each movement, thus the transit operator's expenses can be captured by $k(f)=k_0+k_1 f$ (similar assumptions can be found in Basso and Jara-Díaz, 2012). Therefore, the net profit received by the bus operator is part of the ticket revenue that outweighs the operational expenses:

$$\pi_b=\int_0^T\left[\tau_b(t)x_b(t)-k(f)\right]dt. \tag{6}$$

As the reaction to the change of the space allocation given in Eq.(1) and the boosted travel demand during the peak period, the service frequency and fare should be adjusted during the peak period. The time-dependent frequency and fare considered in this paper are:

$$f(t)=\begin{cases} f_1, & \text{if } 0<t\le t_s, \text{or } t_e<t<T \\ f_2, & \text{if } t_s<t\le t_e \end{cases}.$$

f_1: service frequency at off-peak time.

f_2: service frequency at peak time.

and

$$\tau_b(t)=\begin{cases} \tau_{b1}, & \text{if } 0<t\le t_s, \text{or } t_e<t<T \\ \tau_{b2}, & \text{if } t_s<t\le t_e \end{cases}.$$

τ_{b1}: transit fare at off-peak time.

τ_{b1}: transit fare at peak time.

Preliminaries

We intend to analyze the optimal strategy for the bus operator to adjust the frequency or fare at peak and off-peak periods by considering space allocation, congestion effect and mode choices, under different economic regimes, i.e., profit maximization and total travel cost minimization with the budget constraint.

We firstly investigate the bus optimal design in the steady state. To make the model tractable, the automobile traffic is characterized by the simplified traffic flow model as shown in Figure 1, where the flow-density relation is linear:

$$p(n_a) = \begin{cases} v_f n_a, & \text{if } 0 < n_a < n_c \\ \dfrac{v_f n_c}{n_j - n_c}(n_j - n_a), & \text{if } n_c < n_a < n_j \end{cases} \tag{7}$$

The subsequent speed-density relation is

$$v_a(n_a) = \begin{cases} v_f, & \text{if } 0 < n_a < n_c \\ \dfrac{v_f n_c}{n_j - n_c}\left(\dfrac{n_j}{n_a} - 1\right), & \text{if } n_c < n_a < n_j \end{cases}. \tag{8}$$

With a given space allocation strategy, the steady state is reached when the marginal change in traffic accumulation is constant, i.e., $dn_a(t)/dt = 0$. Then according to Eq.(2), we have the relation between traffic density and demand,

$$n_a = \begin{cases} \dfrac{lx_a}{v_f}, & \text{if } 0 < n_a < n_c \\ n_j - \dfrac{n_j - n_c}{v_f n_c} lx_a, & \text{if } n_c < n_a < n_j \end{cases}$$

and the speed with demand

$$v_a = \begin{cases} v_f, & \text{if } 0 < n_a < n_c \\ \dfrac{lx_a}{n_j - \dfrac{n_j - n_c}{v_f n_c} lx_a}, & \text{if } n_c < n_a < n_j \end{cases}$$

The static equilibrium condition is

$$c_a = c_b \Leftrightarrow \tau_a + \beta \cdot \frac{l}{v_a} = \tau_b + \beta \cdot \frac{l}{v_b} + \alpha \cdot \frac{1}{2f}. \tag{9}$$

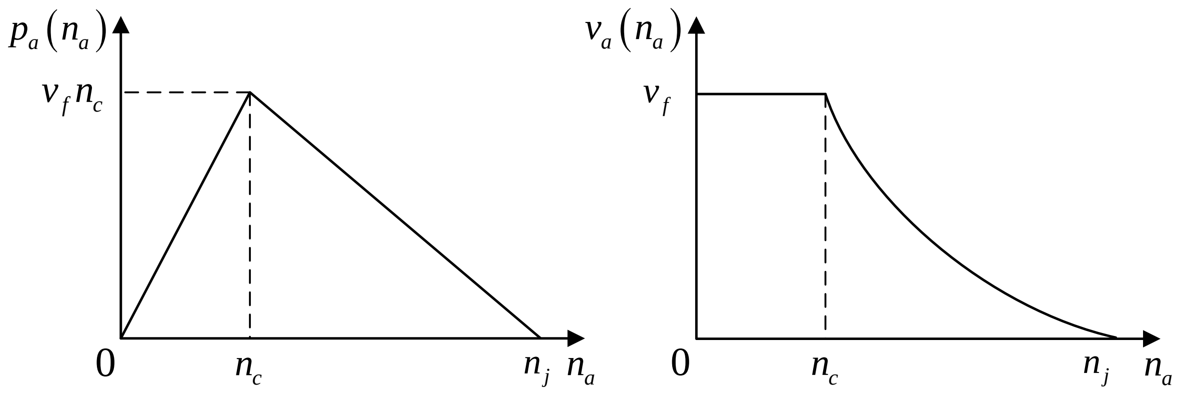

Figure 1. The traffic flow model for the steady-state analysis

When the automobile traffic is under uncongested condition ($0 < n_a < n_c$), the equilibrium demand for the automobile is

$$x_a = D - \frac{v_{b0} - b_1 f - \dfrac{v_f \beta l}{v_f\left(\tau_a - \tau_b - \dfrac{\alpha}{2f}\right) + \beta l}}{b_2}.$$

