Modeling the Morning Commute for Urban Networks with Cruising-for-parking: an MFD Approach

Wei Liu¹, Nikolas Geroliminis
Urban Transport Systems Laboratory (LUTS), École Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland

Abstract

This study examines the morning commute equilibrium with explicit consideration of cruising-for-parking, and its adverse impacts on traffic congestion. The cruising-for-parking is modeled through a dynamic aggregated traffic model for networks or areas: the macroscopic fundamental diagram (MFD). Firstly, we formulate the morning equilibrium solution for a congested downtown network with cruising-for-parking. It is shown that the cruising-for-parking would yield smaller system or network outflow, and thus induce more severe congestion. We then develop a dynamic model of pricing for the network to reduce system travel cost including cruising time cost, moving time (the duration during which vehicles move to the destination but do not cruise for parking yet) cost and schedule delay cost. At the system optimum, the downtown network should be operating at the maximum production of the MFD, but the cruising effect is not fully eliminated. Also, it is shown that the time-dependent toll has a different shape than the classical Vickrey equilibrium fine toll. This analysis is then extended to the bi-modal commuting equilibrium with cruising-for-parking in the auto side. In this case, besides departing earlier to enjoy less cruising time, travelers can take public transportation to avoid the cruising-for-parking. Similarly, the optimal dynamic toll is introduced to reduce traffic inefficiency due to cruising-for-parking and roadway congestion, and realize the bi-modal system optimum. Finally, analytical results are illustrated and verified with numerical studies.

Keyword: morning commute, cruising-for-parking, MFD, pricing

1. Introduction

Parking is not only a headache for commuters in the morning peak, but also a challenging issue for the transport system planners, operators and regulators. Finding a parking space often

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constitutes an appreciable fraction of the total travel time. Due to its inefficiency, the phenomenon of cruising-for-parking is one of the most studied topics in the economics of parking. Understanding the effect of cruising-for-parking for congested networks can improve efficiency in the flow of vehicles and facilitate the development of more equitable strategies as trips with cruising might contribute to congestion more than trips without, e.g. trips with destinations outside the limited parking zones.

This study examines the morning commute equilibrium which explicitly incorporates not only the cruising-for-parking, but also its adverse impacts on traffic congestion and how this interactions re-shape the commuting equilibrium. The impact of cruising-for-parking is modeled through a recently proposed traffic model for networks or areas: the macroscopic fundamental diagram (MFD). By using the MFD approach, one of the advantages is that the downward-sloping part of the curve between traffic flow and density, known as hypercongestion in economic terms, can be modeled. Under the MFD framework, the traffic arriving their destinations or the outflow of the network depends on the traffic accumulation in the system and the average trip length of all the traffic. After taking into account parking, for given traffic accumulation in the downtown network, if parking vacancy goes down over time, it becomes more difficult to find a vacant parking space, and the probability of finding a vacant parking space decreases. It follows that the cruising distance for finding a vacant parking space will increase. This would lead to a decrease in traffic arriving at destination (find a vacant parking space) or the outflow of the downtown network. Furthermore, the increased travel distance will also lead more severe congestion in the network. If we look at the traffic dynamics, given the future traffic inflow, the decreased outflow due to cruising-for-parking would in return intensify the traffic accumulation of the network in the future and create more severe traffic congestion.

2. **Major Framework**

2.1. **MFD representation of traffic dynamics with cruising-for-parking**

Consider a downtown area where congestion is homogeneous distributed over space and exhibits an MFD with low scatter. Denote \( n \) the accumulation (number of the vehicles in the system) of the downtown network or area. The average traveling speed of all the traffic in the area would depend on the accumulation \( n \), i.e., \( v = v(n) \). Let \( P(n) \) be the production (vehicle kilometers traveled per unit time) of the system, where \( P(n) = n \cdot v(n) \). The outflow of the system under
steady state can be approximated by \( o(n) = P(n)/L \), where \( L \) is the average trip length of traffic in the network. The travel time for a trip then is \( \tau(n) = L/v(n) \).

If cruising-for-parking is taken into account, trip length \( L \) would be composed of two parts: moving distance (vehicles move towards their destinations but do not cruise for parking spaces yet), denoted by \( l_m \), and cruising or searching distance (vehicles cruise or search for vacant parking spaces), denoted by \( l_s \). Thus, the trip length is \( L = l_m + l_s \). In this paper, the average moving distance \( l_m \) is assumed to be a constant. The cruising distance \( l_s \), however, will depend on the percentage of available parking spaces, \( p \), and the average distance traveled in each trial a vehicle tries to find a parking space (might be occupied or empty), \( d \). On average, to find an available parking space, the distance traveled is \( l_s = d/p \). The total distance traveled to complete a trip is \( L(p) = l_m + d/p \). The percentage of available parking spaces \( p = 1 - n_p/N_p \), where \( n_p \) is the number of occupied parking spaces and \( N_p \) is the total number of parking spaces or the parking supply in the considered network. After taking into account the cruising-for-parking, the travel time is then \( \tau(n,p) = L(p)/v(n) \), and outflow of the system is \( o(n,p) = n \cdot v(n)/L(p) \).