Then the marginal demand for buses associated with the change of service frequency is

$$\frac{dx_b}{df} = -\frac{dx_a}{df} = \frac{1}{b_2}\left[\frac{\alpha\beta v_f^2 l}{2\left[\left(\left(\tau_a - \tau_b\right)v_f + \beta l\right) f - \dfrac{\alpha v_f}{2}\right]^2} - b_1\right]. \tag{10}$$

The marginal profit for the bus operator is

$$\frac{d\pi_b}{df} = \tau_b \frac{dx_b}{df} - k_1. \tag{11}$$

The optimal frequency for a profit-maximizing bus operator can be derived from Eqs.(10) and (11):

$$f^{opt} = \frac{\sqrt{\dfrac{\alpha\beta v_f^2 l}{2\left(\dfrac{b_2 k_1}{\tau_b} + b_1\right)}} + \dfrac{\alpha v_f}{2}}{\left(\tau_a - \tau_b\right)v_f + \beta l}.$$

When the total demand is high, the lanes for the automobile traffic can be very congested where the density lies in the range of $n_c < n_a < n_j$, the demand for automobile is determined by $g(x_a) = 0$, and

$$g\left(x_a\right)=\tau_b-\tau_a+\beta\left(\frac{l}{v_{b0}-b_1 f-b_2\left(D-x_a\right)}-\frac{n_j-\dfrac{n_j-n_c}{v_f n_c}lx_a}{x_a}\right)+\frac{\alpha}{2f}.$$

It is shown that there exists a unique positive solution to $g\left(x_a\right)=0$, so the equilibrium mode share can be uniquely determined. Then the marginal bus demand with respect to service frequency is

$$\frac{dx_b}{df}=-\frac{dx_a}{df}=\frac{b_1 l+\dfrac{\alpha v_b^2}{2\beta f}}{b_2 l x_a^2-n_j v_b^2}x_a^2. \tag{12}$$

According to Eqs. (11) and (12), the optimal frequency is

$$f^{opt}=\begin{cases}\dfrac{\alpha\tau_b v_b^2 x_a^2}{2\beta\left(k_1\left(b_2 l x_a^2-n_j v_b^2\right)-b_1 l\tau_b x_a^2\right)}, & \text{when } x_a^2>\dfrac{n_j v_b^2}{b_2 l}\\ f^{\min}, & \text{when } x_a^2<\dfrac{n_j v_b^2}{b_2 l}\end{cases}$$

This implies that when the total demand is high and the mode share of bus (endogenously determined by the frequency) is small, the bus operator would provide the service as less frequent as possible, which would possibly give rise to a vicious cycle as described in Bar-Yosef et al., (2013). With the static equilibrium, the optimal pricing strategy can be derived for the objective of profit-maximization following the same logic and the properties of the optimal strategies can be analyzed accordingly.

The developed framework is applied to a case study in the dynamic context. Consider a downtown area with the radius of 5km. A fraction of road space is dedicated to buses only. We simulate an urban road traffic system for a typical morning or evening period. Demand has a symmetric trapezoidal shape with time and the length of peak period is equal to 1h. The setting of the MFD for automobile traffic follows Zheng and Geroliminis (2013).

The preliminary results show that our approach enables the comparison of system performance given different bus service levels. On-going work investigates (i) an optimal service strategy including bus fare as a third decision variable, and (ii) a multi-objective optimization problem where the transit operator adapts their service strategy to satisfy a public mobility goal, e.g. the total travel cost is minimized. We will report these results in the final version of the paper.

References

Ahn, K., 2009. Road pricing and bus service policies. Journal of Transport Economics and Policy 43(1), 25-53.

Bar-Yosef, A., Martens, K., Benenson, I., 2013. A model of the vicious cycle of a bus line. Transportation Research Part B: Methodological 54, 37-50.

Basso, L.J., Guevara, C.A., Gschwender, A., Fuster, M., 2011. Congestion pricing, transit subsidies and dedicated bus lanes: Efficient and practical solutions to congestion. Transport Policy 18(5), 676-684.

Basso, L.J., Jara-Díaz, S.R., 2012. Integrating congestion pricing, transit subsidies and mode choice. Transportation Research Part A: Policy and Practice 46(6), 890-900.

Geroliminis, N., Daganzo, C., 2008. Existence of urban-scale macroscopic fundamental diagrams: some experimental findings. Transportation Research Part B: Methodological 42(9), 759–770.

Mohring, H., 1972. Optimization and scale economies in urban bus transportation. American Economic Review 62(4), 591–604.

Tirachini, A., Hensher, D.A., 2011. Bus congestion, optimal infrastructure investment and the choice of a fare collection system in dedicated bus corridors. Transportation Research Part B: Methodological 45(5), 828-844.

Zhang, F., Yang, H., Liu, W., 2014. The Downs-Thomson Paradox with responsive transit service. Transportation Research Part A: Policy and Practice 70, 244-263.

Zheng, N., Geroliminis, N., 2013. On the distribution of urban road space for multimodal congested networks. Transportation Research Part B: Methodological 57, 326-341.