2.2. Commuting Equilibrium

The purpose of this study is to examine the downtown parking problem in the context of dynamic user equilibrium in the morning commute. Thus, the mentioned accumulation \( n \) and percentage of vacant parking spaces \( p \) will be time-dependent, and travel time, outflow of the system would also be time-dependent. It is assumed a continuum of \( N \) commuters travelling through a network and reach their destination. They have a common desired arrival time \( t^* \). Commuters are assumed to be aware of traffic conditions and parking vacancies after their long term experience, and they choose their departure time to minimize their individual travel cost, which is composed of travel time cost and schedule delay cost. The full trip cost of a commuter by departing from home at time \( t \) is given by

\[
 c(t,t^*) = c_w \cdot \tau(n(t), p(t)) + c_s \cdot (t^* - t - \tau(n(t), p(t))) ,
\]

where \( \tau(n(t), p(t)) \) is the travel time, \( c_w \) is the value of unit travel time, and \( c_s \) is the schedule penalty of unit time. The schedule penalty \( c_s = e \) for a unit time of early arrival, i.e., \( t^* \geq t + \tau(n(t), p(t)) \), while \( c_s = -l \) for a unit time of late arrival, i.e., \( t^* < t + \tau(n(t), p(t)) \). Also, it is assumed that \( e < c_w < l \), which is consistent with empirical studies.
As mentioned, equilibrium requires that no one can reduce its travel cost by unilaterally changing its departure time. By taking the first-order derivative of the individual travel cost given by Eq. (1) with respect to \( t \), and let it be zero, we have the equilibrium condition. For travelers departing at time \( t_s \), travel time is given by \( \tau_s = \tau(n_s, p_0) \) where \( p_0 = 100\% \). With this as the boundary condition, we can derive the equilibrium travel time profile depicted in Figure 1 based on the equilibrium conditions. With the equilibrium travel time profile, we then can estimate the equilibrium time-varying accumulation, percentage of available parking spaces, and outflow of the system according to the MFD based traffic dynamics presented.

![Travel Time vs. Departure Time](image)

**Figure 1.** The equilibrium travel time profile

2.3. System Optimum

The travel delay due to roadway congestion (intense traffic because of both concentrated schedule preference and cruising-for-parking), and increased schedule delay due to competition to enjoy less cruising-for-parking are both deadweight loss of social welfare. We now introduce a time-varying (fine) toll to minimize total travel cost including travel time cost and schedule delay cost, and improve traffic efficiency. It is straightforward to show that, for a single-region system, the total travel cost will be minimized when the downtown network or system is operating at the maximum production of the MFD (of the downtown network), i.e., \( n(t) = n_c \) and \( v(t) = v(n_c) \), and \( P(t) = n_c \cdot v(n_c) \). Let \( T(t) \) be the toll for the commuters departing at time \( t \) or entering into the network at time \( t \), individual full trip cost including the toll can be written as follows:

\[
c(t, t^*) = c_w \cdot \tau(n(t), p(t)) + c_s \cdot (t^* - t - \tau(n(t), p(t))) + T(t).
\] (2)
Suppose under the time-varying toll, the peak starts at $t_{s,1}$, of which the estimation is discussed later. For $t \leq t_{s,1}$ we set $T(t) = T_0$. After $t_{s,1}$, since we maintain $n(t) = n_c$, $dn(t)/dt = 0$. Then after some manipulations from Eq. (2), we have the toll to support $n(t) = n_c$ during the peak depicted in Figure 2.

By utilizing the toll design as discussed in the above, choosing different $t_{s,1}$ will not affect the exact departure/arrival pattern since it is determined by $n(t) = n_c$, but translate that pattern along the time horizon. The travel time cost then would be identical under different $t_{s,1}$. To minimize total travel cost, it suffices to choose an appropriate $t_{s,1}$ to minimize schedule delay cost, and we can prove that in the system optimum, the early arrival traffic $N_e$ should be $l/e$ times as much as the late arrival traffic $N_l$. This is consistent with the case without cruising and that in Vickrey’s model. By utilizing this information, similar procedure as that for estimating User Equilibrium solution can be developed to compute the System Optimum.

![Figure 2. The equilibrium travel time profile](image-url)